SECTION 1

MECHANICS AND HYDRAULICS

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(PART 1)

STATICS

NEWTON'S THREE LAWS OF MOTION

1. Mechanics is that branch of science that treats of forces and their action on bodies to produce equilibrium or motion. Mechanics is usually treated under two main headings: *statics* and *dynamics*.

Statics treats of forces that produce equilibrium; that is, of the conditions that cause a body to be at rest or to be in uniform rectilinear motion when acted on by forces. **Dynamics** treats of the motion and change of motion of bodies when acted on by forces.

2. Force was defined in *Elements of Physics*, and was there shown to be equivalent to a push or a pull. Throughout this Section, under the head of Statics, forces will be considered as equal to equivalent *weights*; that is, a force of, say, 10 pounds will be considered as equivalent to a weight of 10 pounds.

3. Comparison of Forces.—In order to compare forces, it is necessary to know four things regarding every force, viz.:

(*a*) The magnitude of the force (the value of the equivalent weight).

(*b*) The line of action of the force (the right line along which the force tends to move the point of application).

(*c*) The direction along the line of action.

(*d*) The point of application (the point of the body at which the force acts or may be considered as acting).

The necessity for the fulfillment of these four requirements will be made evident by what follows.

4. Representing a Force by a Line.—A right line combined with an arrowhead will completely represent a force. Thus, in Fig. 1, if B is the point of application, the arrowhead indicates that the force acts from B toward A, along BA, the line of action—the line along which the force tends to move the point of application. Now, if the length of the line be such that its length multiplied by some number will give a product having the same value numerically as the number of pounds weight that the force equals, then the line BA represents the force completely. If the length of BA is, say, .97 in., and 1 inch represents 60 pounds, BA represents $.97 \times 60 = 58.2$ lb. the magnitude of the force. The arrowhead shows that the force acts from B toward A; if it acted from A toward B, the direction of the arrowhead would be reversed, and would then point toward B. The line BA is called the **line of action** or **action line,** and like all right lines, is indefinite in extent.

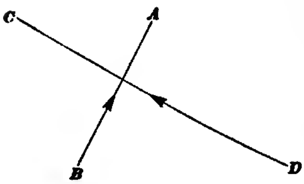

FIG. 1.

To draw a force, first draw the line of action; locate on this line the point of application; place an arrowhead on the line, to indicate the direction in which the force tends to move the point of application; and, lastly measure off in the given direction a length that will represent the magnitude of the force. Thus, suppose several forces are to be laid off to a scale of 60 lb. = 1 in.; if one of these forces were 58.2 lb., its point of application were B (Fig. 1) its line of action BA, and its direction from B toward A, draw a line through B parallel to BA (it will coincide with BA in this case), place the arrowhead as shown, and measure off $58.2 \div 60 = .97$ in. from B; then BA will represent the force. If the magnitude of this force had been 150 lb., the length of the line to the same scale would be $150 \div 60 = 2.5$ in.

Suppose it were desired to draw a force of 114 lb. at right angles to BA, acting toward and at the point C, the scale being the same as before. From C, draw CD at right angles to BA, lay off from C $114 \div 60 = 1.9$ in. $= CD$, draw the arrowhead pointing toward C, and DC represents the force. (The line is read in the same direction that the force acts.)

5. Three Laws of Motion.—The laws connecting force and motion were first stated by Sir Isaac Newton (1642–1727), the discoverer of the law of universal gravitation; they are called *Newton's three laws of motion.* These laws were first stated in Latin, and consequently the wording in English by different authors varies slightly. As here stated, the language is that of J. Clerk Maxwell, one of the greatest of modern mathematicians and scientists.

First Law.—*Every body perseveres in its state of rest or of moving uniformly in a straight line, except insofar as it is made to change that state by external forces.*

This law means that if a body is free to move in any direction and has motion, the direction of motion will be a *straight line* and the velocity will be *uniform.* To change the *direction of the motion* or to change the *velocity* requires that some force or forces outside of the body (*external forces*) act on the body; no force acting within the body (*internal force*) can have the slightest effect in changing the motion of a body, either in direction or velocity. The reason that a locomotive moves is because the steam, an internal force, moves the piston, which causes the connecting rod to turn the crank and with it the drivers; the friction between the drivers and the rails causes the locomotive to move ahead. All the forces are here internal forces *except the friction,* which is an external force. If the rails were perfectly smooth, there would be no friction, and the locomotive would not move. It is to be understood that change of motion here means change either in direction or velocity or both.

The first law of motion is frequently called the law of inertia (see definition of inertia in *Elements of Physics*), and it states that only the action of an external force can change the state of rest or of motion of any body. The law does not apply, of course, to gases, all of which expand and fill vessels of any size, no matter how large, but it does apply to every liquid and solid.

6. Second Law.—*Change of motion is proportional to the impressed force, and takes place in the direction in which the force is impressed.*

This law may otherwise be stated as follows: change of motion is proportional to the acting force, whether it act alone or in combination with other forces, and whether the body be at rest or in motion; and the acting force tends to move the body in the direction of its action line.

According to this law, if a body is acted on by two or more forces, the final result will be the same, however the forces act. That is, the forces may all act simultaneously or one may act, then another, and so on until all have acted. For example, if a stone be thrown in a horizontal direction from a height, say a height of 20 feet, and another stone be dropped from the same height at the same instant, both will strike the ground at the same time, because the acceleration due to gravity being the same and the height of the stones above the ground being the same, gravity acts with the same specific force (force per unit of

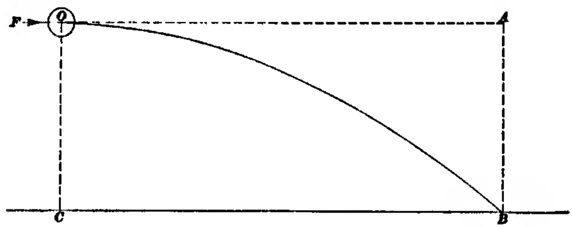

Fig. 2.

mass) on both stones and pulls them to the ground in the same time. The force acting to make one stone move in a horizontal direction is at *right angles* to the force of gravity (a vertical force) and has no influence in altering the effect produced by gravity. Referring to Fig. 2, let O be the stone acted on by the horizontal force F, and let OC be the height of the stone above the ground; let t be the time it takes the other stone to fall through the height OC. Now suppose the force F is just sufficient to cause the stone to strike the ground at B. Draw BA vertical and OA horizontal, the two lines intersecting at A; then $OABC$ is a rectangle, and $AB = OC$; also, $OA = CB$. If gravity did not act, the force F would carry the stone to A in the time t; but since gravity does act during the entire time t and produces a variable velocity (acceleration) downwards, the path of the body will be the curved line OB. One stone travels a much greater distance than the other, but they both travel the same *vertical* distance under the action of gravity.

That both stones will strike the ground at the same time may be easily proved by direct experiment.

7. Third Law of Motion.—*Reaction is always equal and opposite to action, that is to say, the actions of two bodies upon each other are always equal and in opposite directions.*

This law may otherwise be stated thus: to every **action** (force) there is always opposed an equal action (force), called the **reaction,** which has the same line of action as the acting force, but is opposite in direction.

Examples of this law are everywhere. A book rests on a table; the book presses against the table, and the table reacts and presses against the book. This is readily seen in the case of a mass of soft dough or putty; the reaction flattens it out at the surface of contact and changes the shape throughout the mass. One cannot lift one's self by pulling on one's boot straps, because the pressure of the fingers against the straps is balanced by the force (reaction) with which the straps press against the fingers, one set of forces acting upwards and the other set downwards. This explains why, in accordance with the first law, an internal force cannot change the motion of a body. Unless great care is exercised, a person cannot jump from a small row boat in open water; the downward force exerted on the boat has a reaction, but the force opposing the movement of the boat is so small that, unless the jump is a vertical one or very nearly vertical, the boat will move from under him and he will fall into the water. If the boat is immovable, however, then the jump can be made, because the reaction will then be balanced by an equal force holding the boat.

The whole science of mechanics rests on the principles just explained in connection with the three laws of motion; and since frequent applications will be made of these principles, further discussion of them will not be given here.

COMPOSITION AND RESOLUTION OF FORCES

COMPOSITION OF FORCES

8. Definition.—By **composition** of forces is meant the process of finding a single force that will have the same effect on the body as the several forces that are considered as acting on it. Unless otherwise stated, all forces will be considered as acting in the same plane, and their lines of action will be assumed to pass through the center of gravity of the body.

The single force that is equivalent in effect to the action of several forces is called the **resultant** of those forces. The method of finding the resultant of two forces will first be considered.

9. When Two Forces Have the Same Line of Action.—In Fig. 3, let BA and DC represent in magnitude and direction two forces, of 156 and 108 lb. respectively, the lines of action being parallel and the point of application being A'. As indicated by the arrows,

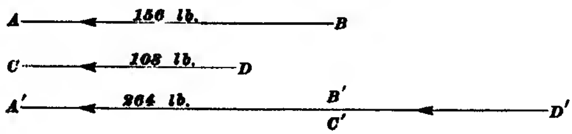

Fig. 3.

both forces act in the same direction. Through A', draw $B'A'$ parallel to BA = 156 lb. If the scale is 1 in. = 80 lb., make $B'A'$ equal in length to 156 ÷ 80 = 1.95 in. Place an arrowhead on $B'A'$, as shown. Since DC has the same direction as BA, produce $A'B'$, lay off $B'D'$ = 108 ÷ 80 = 1.35 in., and place the arrowhead on $D'B'$ = $D'C'$, as shown. Then $D'A'$ = 156 + 108 = 264 lb., is the resultant of the two forces, and it will produce the same effect on the body as the two forces.

If, however, one of the forces, say DC be reversed, so that the two forces act in opposite directions, draw B'A' (Fig. 4) as before; then, if A' is the point of application, lay off B'D' = DC = D'C'. Evidently, D'C' destroys a part of the force B'A', the remaining part D'A' = B'A' − D'C' being the resultant, which is equal in magnitude to 156 − 108 = 48 lb. The result is similar in effect to the action of two forces, one of 156 lb. acting on one side of a body and another of 108 lb. acting in the opposite direction on the other side of the body. The greater force tends to make the body move in the direction in which the force acts, and its value is equal to the original force minus the opposing force. The method of drawing the resultant for this case is indicated in Fig. 4.

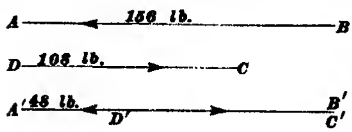

Fig. 4.

If care has been exercised in drawing Figs. 3 and 4, it will be found that the length of $D'A'$ in Fig. 3 is 3.3 in.; and since 1 in.

= 80 lb., $D'A'$ = 3.3 × 80 = 264 lb. Similarly, $D'A'$ in Fig. 4 will be found to have a length of .6 in., and $D'A'$ = .6 × 80 = 48 lb.

10. When Two Forces Have Different Lines of Action.—Let the magnitudes of the two forces be the same as before, both having the same point of application, but with the directions indicated by BA and DC in Fig. 5. If O is the point of application, draw OE parallel to BA; using the same scale as before,

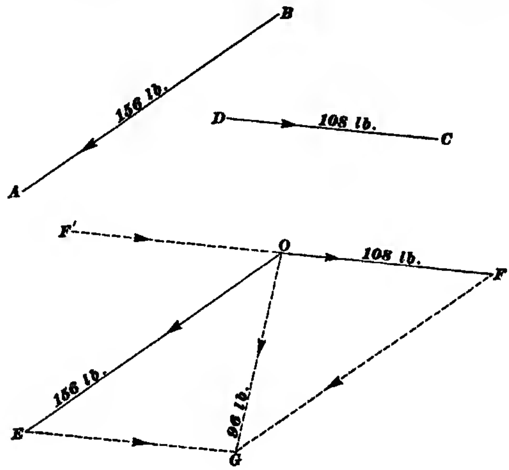

FIG. 5.

make OE = 156 ÷ 80 = 1.95 in., and place the arrowhead as shown. Through O, draw $F'F$. If arrowheads are placed on OF' and OF, it remains to be determined which of these two segments of $F'F$ is to be taken as representing DC. This point is settled by always drawing the two forces so that *both will act toward or both away from the point of application.* Here OF and OE both act away from the point O; but OF' acts toward O, while OE acts away from O; hence, make OF = 108 ÷ 80 = 1.35 in. Now draw EG parallel to OF and FG parallel to OE; they intersect in G, and the four lines make up the parallelogram $OEGF$. From O, the point of application, draw the diagonal OG, and OG will represent the resultant in magnitude, direction, and position; in other words, it represents the resultant completely. This result follows at once from the second law of motion;

because the force BA would carry the body from O to E, and the force DC would carry the body from E to G, EG being equal to $OF = DC$. Measuring OG, its length is found to be 1.2 in.; hence, the magnitude of the resultant is $1.2 \times 80 = 96$ lb., and its direction is from O toward G.

If the direction of one of the forces, as DC, be reversed, draw $OE = BA$ from the point of application O, as before; then, referring to Figs. 5 and 6, the force CD must be laid off in the direction OF', making $OF = CD$ if both forces are to act *away* from O. Complete the parallelogram as shown in Fig. 6 and draw the diagonal OG, which is the resultant, between the two forces OE and OF. Measuring OG, its length is found to be 3.1 in., which multiplied by the scale gives $3.1 \times 80 = 248$ lb., the magnitude of the resultant. The direction of this resultant is from O toward G.

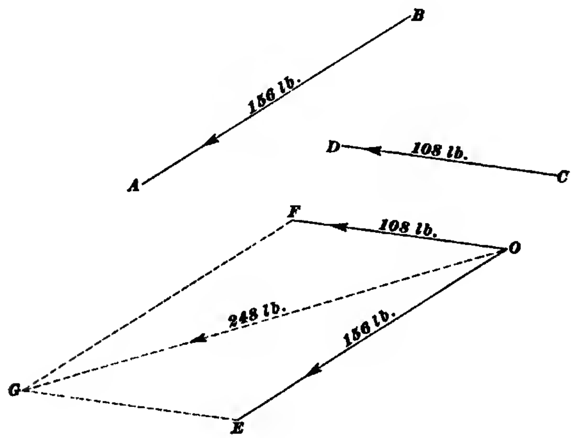

Fig. 6.

11. To understand why the resultant is so much larger when the forces act as in Fig. 6, suppose EOF to be a flexible rope and O to be a round pin; a pull on the end E of 156 lb. and on the end F of 108 lb. will produce a pressure on the pin of 96 lb. in the case of Fig. 5, and the pressure will tend to move the pin in the direction OG, the resultant. In the case of Fig. 6, the pull on the pin is 248 pounds, and tends to move the pin in the direction OG, the resultant. If the two parts of the rope, OE and OF, were

parallel, the resultant would be parallel to both forces, and its magnitude would then be the sum of the two forces, or 156 + 108 = 264 lb. As the ends of the rope spread outwards, the pull on the pin becomes less and less, until when the two parts of the rope become one, their center lines coinciding, as in $F'OF$, Fig. 5, the pressure on the pin becomes 0, and there is no tendency for the pin to move except in the direction of the greater of the two forces acting along the same line. In Fig. 6, the two parts of the rope are more nearly parallel than in Fig. 5; consequently, there is a greater pressure on the pin in the case of Fig. 6 than in the case of Fig. 5.

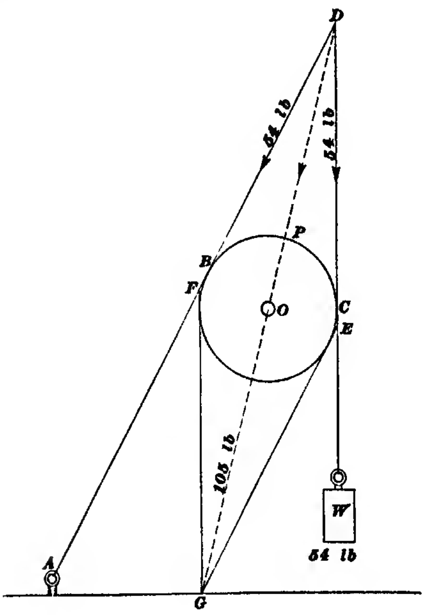

FIG. 7.

EXAMPLE.—Referring to Fig. 7, P is a pulley around which a rope is passed, one end of the rope being fastened to a staple in the floor and the other end having a weight W of 54 lb. attached to it; what is the pressure on the axle O of the pulley and in what direction does it act?

SOLUTION.—The force of 54 lb. is transmitted to every part (section) of the rope, and must therefore exert a pull on the staple A of 54 lb. By the third law of motion, the staple pulls on the rope with an equal and opposite

force (reaction) of 54 lb.; consequently, the part *AB* of the rope is pulled by the staple with a force of 54 lb. in exactly the same manner as though the staple were replaced by a force of 54 lb. acting in the direction *BA*. To draw the parallelogram of forces, it is convenient to produce *AB* and *WC* (*B* and *C* being the points of tangency of the rope and pulley) until they intersect in *D*. Assume *D* to be the point of application and lay off $DF = DE = 54$ lb. If a scale of 1 in. = 30 lb. be selected, $DE = DF = \frac{54}{30} = 1.8$ in. Complete the parallelogram by drawing *EG* parallel to *DF* and *FG* parallel to *DE*; they intersect in *G*; draw *DG*, and it will be the resultant, it will act from *D* towards *G*, and will pass through the center of the axle *O*. Measuring *DG*, its length, in this case, is 3.5 in.; hence, the magnitude of the resultant is $3.5 \times 30 = 105$ lb., and it has the direction *DG* through the center of the axle. *Ans.*

NOTE.—It may happen in some cases that when the lines on a cut are measured accurately, their lengths will be found to differ slightly from the lengths specified in the text. This is caused by the fact that the original drawing was made to a larger scale than that given in the text and the engraver did not reduce to the exact size specified. The measurements recorded in the text are correct, however.

If the reader is doubtful about the correctness of the above reasoning, let him tie a string to a small weight, say the handle of a flatiron; lift the weight by pulling on the end of the string. Now tie the free end of the string to a nail or staple in the floor and raise the weight by means of a round stick, say a broom handle, by allowing the string to pass over the stick. He will note that it will require about twice as much of an effort as when he lifted the weight by pulling on the string.

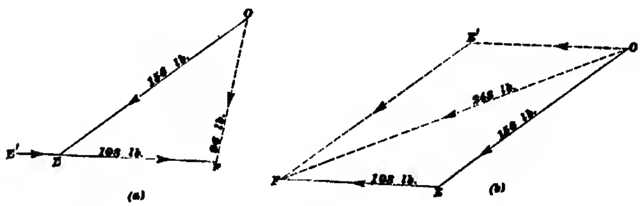

FIG. 8.

12. Triangle of Forces.—Referring to Fig. 5, the diagonal (resultant) *OG* divides the parallelogram into two equal triangles *OEG* and *OFG*; the sides *OE* and *FG* are equal, the sides *OF* and *EG* are equal, and the side *OG* is common. Since *EG* is parallel to *OF*, it must be parallel also to *DC*; hence, if *OE* be drawn parallel to *BA*, *O* being the point of application, and the length of *OE* be made such that it will represent to some scale 156 lb., it will represent the force *BA* fully. Now, having drawn *OE* = *BA* parallel to *BA*, draw *EF* from *E*, Fig. 8 (*a*), parallel

to DC, and make it equal to 108 lb. Joining O and F, the triangle OEF is equal in all respects to the triangle OEG, Fig. 5.; in other words, OF is the resultant of the forces $OE = BA$ and $FE = DC$.

To determine whether $E'E$ or EF shall represent DC, note that in the triangle of forces and the polygon of forces (to be described presently), the sides representing forces follow one another so that, at any common meeting point, as E in Fig. 8 (a), the arrowhead on one force points *toward* the point of intersection and on the other force *away* from the point of intersection. Note that this is contrary to the rule for the parallelogram of forces. Hence, it is necessary to draw the line representing the second force from E to F; then OE points toward E and EF away from E; if drawn from E' to E, both forces point toward E. The application of common sense will show whether two forces are acting so that one tends to increase or decrease the effect of the other.

To determine the direction of the resultant, start with the point of application or the point that corresponds to it in the triangle, the point O in this case, and go around the triangle (as though tracing it) until the starting point O is reached; then make the arrowhead point in the opposite direction. Thus, starting at O, move to E, then to F, then to O; hence, the arrowhead must point in the opposite direction, from O toward F.

Fig. 8 (b) shows the application of the triangle of forces to the case of Fig. 6. OE is parallel and equal to BA, EF is parallel and equal to CD, and OF is the resultant. Either force may be drawn first; thus, drawing OE' parallel and equal to CD, and $E'F$ parallel and equal to BA, OF is the resultant, as before. OE' points toward E', $E'F$ away from E', and the resultant OF points from O toward F, which is opposite to the general direction $OE'FO$ around the triangle.

It will be noted that in the parallelogram of forces, the lines of action of all the forces and of the resultant pass through the point of application; but, in the triangle of forces, the second force does not pass through this point; it is, however, *parallel* to the line of action of the force passing through the point of application.

The triangle of forces is a simpler figure to construct than the parallelogram of forces, and the principle can be better adapted to finding the resultant of more than two forces. It will give the resultant correct in magnitude, direction, and position, and that is all that is required.

13. The Polygon of Forces.—Suppose five forces, all in the same plane, to act on the point O, Fig. 9, in the directions indicated by the arrowheads, and to have the magnitudes indicated. The line of action of the resultant will pass through O, and it is required to determine its magnitude and direction. Adopting *a scale of 1 in. = 200 lb., through any convenient point O, draw*

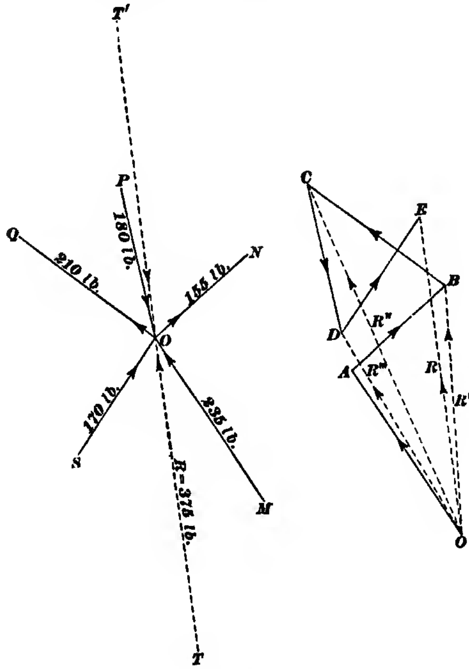

FIG. 9.

OA parallel to one of the forces, say OM = 235 lb. Make the length of OA 235 ÷ 200 = 1.175 in. From A, draw AB parallel to the force ON (any other force might have been selected), and make the length of AB 155 ÷ 200 = .775 in. Selecting another force, say OQ, draw BC parallel to OQ and make its length 210 ÷ 200 = 1.05 in. From C, draw CD parallel to PO and make its length 180 ÷ 200 = .90 in. From D, draw DE parallel to SO and make its length equal to 170 ÷ 200 = .85 in. As there are now no more forces, join E and O, and EO will represent the resultant

in magnitude; its direction will be from O toward E, in the opposite direction to that of the forces around the polygon OAB-$CDEO$. Draw OT through the point of application, make it equal in length to OE, place the arrowhead so it points from T toward O, and TO represents the resultant in magnitude, direction, and position; it will produce the same effect on the body as the five forces MO, ON, PO, OQ, and SO. Finally, check up the sides of the polygon to be sure all the forces are included in direction and magnitude.

That this method of finding the resultant is correct is easily shown. The resultant of MO and ON is $R' = OB$, and its direction is from O to B, combining this resultant with one of the other forces, as OQ, the resultant of $R' = OB$ and $OQ = BC$ is $R'' = OC$, and its direction is from O to C; hence, R'' is the resultant of the *three* forces OA, AB, and BC. Combining R'' with one of the other forces, as PO, the resultant is $R''' = OD$, and its direction is from O to D; hence, R''' is the resultant of the *four* forces MO, ON, OQ, and PO. Finally, combining R''' with the last remaining force SO, $R = OE$ is the resultant of *all* the forces, and its direction is from O to E. Measuring OE, its length is found to be 1.875 in., in this case; consequently, its magnitude is $1.875 \times 200 = 375$ lb.

14. The polygon $OABCDEO$ is called the **force polygon.** When drawing it, it does not matter what force is used to begin with or the order in which the forces are taken; if the drawing is accurately made, the resultant will be of the same length and will have the same direction, the only difference being in the shape of the polygon. Thus, in Fig. 10, the force MO was selected to begin with, as before; then $FG = PO$, $GH = OQ$, $HI = SO$, and $IJ = ON$ were drawn, the resultant being OJ. If the two force polygons $OABCDEO$ and $OFGHIJO$ are drawn to the same scale of forces on the same sheet, it will be found that OJ and OE are parallel and that their lengths are equal. This must be the case, since all five forces may be replaced by the single force TO, the resultant, and the resultant can have but one value.

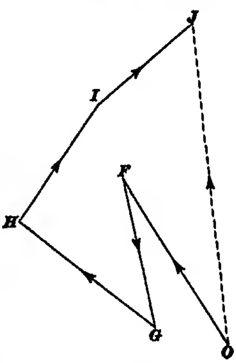

FIG. 10.

EXAMPLE.—Fig. 11 is a scale drawing showing an arrangement of three pulleys over which a rope passes in the manner indicated. What pull at the free end G is required to raise the weight $W = 84$ pounds, and what is the resultant force acting on the axle O'' of the middle pulley?

SOLUTION.—Assuming that there is no friction between the rope and the pulleys and that no force is required to bend the rope, the pull at G is exactly the same as the force exerted by the weight W, or 84 lb. This force is transmitted through the entire rope between W and G and exists at any section between those points. The pull due to W is indicated in the different

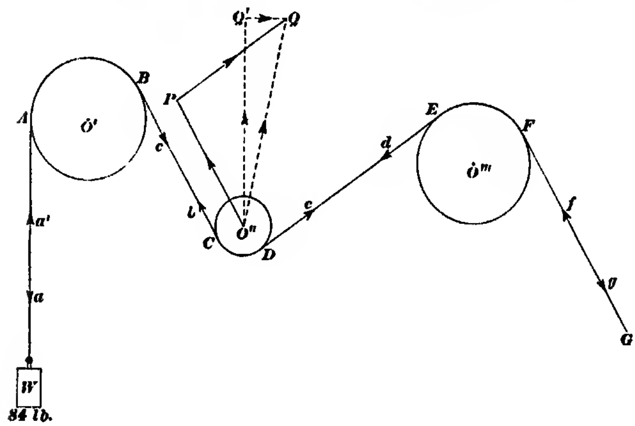

FIG. 11.

parts of the rope by the arrowheads a, b, d, and f; the reactions, or pull due to G are indicated by the arrowheads, g, e, c, and a'. There are, therefore, two forces of 84 lb. each acting on the middle pulley, one along CB from C toward B and the other along DE from D toward E; A, B, C, D, E, and F are the points of tangency of the rope and pulleys. From O'', the point of application, draw $O''P$ parallel to CB; if a scale of, say, 1 in. = 100 lb. be adopted, the length of $O''P$ is $84 \div 100 = .84$ in. From P, draw PQ parallel to DE and make its length the same as $O''P$. Join Q and O'', and $O''Q$ is the resultant in magnitude, direction and position. Measuring $O''Q$, its length is found to be 1.225 in., in this case. Hence, the resultant $O''Q = 1.225 \times 100 = 122.5$ lb. *Ans.*

The same result might have been obtained by means of the parallelogram of forces, but the method of triangle of forces is simpler and the figure is easier to draw.

RESOLUTION OF FORCES

15. Resolving a Force into Two Components.—Let ABC, Fig. 12, be a horizontal plane surface, on which rests an iron

block H. Suppose the surface to be smooth and frictionless and to be hinged at B, so that the part BC can be raised and occupy positions making various angles with the horizontal. Let the weight of the block be 30 lb.; then, when in the position BC, the whole weight of the block presses downwards against BC with a force of 30 lb., and there is no tendency for the block to move in any other direction. If, now, the surface BC be raised to the position BC', carrying the block with it, the block will tend to slide down toward B, and the pressure against the plane will be less than before. If raised still farther, to BC'', there will be a still greater tendency for the block to slide down,

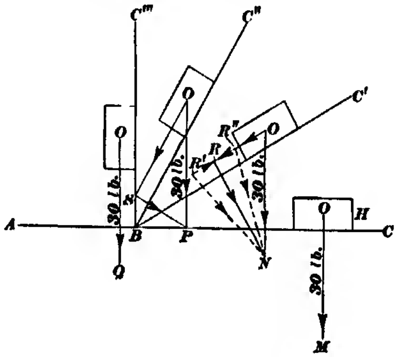

FIG. 12.

and the pressure against the plane will be still less also. When the plane has reached the position BC''' and is vertical, the entire force due to the weight of the block urges it downwards, and there is no pressure against the plane. The only force acting on the block is the force of gravity, which is in this case 30 lb.; but when the plane is in positions C' and C'', there is a force acting parallel to the plane, which tends to move the block downwards. To find this force for position C', draw ON vertical through O, the center of gravity of the block, and make it equal to 30 pounds to some scale; if the scale is 1 in. = 40 lb., the length of $ON = 30 \div 40 = .75$ in. Through O, draw OR parallel to BC', and through N, draw NR perpendicular to BC'; then, ON may be considered as the resultant of two assumed forces, OR and RN. The force OR represents the force urging the block *down* the plane, and the force RN represents the force with which the block

presses against the plane, and both may be measured to the same scale as that used to lay off ON.

The force ON is said to be resolved into two forces, called **components,** OR and RN. The components OR and RN might have been drawn in any direction, so long as they intersect, but if the pressure against the plane and the force acting down the plane are desired, they must be drawn as here described. For instance, if OR' be taken as representing in magnitude the force acting to move the block down the plane, join R' and N; then, OR' and $R'N$ are components of ON. But $R'N$ may be resolved into the two components $R'R$ and RN, $R'R$ coinciding with the action line OR and destroying the portion RR' of OR', thus leaving the force acting down the plane as OR, the value previously found. Similarly, if OR'' be taken as the force acting down the plane, join R'' and N; then $R''N$ may be resolved into the two components $R''R$ and RN. Since $R''R$ and OR'' have the same line of action and act in the same direction, the total force urging the block down the plane is $OR'' + R''R = OR$, as before.

For the position C'', draw OS parallel to and PS perpendicular to BC''; then OS is the component of the force OP that urges the block down on the plane and SP is the component that presses the block against the plane. For the position C'', OQ is the component urging the block down the plane; its value is the total force, 30 lb., and there is no component perpendicular to the plane; in other words, this component is 0, because the component parallel to the plane coincides with the resultant.

16. The foregoing serves to explain why it is, in general, harder to push a wheelbarrow than to pull it. Thus referring to Fig. 13, let the circle represent the wheel, O the center of the axle, and AO the center line of the handles; suppose the ground MN to be level and that $A'O$ is parallel to MN. Usually, the center line of the handles is above the horizontal $A'O$, in which case, let BO represent to some scale the force exerted in pushing the barrow with its load; its direction is indicated by the arrowhead a. Resolve this force into the horizontal and vertical components BC and CO, which act in the directions indicated by the arrowheads b and c. The load carried by the barrow acts downward also; hence, this load is increased by a force represented in magnitude by the component CO. If, on the contrary, the barrow is *pulled*, and OD, acting in the direction indicated by the

arrowhead a', represents the force exerted through the handles in pulling it, resolve OD into the horizontal and vertical components ED and OE, which act in the directions indicated by the arrowheads b' and c'. Here OE acts upwards, and counteracts a part of the load, which acts downwards; this makes the force exerted through the handles less than when pushing the barrow.

If the center line of the handles were horizontal, it would evidently make no difference whether the barrow were pushed or

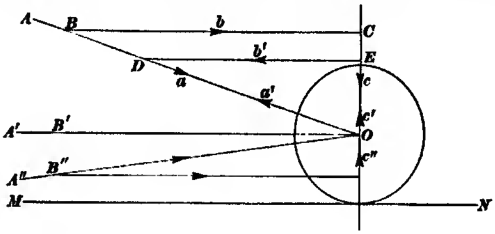

Fig. 13.

pulled, since there would then be no vertical component, the entire force acting in a horizontal direction. If the line of action were in the position $A''O$, below the horizontal, the conditions would be reversed and it would be easier to push than to pull; the vertical component then acts upwards against the load, when pushing, as indicated by the arrowhead c''.

17. When several forces act on a body and their lines of action all pass through a common point, the forces are said to be **concurrent** and are called **concurrent forces.** If the forces are concurrent, they can always be replaced by a single resultant. Thus, the five forces in Fig. 9 are concurrent and can be replaced by the single resultant TO, which is also concurrent with the five forces at the point O.

Since any number of concurrent forces has a single resultant, it follows that a single force may be resolved into any number of components; thus, if it were so desired, the resultant OE in Fig. 9 might be resolved into the forces OA, AB, BC, CD, and DE; then assuming a common point of application O, the forces may be drawn as indicated in the left-hand part of the figure. Usually, however, a force is resolved into two components only, and it usually happens that the total force acting in some particular direction is required. In such case, from one end of the

given force, a line is drawn *parallel* to the required direction and from the other end, a line is drawn *perpendicular* to the line first drawn. The distance from the point of intersection to the end of the line representing the force, measured in the required direction, is the magnitude of the desired component measured to the same scale as the given force. Thus, in Fig. 11, if it were desired to find the force tending to lift the middle pulley vertically, find the resultant $O''Q$ as before; from O'', draw $O''Q'$ vertical (the desired direction) and from Q, draw QQ' perpendicular to $O''Q'$; then $O''Q' = 118$ lb. is the force that tends to move the pulley vertically upward, and $Q'Q = 33$ lb. is a force that tends to move the pulley sidewise in a horizontal direction. The action of both these component forces tends to move the pulley along the line $O''Q$, and it would move along this line if the pulley were not restrained, that is, kept in place by some means, in this case, the bearings that enclose the axle of the pulley.

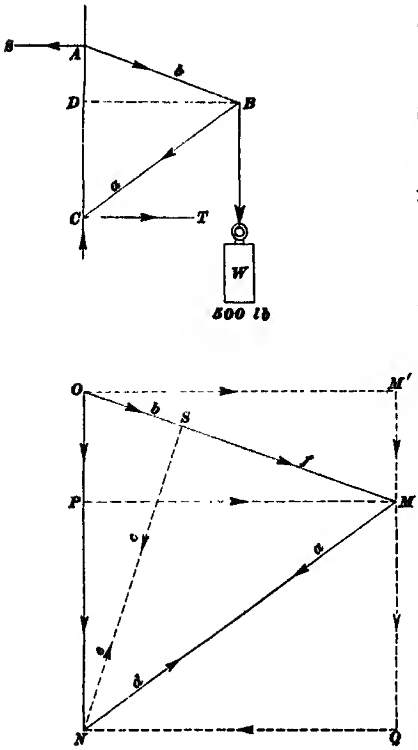

FIG. 14.

EXAMPLE.—Referring to Fig. 14, ABC is a bracket attached to a vertical wall; from the vertex B, a weight of 500 lb. is suspended, what forces act in the arms AB and BC and in what direction?

SOLUTION.—Selecting a scale of say 1 in. = 250 lb., draw ON vertical and make its length $500 \div 250 = 2$ in. From O, draw OM parallel to AB;

from N, draw NM parallel to CB; then OM represents (to the same scale) the force acting in the arm AB, and its direction is from A to B; MN represents the force acting in the arm CB, and its direction is from B to C. There is, therefore, a *pull* in the arm AB and a *push* in the arm CB. Measuring OM and MN and multiplying by the scale, 250, the force in AB is found to be 515 pounds, and the force in CB is found to be 590 pounds. *Ans.*

Referring again to Fig. 14, if OM be resolved into its horizontal and vertical components PM and OP, and MN be also resolved into its horizontal and vertical components QN and MQ, it will be noted that MQ is parallel and equal to PN, and since OP and MQ act in the same direction, the total downward force due to the components OM and MN is $OP + PN$ $(= MQ)$ $= ON = 500$ lb., as it should.

That the total force acting in AB is represented by OM is easily shown. The force acting in AB due to the weight W of 500 lb. is found by drawing OS parallel to AB (coinciding with OM) and NS perpendicular to OS; then $W(= ON)$ exerts a force OS, acting from O to S as indicated by the arrowhead b, in the arm AB. But this arm is also acted on by a component of the force acting in the arm CB. Considering ON $(= W)$ and OS as two separate and distinct forces, their resultant is SN, which acts from S toward N, as indicated by the arrowhead c. This resultant force SN may be resolved into the two components SM and MN, which act in the directions indicated by the arrowheads f and a. Then, the total force acting in AB is equal to $OS + SM = OM$. That the force acting in OM is greater than that due to the weight W will be apparent when it is considered that the weight W produces a downward force in MN, acting from M toward N; this produces a reaction at N that acts from N toward M, which can be resolved into the two components NS and SM. In a similar manner, it can be shown that the total force in BC acting from B to C is represented by MN.

Note that PM, the horizontal component of OM, and QN the horizontal component of MN, are equal in magnitude, but opposite in direction; this fact will be referred to later. (Art. **29.**)

18. The Equilibrant.—Referring again to Fig. 9, the resultant of the five concurrent forces is TO. Produce TO to T', making OT' equal to TO. The action of the five forces, as represented by the resultant TO, tends to move the body along the line OT'. If, now, a force $T'O$ acting from T' toward O and equal in magnitude to TO be applied to the body, it will counteract the result-

ant TO completely, and the body will have no tendency to move, that is, it will be in equilibrium under the action of the six forces MO, ON, PO, OQ, SO, and $T'O$. This force $T'O$, required to produce equilibrium, is called the **equilibrant**; it is always equal and opposite to the resultant. In the force polygon, the force $T'O$ will be represented by EO, and its direction will be represented by an arrowhead pointing from E toward O. There will then be no resultant, because the polygon is *closed* and there will be no side to draw to complete it. In Fig. 10, the equilibrant is JO, acting from J toward O. Note that the equilibrant has the same general direction around the polygon as the other forces.

Whenever the force polygon closes, there is no resultant; but when the polygon does not close, it must be made to close, as in Figs. 9 and 10, and the closing side is the resultant. A force equal and opposite to this is the equilibrant. Therefore, *to produce equilibrium, the force polygon must close.* This statement is a very important law.

QUESTIONS

(1) When rowing a boat, what causes the boat to move? Explain in accordance with the laws of motion the effect produced by the oars.

(2) What is the force called that is equal and opposite to the resultant, and how is it determined?

(3) Show that the principles governing the composition and resolution of forces are a direct consequence of the second law of motion.

(4) What is meant by concurrent forces, and when are forces said to concur?

(5) State the third law of motion, and give a practical illustration showing how it is applied.

(6) Suppose four forces to concur at a given point. Draw lines to indicate their directions (which may be selected at pleasure), and mark on them their magnitudes (state the scale chosen); then construct the force polygon and find the magnitude and direction of the equilibrant.

(7) Draw a line in any direction except vertically or horizontally, make its length represent a force to some convenient scale, and resolve this force into two components, one component perpendicular to the other and equal in magnitude to one-sixth the force; what is the value of the other component? *Ans.* .986 given force.

MOMENTS AND COUPLES

MOMENTS

19. Turning Force or Torque.—Fig. 15 shows a round iron bar of uniform cross section and density throughout and balanced on a knife edge over its center of mass (center of gravity), thus making $OA = OB$. At points C and D, near either end, and equally distant from O, equal weights W and Z are suspended. The weight Z tends to make the bar revolve about O as a center in the direction of the hands of a watch; the weight W tends to cause the bar to rotate about the same center O in a direction opposite to that of the hands of a watch.

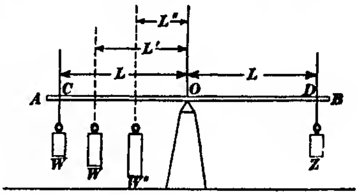

FIG. 15.

Since the two weights are equal and their distances from O are also equal, the tendency to rotate in one direction is counteracted by an equal tendency to rotate in the other direction, with the result that the bar and the weights are in equilibrium.

20. When looking at a revolving body, the plane in which it revolves, called the *plane of rotation,* is assumed to be perpendicular to the line of vision (like the dial of a watch or clock); if the rotation is in the direction of the hands of a watch or clock, it is called **clockwise** or **right-hand rotation**; but, if in the opposite direction, it is called **counterclockwise** or **left-hand rotation.** Further, right-hand rotation is usually considered as positive or + and left-hand rotation as negative or −. In Fig. 15, Z tends to produce right-hand rotation, and W tends to produce left-hand rotation.

21. If W, Fig. 15, be moved to the position W', it is evident that the tendency to right-hand rotation will be greater than the tendency to left-hand rotation; but, by increasing the weight of W until the bar again balances, the two rotative effects will again be equal, and since they are opposite in direction, the system will be in equilibrium.

As can be readily proved by experiment, the turning forces will be equal numerically when the weight on one side of O multi-

plied by its distance from O equals the weight on the other side multiplied by its distance from O; that is,

$$Z \times L = W \times L = W' \times L' = W'' \times L'' = \text{etc.}$$

Hence, to find W' when Z, L, and L$'$ are known, $ZL = W'L'$, from which

$$W' = \frac{ZL}{L'}.$$

For example, if $Z = 24$ lb., $L = 15$ in., and $L' = 12$ in.,

$$W' = \frac{24 \times 15}{12} = 30 \text{ lb.}$$

The product $Z \times L$ is called the **moment** of Z about O as a center; the moment of W about O is WL; of W' about O is $W'L'$; etc. The point O is called the **center of moments** (sometimes called the **origin of moments**), and is the point about which the force is supposed to turn the body.

22. Unit of Measurement of a Moment.—The moment of any force about any point assumed as the center of moments is the product of the force by the length of the perpendicular drawn from the center (origin) to the line of action of the force. In Fig. 15, the lines of action of the various forces are all vertical and the perpendiculars from O, the origin, are consequently horizontal. In Fig. 16, four forces A, B, C, and D are represented in direction, position, and magnitude, the last being indicated by the full lines. Suppose that it were required to find the moments of these forces about a specified center of moments O. Draw Oa perpendicular to the line of action of A, Ob perpendicular to the line of action of B, Oc perpendicular to the line of action of C, and Od perpendicular to the line of action of D; then, denoting right-hand rotation by $+$ and left-hand rotation by $-$, the moment of A about O is $+A \times Oa$; of B, $+B \times Ob$; of C, $-C \times Oc$; and of D, $-D \times Od$.

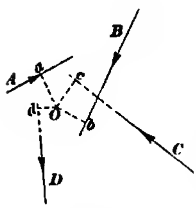

FIG. 16.

When the English system of units is used, forces are generally measured in pounds or tons and distances in inches or feet; hence, the unit employed for measuring **torques** (moments of forces) is the inch-pound, the foot-pound, or the foot-ton. Since these same units are used for measuring work and energy, some writers express the unit of torque as *pound-inch*, *pound-foot*, or *ton-foot*, to

distinguish these units from those used in measuring work and energy. The names of the units, then, have entirely different meanings, according to whether the unit of linear measure precedes or follows the other unit; thus, the foot-pound means the product of a force by the distance through which it acts, while the pound-foot means the product of a force and the perpendicular distance between the action line of the force and the center of moments. The term foot-pound, however, is frequently used, irrespective of the manner in which the force acts.

23. Condition for Equilibrium.—If the forces acting on a body are not concurrent, they will cause the body to rotate, unless the *sum* (algebraic sum) of the moments is 0; thus, in Fig. 15, if $W = Z, Z \times L - W \times L = 0$, and the bar, with its two weights, is in equilibrium. If $ZL - W'L' = 0$ or if $ZL - W''L'' = 0$, the bar, with its weights, is still in equilibrium. In Fig. 16, suppose all four forces to act in the same plane and that this plane is horizontal; suppose also that O is some point in a body acted on by the four forces; then the rotative effect (torque) about O is determined by the equation $A \times Oa + B \times Ob - C \times Oc - D \times Od = 0$. If the left-hand member is equal to 0, the body is in equilibrium, insofar as any turning effect is concerned; but if it is not equal to 0, then the value of the left-hand member will be the turning effect about the point O, and its sign will indicate whether the body tends to turn clockwise or counterclockwise. For example, suppose $A = 22$ lb., $B = 35$ lb., $C = 30$ lb., and $D = 26$ lb.; also, suppose $Oa = 10$ in., $Ob = 12$ in., $Oc = 9$ in., and $Od = 5.5$ in.; then, $22 \times 10 + 35 \times 12 - 30 \times 9 - 26 \times 5.5 = 220 + 420 - 270 - 143 = +227$ in.-lb. Since the sign of the moment is +, the body tends to turn clockwise. The sum (algebraic) of all the moments is called the **resultant moment**; hence, if the resultant moment is zero (0), the body has no tendency to rotate.

24. It matters not where the origin of moments is taken, the resultant moment always has the same value. This is evident, since the resultant moment must (or ought to) equal the resultant moment when the origin is taken at the center of mass (center of gravity); otherwise, changing the origin would change the torque produced by the forces without changing the magnitude or direction of the forces or their distances from the center of mass, which is absurd. As an example, refer to Fig. 15. Here

there is a system of three forces (not considering the weight of the bar), viz., W and Z acting downwards and the reaction of the knife edge O acting upwards; the reaction is evidently equal to the sum of W and Z. Suppose the origin of moments be taken in the center line of the bar and a in. from C, the point of intersection of the center line of AB and the action line of W; denote this point by C' and suppose it to be located between O and O. The distance of O from C' is $L - a$; the distance of D from C' is $L - a + L = 2L - a$; the weight Z tends to produce a positive rotation about C' as a center, and the weight W and reaction R at O tend to produce negative rotation about C'. Therefore,

$$Z(2L - a) - R(L - a) - Wa$$

is the resultant moment. But $Z = W$ and $R = 2W$; substituting these values for Z and R, the resultant moment is
$W(2L-a)-2W(L-a)-Wa=2WL-Wa-2Wl+2Wa-Wa = 0$, which is the same result as was obtained before; that is, the system is in equilibrium.

In Fig. 16, if the origin be taken at some point other than O, the value found for the resultant moment will be exactly the same as when the origin is taken at O.

25. The perpendicular from the origin of moments to the action line of the force is called the **arm of the force** or, frequently, the **moment arm.** If the origin be taken any where on the action line of a force, the moment of that force will be zero, because the arm will be zero, and the moment, which is the product of a force and its arm, will be the product of a force by 0.

26. A body is in *complete equilibrium* when (a) the resultant of all the forces is equal to zero, and (b) when their resultant moment is equal to zero. If the resultant moment equals zero, but the resultant of the forces is not equal to zero, then, if free to move, the body will move in a straight line, along the action line of the resultant, and every point of the body will describe a right line parallel to the line described by the center of gravity of the body; the body is then said to have a movement of **translation.** If the body is free to move and the resultant of the forces acting on it is zero, but the resultant moment is not zero, the body will have no movement of translation; it will simply rotate about an axis passing through its center of gravity. If neither the resultant force nor the resultant moment is equal to zero, and

the body is free to move, the body will move in a straight line, along the action line of the resultant, and will rotate as it moves; it is then said to have a combined movement of rotation and translation. The movement of translation in the last case is not affected in any way by the movement of rotation, and vice versa. The foregoing statements are best exemplified by means of an example.

Fig. 17 (a) shows a brick that is acted upon by three forces U, T, and S, all acting in the plane MN, which is shown as

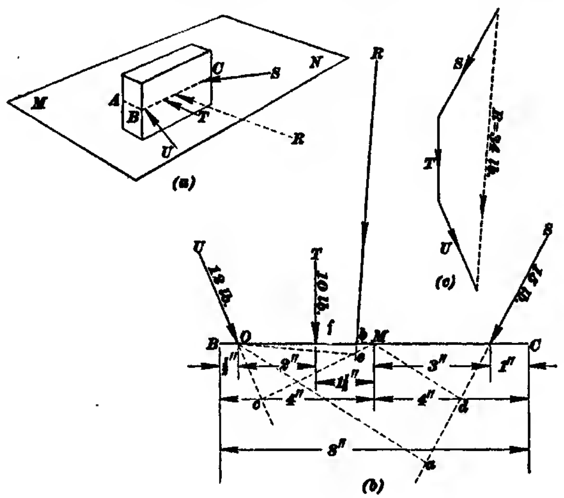

FIG. 17.

a plane of symmetry. The line BC is 8 in. long, and the magnitudes, positions, and directions of the forces with reference to this line are indicated in Fig. 17 (b). The force polygon is shown in Fig. 17 (c), the resultant R being determined in magnitude and direction, and it now remains to determine its position with reference to the other forces. Take O, the point of intersection of the action line of U with BC as the origin of moments; the moment of U will then be zero. The sum of the moments of the three forces about O is $U \times 0 + T \times 2 + S \times Oa$; measuring Oa, the arm of S with reference to O, it is found to be 5.63 in.; hence, the sum of the moments is $0 + 10 \times 2 + 15 \times 5.63 = 104.45$ lb.-in. This must equal the moment of the resultant

about O. The value of the resultant, as determined by measurement from the force polygon is 34 lb. Since its arm is not known, represent it by x; then,

$$34 \times x = 104.45, \text{ or } x = \frac{104.45}{34} = 3.07 \text{ in.}$$

From O, draw a line Oe perpendicular to R or to a line parallel to R, the resultant, and lay off Oe = 3.07 in.; through e, draw a line parallel to R, make its length equal 34 lb. to the scale used, and it will represent the resultant in magnitude, direction, and position.

It will be observed that the resultant intersects the line BC between M, the middle point, and the end B. As shown in Fig. 17 (b), this will tend to make BC revolve counterclockwise about M; but as shown in Fig. 17 (a), it will tend to revolve the brick clockwise (when the dial of the clock is horizontal and face up) about the center of gravity of the brick, which is directly under the point M in Fig. 17 (b). This may also be proved by calculation and measurement. Thus, in Fig. 17 (b), taking M as the center of moments, the arm of U is Mc = 3.19 in.; the arm of T is Mf = 1.5 in.; the arm of S is Md = 2.6 in.; and the arm of R is Mb = 0.42 in.; then, the sum of the moments of the forces is

$$-12 \times 3.19 - 10 \times 1.5 + 15 \times 2.6 = -14.28$$

the negative sign showing that the brick tends to revolve counterclockwise about M. It will be observed that the moment of the resultant is $-34 \times .42 = -14.28$, which is the same as the resultant moment previously found. The effect produced by the three forces acting on the brick is to make it move so that the path of its center of gravity will coincide with the action line of the resultant R; at the same time, the brick will revolve clockwise about its center of gravity, when viewed from a point to the right of the face BC, Fig. 17 (a). The value of the force causing the movement of translation of the center of gravity is 34 lb., and the value of the moment or torque causing rotation is 14.28 lb.-in.

27. Resultant of Parallel Forces.—The principles employed in the example of the last article are used in finding the position of the resultant of parallel forces, a problem that is constantly arising in connection with the loads on beams and girders. The method can be best understood by application to a specific case.

Referring to Fig. 18, let AB be a beam 24 ft. long, supported at its ends and carrying five loads, M = 420 lb., N = 280 lb., P = 160 lb., Q = 300 lb., and S = 640 lb., the loads being in the positions indicated. Suppose the beam is in a horizontal position, is of uniform cross-section throughout, and that it weighs 32 lb. per foot. It is required to find the position of the resultant and the reactions R_1 and R_2 of the supports.

The beam is acted on by 8 forces, the five forces just mentioned, the weight of the beam, all of which act vertically downwards,

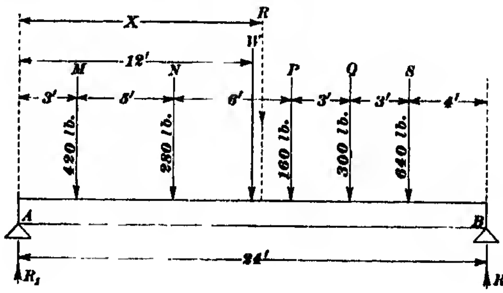

Fig. 18.

and by the reactions of the two supports, which act vertically upwards. The weight of the beam may be considered as a force equal in magnitude to the weight, and whose action line passes through the center of gravity of the beam; and since the beam has a uniform cross-section, the center of gravity will lie in its middle section, equally distant from either end. The weight of the beam is represented by W in the figure, situated 12 ft. from either end, and its magnitude is $32 \times 24 = 768$ lb. Since M, N, P, Q, S, and W are parallel and they all act in the same direction, the magnitude of their resultant is equal to their sum, or $420 + 280 + 160 + 300 + 640 + 768 = \text{R} = 2568$ lb. In order that the beam may not move downward under the action of this resultant force, the sum of the reactions R_1 and R_2 must equal the resultant, or $R_1 + R_2 = 2568$ lb. To find the value of the reactions, take a point on one of the reactions, say R_1, as the origin of moments; then the moment of this reaction will be zero and the other reaction can be found from equation

$$420 \times 3 + 280 \times 8 + 160 \times 14 + 300 \times 17 + 640 \times 20 + 768 \times 12 - \text{R}_2 \times 24 = 0 \qquad (1)$$

Solving this equation for R_2, $R_2 = \frac{32856}{24} = 1369$ lb. The sum of the moments of all the forces equals zero, because the system is in equilibrium. Since $R_1 + R_2 = 2568$, $R_1 + 1369 = 2568$, and $R_1 = 2568 - 1369 = 1199$ lb. Or, R_1 may be found by taking the origin of moments on the action line of R_2, in which case,

$$R_1 \times 24 - 420 \times 21 - 280 \times 16 - 160 \times 10 - 300 \times 7 - 640 \times 4 - 768 \times 12 = 0 \qquad (2)$$

from which, $R_1 = \frac{28776}{24} = 1199$ lb., as before. Now, to find the position of R, the resultant, take some convenient point as the origin of moments, say a point on R_1. The sum of the moments about this point was found above to be 32,856 lb.-ft. for the downward forces; this must equal the resultant R multiplied by the arm, which is the normal distance from R_1 to R; representing the arm by x, $R \times x = 32{,}856$, or $x = \frac{32{,}856}{2568} = 12.794$ ft. $= 12$ ft. $9\frac{1}{2}$ in. very nearly. Had the origin been taken at any other point, say on N, $R \times x = 768 \times 4 + 160 \times 6 + 300 \times 9 + 640 \times 12 - 420 \times 5 = 2568 \times x$, from which $x = 12{,}312 \div 2568 = 4.794$ ft., the distance of R from N; the distance from R_1 is $4.794 + 8 = 12.794$ ft., as before.

The position of the resultant R may be found in a somewhat easier manner by considering the reactions R_1 and R_2 instead of the loads. Thus, taking the origin of moments on R_1, the forces acting on the beam may be considered as R_1, R_2, and R, which produce equilibrium, and

$$R_1 \times 0 + R \times x - R_2 \times 24 = 0;$$

from which, since $R = 2568$, and $R_2 = 1369$,

$$x = \frac{1369 \times 24}{2568} = 12.794 \text{ in.},$$

the same result as previously found.

COUPLES

28. Moment of a Couple.—When a body is acted on by two *equal* parallel forces acting in *opposite* directions, the two forces are said to form a **couple** or to constitute a couple. Thus, in Fig. 19 (*a*), the forces P and Q are parallel and equal and they act

in opposite directions; hence, they form a couple. The perpendicular distance AB between the forces is called the **arm of the couple,** and is denoted by the dimension a.

The **moment of a couple** is the resultant moment of the two forces about some point as the origin of moments. Taking O as the origin, the moment of the couple is

$$P \times OA + Q \times OB$$

Since $Q = P$, this expression becomes

$$P \times OA + P \times OB = P(OA + OB) = P \times AB = Pa$$

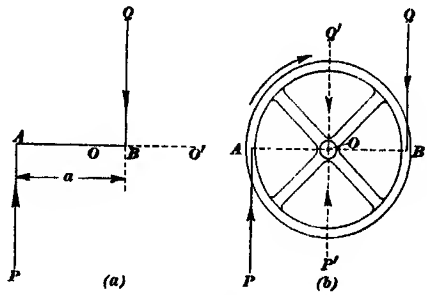

FIG. 19.

If O' be taken as the origin, the moment of the couple is

$$P \times O'A - Q \times O'B$$

Since $P = Q$, this expression becomes

$$P \times O'A - P \times O'B = P(O'A - O'B) = P \times AB = Pa$$

Therefore, *the moment of a couple is equal to the product of one of the equal forces and the arm of the couple.*

A practical illustration of a couple is shown in Fig. 19 (b), which may be considered as representing the steering wheel of an automobile, the hands are supposed to be at A and B; then, when one hand pulls down as much as the other pushes up, two equal and opposite forces are exerted on the wheel, forming a couple whose arm is the diameter of the wheel.

29. Couples Produce Rotation Only.—The only effect produced on a body by the action of a couple is rotation; it has no tendency to move the center of gravity of the body and, therefore, produces no movement of translation. No single force and no combination of concurrent forces (which will have, of course, a

single resultant) can produce equilibrium in a body acted on by a single couple; the rotative action may be destroyed, but the force will produce a movement of translation, and the body will not be in equilibrium. The only way that equilibrium can be produced in a body acted on by a single couple is to introduce another couple having an equal moment and tending to rotate the body in the opposite direction.

Another illustration of a couple is afforded by the conditions illustrated in Fig. 14. The moment of W about C as the origin is equal to $W \times BD$. In the force polygon, OM' ($= PM$) and QN, which are parallel components acting in opposite directions, constitute a couple that is equivalent to the couple produced by the moment of W (see Art. **30**), the arm being AC; hence, $OM' \times AC = W \times BD$. This couple tends to produce right-hand rotation; but it is resisted and equilibrium is produced by the reactions at A and C, indicated by the arrows S and T, which act in opposite and parallel directions, and produce a couple whose arm is AC, its moment being $S \times AC$ or $T \times AC$, S and T being equal.

The reason that the moment of the couple produced by W is equal to $OM' \times AC = QN \times AC$ is that the force acting in AB can be resolved into the two components OM' and $M'M = OP$; the force acting in BC can be resolved into the two components NQ and $MQ = PN$. OM' and QN are equal and opposite parallel forces, and they constitute a couple whose arm is the distance AC. Likewise, the sum of the forces OP and PN is $ON = W$, which acts at N and creates a reaction NO whose magnitude is equal to W. This force (reaction) is equal, parallel, and opposite to the force W acting at B, and the two constitute a couple whose arm is the distance BD, and the moment of which is $W \times BD$. This is not a different couple from the one previously mentioned, but another expression for the turning effect produced by the load W. Consequently, $OM' \times AC = W \times BD$ = moment of couple produced by W.

30. Difference between a Moment and a Couple.—Referring to Fig. 20, let P be a force; then the moment of this force about O as the origin is $P \times a$. Through the origin O, draw Q and Q' to represent two equal and opposite forces parallel to P and both equal to P. Since Q and Q' are equal and opposite and concurrent, they have no effect in moving the body, and the body will be in the same state of rest or motion whether acted on by these

forces or not. But, the force Q and the force P constitute a couple whose moment is $P \times a$, and the force Q' tends to move the body in the direction indicated by the arrowhead. The moment of the couple is the same as the moment of the force; hence, the moment of the force P is *equivalent to a couple having an equal moment and a force equal and parallel to P acting through the origin of moments.* The two forces constituting the couple are equal to P, and the arm of the couple is the perpendicular distance from the origin to the force P.

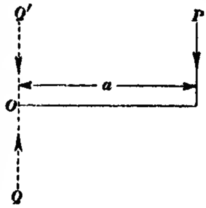

FIG. 20.

It will thus be seen that a *moment* tends to produce both *rotation and translation,* while a *couple* produces *rotation only.* For instance, referring to Fig. 19 (*b*), take O as the origin of moments and suppose only one of the forces, say Q, acts on the wheel. The turning force (torque) produced by Q is $Q \times OB$, and Q also produces a pressure $+ Q'$ on O that tends to move the entire wheel in the direction of the arrowhead on Q'. If P only acts on the wheel, the torque produced by P is $P \times OA$, and P also produces a pressure $- P'$ on O that tends to move the wheel in the direction of the arrowhead on P'. If both forces act at the same time, the resulting effect is $Q \times OB + Q' + P \times OA - P' = P(OA + OB) = P \times AB$, since $Q = P$ and $Q' = P'$. Q' and P' are given opposite signs because they act in opposite directions, and since both act on the same point, they destroy each other, leaving the couple to act on the wheel. When either force acts separately, it tends to rotate the wheel and to move it in the direction of the force; but when both forces act together, they tend only to rotate the wheel about its center.

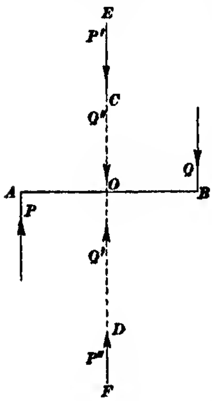

FIG. 21.

If one of the parallel forces is greater than the other and the center of moments (origin) is taken *midway* between the two forces, the action of the two forces produces a couple, whose moment is one-half the sum of the two forces multiplied by the

perpendicular distance between the action lines of the forces, and a parallel force, whose value is equal to the difference of the forces, acting at the center of moments in the direction of the greater force. Thus, referring to Fig. 21, let P and Q be parallel forces, acting as shown, and let AB be the perpendicular distance between their action lines. Then, if $OA = OB$, the moment $Q \times OB$ is equivalent to the couple formed by Q and Q', whose arm is $OB = \frac{AB}{2}$, and the downward force $Q'' = Q$. The moment $P \times OA$ is equivalent to the couple formed by P and P', whose arm is $OA = \frac{AB}{2}$, and the upward force $P'' = P$. The resultant about O as the center of moments is (taking upward forces as + and downward forces as −)

$$Q \times \frac{AB}{2} - Q'' + P \times \frac{AB}{2} + P'' = (Q + P)\frac{AB}{2} - (Q - P)$$
$$= \tfrac{1}{2}(Q + P) \times AB - (Q - P)$$

The negative sign before the parenthesis simply indicates that when Q is greater than P, the resultant force acts downwards, which is the direction of the greater force Q.

QUESTIONS

(1) What is (*a*) a moment; (*b*) a couple? (*c*) what effect do they produce on the body on which they act?

(2) What is the difference in the effects produced by a moment and a couple?

(*c*) Suppose three forces, not concurrent to act on a body; will the resultant have the same or a different value than if they concurred, all three forces acting in the same or parallel planes?

SIMPLE MACHINES

THE LEVER

31. Classes of Simple Machines.—A machine may be defined as any contrivance for altering the position of a body. If the position of a body is changed, the position of its center of gravity is also changed; that is, the position in space occupied by the center of gravity has been changed. The *path* of a body from one position to another is always taken as the line described by its

center of gravity. A machine may also cause a body to rotate, and if the position of the body is not changed it rotates about an axis passing through the center of gravity.

A machine that consists of but one moving part is called a **simple machine.** Machines containing more than one moving part, no matter how complicated they may be, consist of combinations of two or more simple machines.

Simple machines may be divided into the following six classes or types: *levers*, *pulleys* (including *gears*), the *wheel and axle*, *inclined planes*, *wedges*, and *screws*. As will subsequently appear, there are really but two classes—the lever and the inclined plane—the other four types being but modifications of these two.

32. Classes of Levers.—A lever is a rigid bar or frame which turns about a point, knife edge, or pin (when jointed) under the action of a force (called the **power**) and in turning moves a body

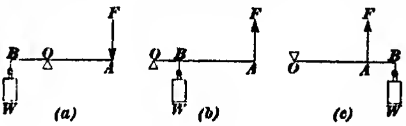

FIG. 22.

(called the **weight** or **load**). The knife edge, pin, or other bearing, about which the rotation occurs, is called the **fulcrum.** The perpendicular distance from the fulcrum to the line of action of the force is called the **power arm**, and the perpendicular distance from the fulcrum to the line of action of the weight or load is called the **weight arm.** In Fig. 22 (*a*), AB is a lever, which turns about O under the action of F, thus moving W. Here O is the fulcrum, F is the force or power, and W is the weight or load; hence, OA is the power arm and OB is the weight arm.

In accordance with the relative positions of the fulcrum, power, and weight, levers are divided into *first class*, *second class*, and *third class*. When the fulcrum is between the power and the weight, as in (*a*), Fig. 22, the lever is of the **first class**; a common example is a pair of shears or pincers. Here there are really two equal levers, which consist of two blades or jaws and handles that turn on the pin that connects them. The pin is the fulcrum, the power (force) is applied to the handles, and the body (or object) is cut by the blades or squeezed by the jaws, and constitutes the weight or load.

When the fulcrum is at one end of the lever and the power is applied at the other end, the lever is of the **second class**; see Fig. 22 (*b*). A common example is a lemon squeezer or a nut cracker, another case of two equal levers, which are joined by a pin at one end; the force is applied at the other end, and the lemon or nut, which is placed between the pin (fulcrum) and the line of action of the force (power), takes the place of the weight or load. This figure also shows the principle of the lighter bar (lever) on a beater. The pin is represented by O, the weight of the roll by W (usually at or near the center of the lever), and screw and hand wheel by F. This class of lever is also used on the paper-machine presses.

When the fulcrum is at one end of the lever and the weight or load or the other end, with the power between, the lever is of the **third class**; see Fig. 22 (*c*). A common example is a pair of tongs, which consist of two levers joined by a pin at one end, the load to be lifted being held by the pressure of the other two ends; the power (squeeze) that forces the two free ends together is applied between the ends.

33. Analysis of the Lever.—Fig. 23 represents a lever of the first class. It consists of a straight bar having a uniform rectangular cross-section and turning on a pin O, which acts as a fulcrum. Holes drilled at A and B permit pins to be placed in them, from which links can be suspended. When weights W and P are attached to the other ends of these links, they will hang in such manner that the forces they represent will act vertically and will therefore be parallel. All forces are considered as acting through the centers of the holes and pins, the weight of the lever being neglected for the present. Draw the horizontal line $A''OB''$ through O.

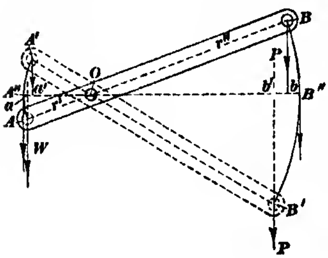

Fig. 23.

Taking O as the origin of moments, the necessary condition for equilibrium is $P \times Ob - W \times Oa = 0$, or $P \times Ob = W \times Oa$; that is, *the power multiplied by the power arm equals the weight multiplied by the weight arm.* This is the law of the lever, and applies in all cases, whatever the class, as will be shown presently.

If the power be applied at B, and it is desired to find what weight W can be raised by application of a given force (power) P with the lever in the position shown, solve the above equation for W, obtaining

$$W = P \times \frac{Ob}{Oa} \qquad (1)$$

Or, if the weight is known and it is desired to find the power required, solve the equation for P, obtaining

$$P = W \times \frac{Oa}{Ob} \qquad (2)$$

Expressed in words, *the product of either force and its arm divided by the other arm gives the other force.*

34. As the end B moves downwards the end A moves upwards, and when the lever reaches the position shown by the dotted lines,

$$P = W \times \frac{Oa'}{Ob'} \qquad (a)$$

When the center line of the lever is horizontal and occupies the position $A''B''$,

$$P = W \times \frac{OA''}{OB''} \qquad (b)$$

It can easily be proved by geometry that the three ratios in the last three expressions for P are equal; that is,

$$\frac{Oa}{Ob} = \frac{Oa'}{Ob'} = \frac{OA''}{OB''}$$

For, since the triangles OaA and ObB are similar right triangles, $\frac{Oa}{Ob} = \frac{OA}{OB}$; but $OA = OA''$ and $OB = OB''$; consequently, $\frac{Oa}{Ob} = \frac{OA''}{OB''}$. In the same manner, it can be shown that $\frac{Oa'}{Ob'} = \frac{OA''}{OB''}$. Since the two left-hand members of these two equations are equal to the same thing, they are equal to each other, and

$$\frac{Oa}{Ob} = \frac{Oa'}{Ob'} = \frac{OA''}{OB''}.$$

In other words, when the lines of action of the power and weight are parallel, it is not necessary to measure the perpendicular distance from the fulcrum to the action lines; simply measure on a straight line through the fulcrum the distances from the fulcrum

to the points of application of the power and weight—the distances OA and OB; then,

$$P = W \times \frac{OA}{OB}, \text{ and } W = P \times \frac{OB}{OA}.$$

The distances OA and OB are called the **lever arms.**

EXAMPLE.—The distance from the end of a crowbar to the point (line) on which it rests is $1\frac{1}{2}$ in., and the entire length of the crowbar is 5 ft.; if a downward pressure of 120 lb. be applied to the end of the long arm, what pressure will be exerted at the end of the short arm, that is, what load can be lifted by the crowbar?

SOLUTION.—The length of the crowbar is 5 ft. = 60 in.; the length of the long arm is 60 − 1.5 = 58.5 in. The long arm in this case is the power arm and the short arm is the weight arm; hence,

$$W = 120 \times \frac{58.5}{1.5} = 4680 \text{ lb. } \textit{Ans.}$$

35. Referring again to Fig. 23, angle $AOA' = BOB'$. Let θ (Greek letter *theta*) be the measure of these angles in radians; then, the lengths of the arcs AA' and BB' are $OA \times \theta$ and $OB \times \theta$, respectively, and their ratios are $\frac{OA \times \theta}{OB \times \theta} = \frac{OA}{OB}$. In other words, the ratio of the lengths of the arcs passed through (the distances passed through) by the points of application of W and P is equal to the ratio of the lever arms of W and P. Hence, the law of the lever may be stated thus:

The power multiplied by the distance through which it moves is equal to the weight multiplied by the distance through which it moves.

It is to be understood that the paths moved through by the two points of application must be similar in their nature; therefore, it is usual to measure them in vertical lines. With this understood, suppose the load in the example of the last article raised the weight $\frac{1}{4}$ in., how far would P move downwards? Applying the law just stated, the power is 120 lb., the weight is 4680 lb., the distance moved through by the weight is .25 in., and, letting x represent the distance moved through by the power,

$$120 \times x = 4680 \times .25, \text{ or } x = \frac{4680 \times .25}{120} = 9.75 \text{ in.}$$

Note that *what is gained in power is lost in distance;* that is, although the power is increased from 120 lb. to 4680 lb., the distance is decreased from 9.75 in. to .25 in. This fact is universally true of any machine, no matter how complicated it may be. The law just given is also true of any machine, provided all resistances due to friction, etc. are neglected.

The foregoing conclusions may also be obtained by applying the principle of work. Thus, a machine merely alters the manner of doing work; no machine can give out more energy (work) than it receives or is expended on the machine. The work done by the lever is the lifting of the weight through a distance, and the work expended in doing this is the power exerted through a distance. In the case last cited, the power of 120 lb. is exerted through a distance of 9.75 in., and the work expended is $9.75 \times 120 = 1170$ in.-lb.; the work done by the lever is $.25 \times 4680 = 1170$ in.-lb. Therefore, as before, the power multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves. The weight may be replaced by a resistance; in which case, the force multiplied by the distance through which it acts is equal to the resistance multiplied by the distance through which it is overcome.

36. If two levers are joined by a pin on which they can turn, as indicated by the full and dotted lines in Fig. 23, and the two ends B and B' are forced toward each other, the action is exactly the same as that of a pair of pincers, and an object held between A and A' will be squeezed or compressed. The pressure exerted on the object can be found by applying formula (1), Art. **33**, because one of the levers may be regarded as fixed, the other one moving, the fixed lever furnishing the reaction.

If the arms OA and OA' have cutting edges, the action of the levers is then the same as that of a pair of shears such as are used for cutting tin and sheet metal, where great force must be applied to the blades. If the arms OB and OB' have cutting edges and the power is applied at A and A', the action is that of a pair of ordinary shears such as are used for cutting paper, cloth, etc., where but little power is applied to the blades. Note that in the first case, the cutting is slow, since the blades have only a slight movement; in the second case, the cutting is fast, because the blades are long and move through a great distance as compared with the distance moved through by the handles. Here, again, what is gained in power is lost in distance, and vice versa.

37. Let S_p be the distance moved through by the power and S_w the distance moved through by the weight; then, $P \times S_p = W \times S_w$, from which,

$$W = P \times \frac{S_p}{S_w} \qquad (1)$$

If time be considered, it is evident that P moves through S_p in the same time that W moves through S_w. Letting t represent the time, the velocity of P is $\frac{S_p}{t}$, the velocity of W is $\frac{S_w}{t}$, and the ratio of the velocities is $\frac{\frac{S_p}{t}}{\frac{S_w}{t}} = \frac{S_p}{S_w} = r$, in which r is the value of the ratio.

The ratio $r = \frac{S_p}{S_w} = \frac{\text{distance power moves}}{\text{distance weight moves}}$ is called the **velocity ratio**; and *in any machine, the weight that can be lifted or the resistance that can be overcome is always equal to the power multiplied by the velocity ratio;* that is,

$$W = rP \qquad (2)$$

Consequently, if the velocity ratio of a machine is known, the resistance that can be overcome by the application of a force P is equal to the product of the velocity ratio and P (neglecting frictional resistances). Further, if the velocity ratio is known and it is desired to know what power is required to overcome a certain resistance, divide the resistance by the velocity ratio. Thus,

$$P = \frac{W}{r} \qquad (3)$$

The velocity ratio is always equal to the power arm divided by the weight arm (see Art. **35**).

EXAMPLE.—When the velocity ratio of a certain machine is 13.6, what force is required to overcome a resistance of 968 lb.?

SOLUTION.—Applying formula (3),

$$P = \frac{968}{13.6} = 71.18 - \text{lb.} \quad \textit{Ans.}$$

38. Fig. 24 (a) represents in diagrammatic form a lever of the second class and Fig. 24 (b) represents similarly a lever of the third class. The laws and principles just given for a lever of the first class also apply to levers of the second and third classes. For, taking the fulcrum O as the origin of moments, the condition of equilibrium for (a) when the lever is in the position OB' is $P \times OB'' - W \times OA'' = 0$, or $P \times OB'' = W \times OA''$. In ($b$), $P \times OB'' = W \times OA''$. Therefore, in both cases, the power multiplied by the power arm equals the weight multiplied by the weight arm, which is the same result as was obtained for

the case of Fig. 23. The other laws and principles may be obtained in the same manner as was done in connection with Fig. 23. It is to be noted that the velocity ratio is equal to $r = \frac{OB}{OA}$ in both cases; but in (*a*), r is always greater than 1, while in (*b*), r is always less than 1. Moreover, OB in (*a*) and OA in (*b*) represent the entire length of the lever. Hence, with a lever of the second class, the power is always less than the weight (or resistance); in a lever of the third class, the power is always greater than the weight; and in a lever of the first class, the power may be greater than, equal to, or less than the weight, according to whether the power is applied to the long arm, whether the arms are equal, or whether the power is applied to the short arm.

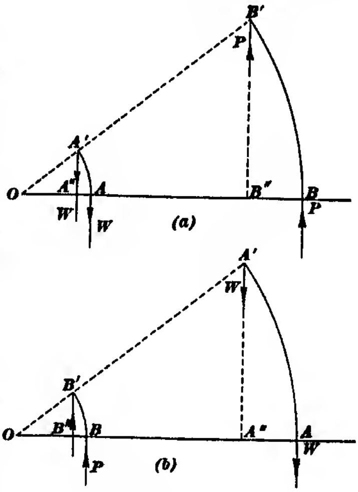

FIG. 24.

39. If in Fig. 24 (*a*), OB and OB' represent the center lines of two levers that are hinged at O, the action of bringing them together corresponds exactly to that of a nut cracker, the nut being placed at A. If the length OB of the levers is 6 in. and the nut is placed at A, 1 in. from O, a pressure of 12 pounds at the ends of the handles will produce a pressure (squeeze) on the nut of $12 \times 6 = 72$ lb. Here $P = 12$ lb. and $r = \frac{6}{1} = 6$.

Similarly, if OA and OA' in (*b*) represent the center lines of two levers hinged at O, the application of a force between O and A and O and A' corresponds exactly to the action of a pair of tongs.

A practical example of a lever of the second class is the lever of a safety valve, shown diagrammatically in Fig. 25. OAB is the lever, V is the valve, VA is the valve stem, and W is a weight (here to be considered as the power), which can be placed

anywhere between A and the free end of the lever; the lever is hinged at O. The steam pressure underneath the valve tends to force it upwards, and this is resisted by the weight, which acts through the lever and tends to force the valve down. Whenever the upward force acting at A exceeds the downward force at A, the valve opens, steam escapes, and its pressure no longer increases; this is why it is called a safety valve.

EXAMPLE.—Referring to Fig. 25, the distance OA of the valve stem from O is $4\frac{1}{2}$ in.; the diameter of the valve seat pressed against by the steam is 4 in., the steam pressure is required not to exceed 70 lb. per sq. in., and the weight of the ball hung at B is 90 lb.; at what distance OB from B should the ball be hung, neglecting the weight of the lever and valve?

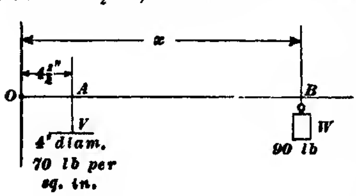

FIG. 25.

SOLUTION.—First find the total pressure exerted on the bottom of the valve. The area of the valve touched by the steam is $.7854 \times 4^2 = 12.5664$ sq. in. The steam pressure being 70 lb. per sq. in. the total upward pressure on the lever at A is $12.5664 \times 70 = 879.648$ lb. Letting x represent the distance OB, the power arm, the power is $W = 90$ lb., the weight is the force to be exerted at $A = 879.648$ lb., and the weight arm is 4.5 in. Then, applying the principle of the lever, power multiplied by power arm = weight multiplied by weight arm,

$$90 \times x = 879.648 \times 4.5, \text{ or } x = \frac{879.648 \times 4.5}{90} = 44 \text{ in.} \quad \textit{Ans.}$$

It will be noticed that if it were required to find what steam pressure will raise the valve when the weight is placed at a certain distance from O, the lever is then of the third class, since the power will be between the fulcrum and the weight. The above example may be solved just as readily by applying the principle of moments. Thus, for equilibrium, letting y = the total steam pressure and taking O as the origin, $y \times 4.5 - 90 \times 44 = 0$; from which $y = 879.648$ lb., as before, the weight being 44 in. from O.

40. Observe that a lever must be acted upon by at least three forces: the power; the weight, load, or resistance; and the reaction of the fulcrum. When the weight of the lever is considered, levers of the second and third classes have at least four forces acting upon them, and a lever of the first class has at least five forces acting on it, since in the latter case, the weights of the two lever arms act to turn the lever about the fulcrum in opposite

directions. The safety valve lever in Fig. 25 has five forces acting on it: the steam pressure, which acts upwards and represents W; the weight, which acts downwards and represents P; the reaction R of the fulcrum, which acts downwards, the weight L of the lever, which acts downwards at the center of gravity; and the weight V of the valve, which acts downwards at the center of the stem. Suppose the weight W of the lever in Fig. 25, is 36 lb., that its length is 48 in., and that it is of uniform cross-section throughout, and suppose the weight V of the valve and stem is $3\frac{1}{2}$ lb. Then, taking O as the center of moments, the moment of R will be 0, the distance of the center of gravity of the lever from O is $48 \div 2 = 24$ in. Equilibrium will occur when

$$+3.5 \times 4.5 + 36 \times 24 + 90 \times x - 879.648 \times 4.5 = 0$$

Solving for x, $x = \frac{3078.666}{90} = 34.21$ in., very nearly. The value previously found for x was 44 in., in Art. **39**. The difference, $44 - 34.21 = 9.79$ in. shows the error caused by neglecting the weight of the lever and the valve. The reason for the error being so large in this particular case is because the weight of the lever is very large compared with the load W. In most cases that arise in practice, the weight of the lever is small compared with the load, and the error is also small—so small that it can usually be neglected.

Whenever the lever is acted upon by more than three forces and all are taken into consideration, the method of moments must be used to find W or P.

41. Straight and Bent Levers.—If the action lines of the weight (load or resistance) and the power are parallel, as in all cases previously considered, the lever is called a **straight lever,** because the perpendiculars drawn from the fulcrum to the lines of action are parts of a straight line. The shape of the lever itself is not considered, and the lever may be straight, bent or curved. In Fig. 26, (*a*) and (*b*) are straight levers, because OA and OB lie in the same straight line, the action lines of P and W being parallel. In (*a*), the lever itself $A'OB'$ is curved, while in (*b*) it is bent, forming what is called a *bell-crank* $A'OB'$. The levers shown at (*c*) and (*d*) are bent levers, because the perpendiculars OA and OB make the broken line AOB. In (c), the lever itself $A'OB'$ is also bent, but in (*d*), the lever OAB' is straight.

In all cases of levers, whether bent or straight, OA and OB,

Figs. 24–26, are called the weight arm and power arm respectively, and the power multiplied by the power arm equals the weight multiplied by the weight arm, neglecting the weight of the lever.

EXAMPLE.—Referring to the example of Art. **34**, suppose the weight of the crowbar were 30 lb. and that its center of gravity were 23 in. from the end nearest the fulcrum; what pressure would be exerted at the weight end by reason of a force of 120 lb. exerted at the other end?

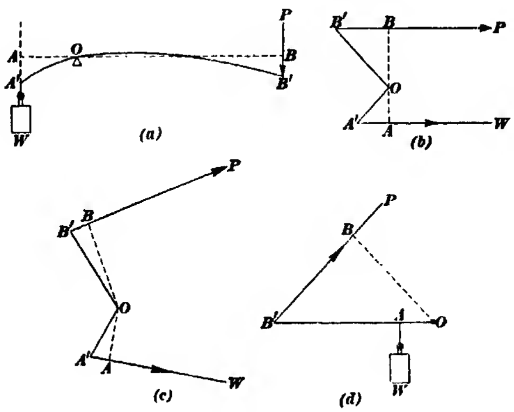

FIG. 26.

SOLUTION.—The fulcrum is 1.5 in. from the weight end and 58.5 in. from the power end; the weight of the bar acts at a distance of 23 − 1.5 = 21.5 in. from the fulcrum. The three forces mentioned are supposed to be parallel. Then,

$$120 \times 58.5 + 30 \times 21.5 - W \times 1.5 = 0$$

$$\text{Solving for } W, \; W = \frac{7665}{1.5} = 5110 \text{ lb. } \quad \textit{Ans.}$$

The result obtained in Art. **34** when the weight of the crowbar was neglected was 4680 lb.; hence, the error was 5110 − 4680 = 430 lb. Here the weight of the crowbar acts in the same direction as the power and thus increases the power. The result obtained is not quite correct, because the weight of the weight arm was not considered and acts in the opposite direction. The result obtained is thus a little too large, but the error is so small that it may be neglected.

42. Compound Levers.—When a series of levers is so arranged that a force applied to one is transmitted to another, the arrange-

ment is called a compound lever, and is commonly seen on paper machines, calenders, etc. A system of three levers acting as a compound lever is shown diagrammatically in Fig. 27. The levers in this case are all straight levers of the second class, such as are used for adding pressure to calenders, etc. and O', O'', and O''' are their fulcrums; the power is applied at P and the weight or pressure is exerted at W. The free end of the power arm of the second lever is connected to the power arm of the first lever by the link $W'P'$; the link $W''P''$ connects the second and third levers in a similar manner. The power arms of the levers are represented by a', a'', and a''' and the weight arms by b', b'', and b'''. The pressure exerted downward, as in this case,

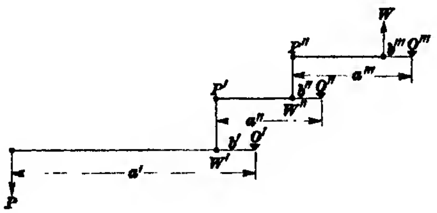

FIG. 27.

or the weight that could be raised by the first lever is (neglecting friction and the weight of the lever) $W' = P \times \frac{a'}{b'}$, which becomes the force P' acting on the end of the second lever; hence, the second lever could exert a pressure, or raise a weight $W'' = P' \times \frac{a''}{b''} = P \times \frac{a'}{b'} \times \frac{a''}{b''}$, which becomes the force P'' acting on the third lever. The third lever, therefore, can exert a pressure, or raise a weight $W''' = P'' \times \frac{a'''}{b'''} = P \times \frac{a'}{b'} \times \frac{a''}{b''} \times \frac{a'''}{b'''} = W$. Clearing this equation of fractions,

$$P \times a' \times a'' \times a''' = W \times b' \times b'' \times b''',$$

that is, in any compound lever, the product of power and all the power arms is equal to the product of the weight and all the weight arms. Note that the links $W'P'$ and $W''P''$ exert no influence on the ratio of P and W; they merely transmit the force from one lever to the next.

43. The velocity ratio of the compound lever is equal to the product of all the power arms divided by the product of all the weight arms; denoting the velocity ratio by r,

$$r = \frac{a' \times a'' \times a''' \times \text{etc.}}{b' \times b'' \times b''' \times \text{etc.}}$$

If, for some reason, it is not practicable to measure the lengths of the power and weight arms, but the distance moved by W when P moves a certain distance is known, then,

$$r = \frac{\text{distance power moves}}{\text{distance weight moves}}$$

Observe that this last equation is the same as was given in Art. **37**; it is true of any machine. That it must be true follows at once from the principle of work; viz., a machine can give out no more work than is imparted to it. The work done in operating the machine is equal to the power multiplied by the distance through which it moves; the work done by the machine is equal to the weight multiplied by the distance through which it moves; if friction, the weight of the moving parts, etc. be neglected, these two works must be equal, and

$$\text{power} \times \text{distance moved} = \text{weight} \times \text{distance moved}$$

whence,

$$W = \text{weight} = \text{power} \times \frac{\text{distance power moves}}{\text{distance weight moves}} = P \times r,$$

from which,
$$\frac{W}{P} = r.$$

But, from the last equation of Art. **42**,

$$\frac{W}{P} = \frac{a' \times a'' \times a'''}{b' \times b'' \times b'''}$$

Therefore,
$$r = \frac{a' \times a'' \times a''' \times \text{etc.}}{b' \times b'' \times b''' \times \text{etc.}}$$

Example.—If the lengths of the power arms of a compound lever are 25 in., 20 in., 36 in., and 28 in., and the lengths of the corresponding weight arms are 7 in., 6 in., 12.5 in., and 5 in., (*a*) what theoretical weight will a force (power) of 60 lb. raise? (*b*) what is the velocity ratio? (*c*) if the power moves 64 in., how far will the weight move?

Solution.—(*a*) From the last equation of Art. **42**,

$$60 \times 25 \times 20 \times 36 \times 28 = W \times 7 \times 6 \times 12.5 \times 5$$

from which, $W = 60 \times \frac{25 \times 20 \times 36 \times 28}{7 \times 6 \times 12.5 \times 5} = 60 \times 192 = 11{,}520$ lb. *Ans.*

(*b*) The velocity ratio is the value of the above fraction, or 192. *Ans.*

(*c*) Since $r = \frac{\text{distance power moves}}{\text{distance weight moves}}$, $192 = \frac{64}{\text{distance weight moves}}$

from which, distance weight moves $= \frac{64}{192} = \frac{1}{3}$ in. *Ans.*

When the word "theoretical" is used in such expressions as theoretical power, theoretical weight, etc., it means the power, weight, etc. when all hurtful resistances are neglected. In this example, it means to neglect friction, weight of levers, etc.

As another example showing a practical application of a compound lever, see Fig. 28, which represents the mechanism for moving the valves, reversing the engine, etc. of a locomotive.

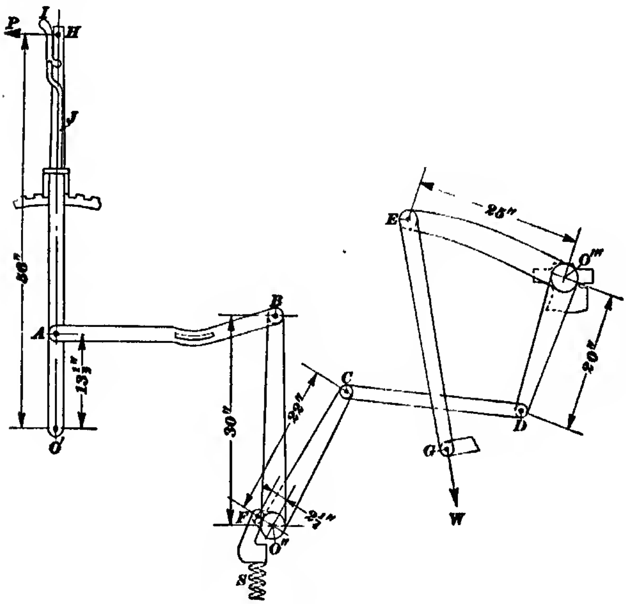

FIG. 28.

When I is pushed against H, the latch J is raised, and the handle H is then free to turn in either direction about the pin O'. Suppose it moves in the direction of the arrow P. The link AB is connected to the handle H at A, and when H moves AB moves also; this makes the handle a lever of the second class with fulcrum at O'. BO'' is keyed to a small shaft, to which is also keyed the cranks CO'' and FO'', all three cranks being moved (turned) when the link AB moves. The link CD connects the crank CO'' with the bell crank $DO'''E$, and this latter raises the link EG against the resistance offered by the valves, etc., and which here

corresponds to the weight W. S is a spring pressing against F and resisting the effort of P on H to move H. Now neglecting the weights of the levers, all frictional resistances, etc. suppose the resistance offered by the valves, etc. is equivalent to a weight W of 420 lb. and that the resistance offered by the spring is 180 lb., what force P is required to move the handle H?

Using the dimensions given in the figure, the power arm of H is 56 in., and the weight arm is 13.5 in. The arms BO'' and CO'' move through arcs of circles that are proportional to the radii CO'' and BO''; they constitute what is virtually a bent lever whose weight arm and power arm are equivalent (proportionally) to the distances CO'' and BO'', respectively. The same is true of the bell crank, the power arm being DO''' and the weight arm $O'''E$. Hence, neglecting the spring S for the present, the whole arrangement is a compound lever whose power arms have lengths of 56 in., 30 in., and 20 in., and whose weight arms have lengths of 13.5 in., 22 in., and 25 in. Therefore, the velocity ratio is $r = \frac{56 \times 30 \times 20}{13.5 \times 22 \times 25} = 4\frac{52}{99}$. Since $W = rP$, $P = \frac{W}{r} = 420 \div 4\frac{52}{99} = 92.81$ lb. = pull on handle required to move the valves, etc. But, before the valves can move, a resistance of 180 lb. due to the spring S must be overcome. The spring is actuated by the crank $O''F$ (which with $O''B$ makes another bell crank), the link AB, and the handle H; this combination makes another compound lever whose power arms are $O'P = 56$ in. and $O''B = 30$ in., and whose weight arms are $O'A = 13.5$ in, and $O''F = 2\frac{1}{4} = 2.25$ in. The velocity ratio of this compound lever is $\frac{56 \times 30}{13.5 \times 2.25} = 55\frac{25}{81} = r'$. Whence, $P = \frac{W}{r'} = 180 \div 55\frac{25}{81} = 3.25$ lb. Therefore, the total force P is $92.81 + 3.25 = 96.06$ lb.

As will be noted, the entire system of levers consists of two compound levers, one actuating the valves and the other the spring.

EXAMPLES

(1) (*a*) How many classes of levers are there? (*b*) Give an example of a lever of the second class not mentioned in the text? (*c*) Can the velocity ratio of a lever of the third class be greater than 1?

(2) (*a*) If the fulcrum is at one end of the lever and the power at the other end, with the weight between, to what class does the lever belong? (*b*) If the

length of this lever is 7 ft. and the distance from the fulcrum to the weight is 6 in.; what power will be required to lift a load of 672 lb.? *Ans.* 48 lb.

(3) (*a*) What is a straight lever? (*b*) a bell crank? (*c*) a compound lever? Illustrate by a sketch.

(4) The arms of a bell crank make an angle of 120° with each other; one arm is 4.5 in. long and the other is 11.5 in. long. If the action lines of the power and weight are parallel and the weight is moved by the short arm, what weight can be moved by application of a power of 27 lb.? To what class does this lever belong? *Ans.* 69 lb.

(5) (*a*) To what is the velocity ratio of a lever or any other machine equal? (*b*) what is the velocity ratio of the bell crank in Question (4)? *Ans.* (*b*) $2\frac{5}{9}$.

(6) The lengths of the power arms of a compound lever are $9\frac{1}{2}$ in., 12 in., 8 in. and of the weight arms $3\frac{3}{4}$ in., $4\frac{1}{4}$ in., 3 in.; what is (*a*) the velocity ratio? (*b*) what power must be applied to raise a load of 1800 lb.?

Ans. { (*a*) $19\frac{19}{255}$
(*b*) 94.367 lb.

(7) If a beater roll weighing 12,000 lb. with the shaft and pulley is placed so that each end of the shaft rests at the center of a lever of the second class, what weight is supported by each fulcrum? Consider the roll to be midway between the bearings. *Ans.* 3000 lb.

(8) What power would be required on each side of the beater if the center of gravity of the roll, shaft and pulley, which weigh 13,000 lb. is 5 ft. from the center of one bearing and 8 ft. from the center of the other, supposing the center of shaft to be 30 in. from the fulcrum and power to applied 70 in. from the fulcrum? Each lever (lighter bar) is assumed to weigh 300 lb., to be of uniform cross section, and to be 70 in. long.

Ans. { 2293 lb.
3579 lb.

(9) Extra pressure is put on a paper mill calender by means of compound levers composed of two simple levers, each lever being of the second class and having a power arm of 65 in. and a weight arm of $6\frac{1}{2}$ in. (*a*) What pressure is added on each side of the machine if the weight (power) is 120 lb.? (*b*) What is the extra pressure per inch width of the calender, if the rolls are 60 in. long at the face?

Ans. { (*a*) 12000 lb.
(*b*) 400 lb.

THE PULLEY

44. The Fixed Pulley.—A pulley is a wheel, which may be rigidly connected to an axle, so that when the pulley turns the axle turns also, or it may turn freely on the axle. The pulley is usually grooved around its circumference, and a rope, cord, or chain passes over the pulley and lies in the groove; if the pulley is not grooved, a band or belt is used. Fig. 29 (*a*) shows what is termed a **fixed pulley,** because it has no movement of trans-

lation—its axis always remains in the same relative position. S is the pulley; M is the axle; the part T, whatever its shape, that holds the pulley and contains the bearing for the axle to turn in is called the **block** (in machinery usually called the *hanger*); the pulley itself is frequently called the **sheave**; the entire combination of block, sheave, and rope is called the **tackle**, though commonly called the **block and tackle.**

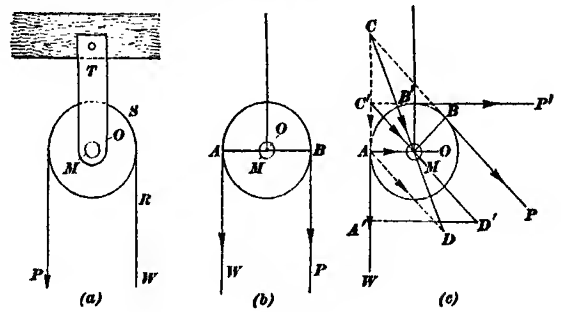

FIG. 29.

45. When the fixed pulley is used to raise loads, as indicated diagrammatically in Fig. 29 (*b*), the load is suspended from one end of the rope and the power is supplied at the other end. The arrangement is essentially the same as that of a lever of the first class with equal arms, the axis O being the fulcrum, the diameter AB being the lever, and the radii OA and OB being the lever arms. Taking O as the origin of moments, $P \times OB = W \times OA$; or, since $OB = OA$, $P = W$. Therefore, no matter what the diameter of the pulley, if friction and other hurtful resistances are neglected, the velocity ratio of a fixed pulley is always $r = \frac{W}{P} = 1$, since W always equals P. The only effect produced by a fixed pulley is to change the *direction* of the force. Thus, in the figure, the force instead of acting upwards to raise the weight W, acts downwards.

Furthermore, it makes no difference whether the action lines of P and W are parallel or not, P always equals W. For instance, referring to Fig. 29 (*c*), W still acts vertically, but P acts in the direction BP. Taking O as the origin of moments, $P \times OB = W \times OA$, or $P = W$, since $OB = OA$. Again, the distance moved by W must necessarily equal the distance moved by P;

hence, the velocity ratio is 1, and $P = W$. The same is true when P acts in the direction $B'P'$ or in any other direction that will keep the rope in contact with the pulley. The resultant pressure on the bearing, however, is different for different directions of the action line of P. Thus, in (*b*), it is $P + W = 2P$; in (*c*), if CA represent W and CB (AD) represent P, the resultant force on the bearing at M is CD; and if $C'A'$ represent W and $A'D'$ represent P, the resultant is $C'D'$. Since $C'D'$ is the hypotenuse of a right triangle having equal legs, assuming that C'P' is horizontal and C'W is vertical, $C'D' = C'A' \times \sqrt{2} = \sqrt{2} \times P$. Evidently, C'D' is less than CD.

46. The Movable Pulley.—A movable pulley is one that moves when the load moves, the load being suspended from the block,

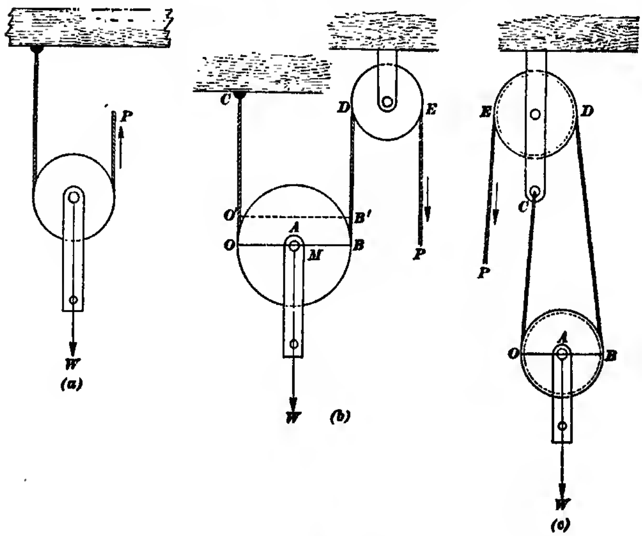

FIG. 30.

see Fig. 30 (*a*). One end of the rope is attached to a beam or other point of support and the other end is *free*. When a force P is applied to the free end, it moves up, and the pulley and weight also move up. The distance moved by P will be twice that moved by W; because, suppose that the pulley be raised (lifted) with its load say 6 in., P remaining stationary. Then, the diameter BO in (*b*), will occupy the position $B'O'$, and $B'B$

$= O'O = 6$ in.; in other words, the slack in the rope will be $6 + 6 = 12$ in., and to take up this slack, it will be necessary for P to move 12 in. But W has moved only 6 in.; hence, the velocity ratio is $r = \frac{12}{6} = 2$, and $W = 2P$.

This same result may be arrived at in another way. Suppose that the pulley were replaced by a lever whose center line is the diameter OB. The power acts at B, the weight at A and the fulcrum is at O; hence, the lever will be of the second class. Taking O as the origin of moments, $P \times OB = W \times OA$, from which, $W = P \times \frac{OB}{OA} = P \times 2 = 2P$, since $OB = 2 \times OA$.

47. Movable pulleys are usually arranged so that the free end of the rope passes over a fixed pulley, as shown at (*b*) or (*c*), Fig. 30. The block of the fixed pulley is called the **fall block.** One end of the rope may be attached to the fall block, as at (*c*) or it may be attached at any fixed point, as at (*b*); in either case, P will move through twice the distance that W moves, and the sheave (pulley) in the fall block has no effect on P other than to change its direction.

Note particularly that in the case of the fixed pulley, Fig. 29, the weight is sustained by only one part of the rope, while in the case of the movable pulley, Fig. 30, the weight is sustained by two parts of the rope, OC and BD. The free end EP is not considered; it is a part of BD.

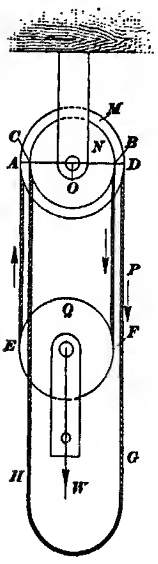

Fig. 31.

48. The Differential Pulley.—The differential pulley is shown in Fig. 31. There are three sheaves, M, N, and Q, M and N being in the fall block and Q in the movable block. Because of the heavy loads that it lifts, chains are generally used instead of ropes in the tackle of this class of pulleys. Sheaves M and N are keyed to the same axle, and when M turns, N and the axle turn also. The chain is an endless one, but the part DG corresponds to the free end. Beginning at G and going upwards, the chain passes over the pulley M (which is fixed), then passes down around the movable pulley, up over

hence, the velocity ratio is 1, and $P = W$. The same is true when P acts in the direction $B'P'$ or in any other direction that will keep the rope in contact with the pulley. The resultant pressure on the bearing, however, is different for different directions of the action line of P. Thus, in (*b*), it is $P + W = 2P$; in (*c*), if CA represent W and CB (AD) represent P, the resultant force on the bearing at M is CD; and if $C'A'$ represent W and $A'D'$ represent P, the resultant is $C'D'$. Since $C'D'$ is the hypotenuse of a right triangle having equal legs, assuming that C'P' is horizontal and C'W is vertical, $C'D' = C'A' \times \sqrt{2} = \sqrt{2} \times P$. Evidently, C'D' is less than CD.

46. The Movable Pulley.—A movable pulley is one that moves when the load moves, the load being suspended from the block,

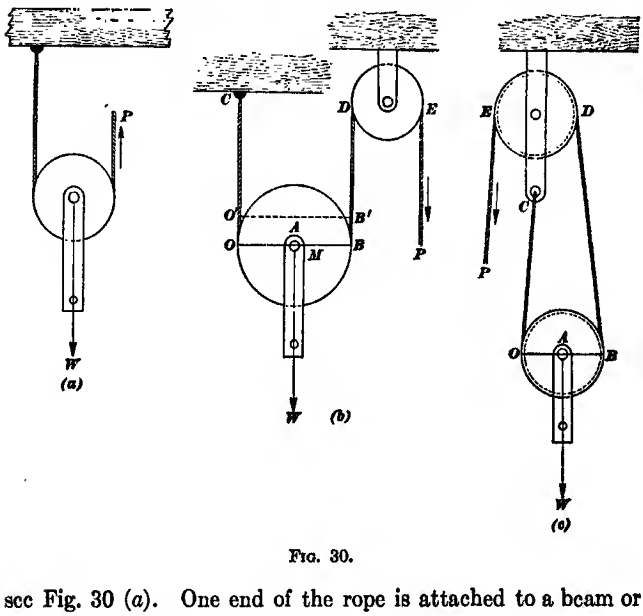

Fig. 30.

see Fig. 30 (*a*). One end of the rope is attached to a beam or other point of support and the other end is *free*. When a force P is applied to the free end, it moves up, and the pulley and weight also move up. The distance moved by P will be twice that moved by W; because, suppose that the pulley be raised (lifted) with its load say 6 in., P remaining stationary. Then, the diameter BO in (*b*), will occupy the position $B'O'$, and $B'B$

from which, $P = \dfrac{W(R - r)}{2R}$

and $W = \dfrac{2PR}{R - r}$

It may be mentioned that the chain is kept from slipping by means of notches cut into, or by teeth projecting from, the circumference of the sheaves.

Example.—What load can be raised with a differential pulley if the diameter of the larger sheave is 13 in. and of the smaller sheave 12 in. by a force P of 60 lb., neglecting the weight of the movable block and all hurtful resistances? What is the velocity ratio?

Solution.—If desired, the diameter may be substituted in formulas (1), (2), and (3) in place of the radius, since $\dfrac{R}{R-r} = \dfrac{\frac{D}{2}}{\frac{D}{2} - \frac{d}{2}} = \dfrac{D}{D-d}$, in which D and d are the diameters of the larger and smaller sheaves. Therefore, applying formula (3),

$$W = \frac{2 \times 60 \times 13}{13 - 12} = 1560 \text{ lb. } \textit{Ans.}$$

and, by formula (1),

$$r_v = \frac{2 \times 13}{13 - 12} = 26 \quad \textit{Ans.}$$

or, $$r_v = \frac{W}{P} = \frac{1560}{60} = 26.$$

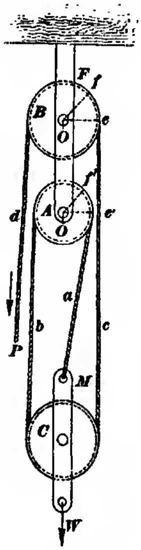

Fig. 32.

49. Combination of Pulleys.—When the blocks contain two or more sheaves and a *single* rope passes around all of them, the arrangement is called a **combination of pulleys.** The simplest form of such an arrangement is shown in diagrammatic outline in Fig. 32. The fall block F contains two sheaves A and B, and the movable block M contains one sheave C. One end of the rope is attached to the movable block; the rope then passes over sheave A around sheave C, and up over sheave B, which simply changes the direction of the power P.

It will be noted that the load is supported by three parts of the rope, designated by a, b, and c. The free end d does not count, since it is merely an extension of the part c; it changes the direction of P from a force acting upwards to one acting downwards. Each of the parts a, b, and c sustains an equal part of the load, or $\frac{W}{3}$; consequently, $P = \frac{W}{3}$ and $W = 3P$.

The velocity ratio is $\frac{W}{P} = \frac{3P}{P} = 3$. If, therefore, the weight to be lifted is 450 lb., the theoretical force P that is required to raise it is $\frac{450}{3} = 150$ lb.

This same result may be obtained as follows: Suppose the block C to be raised a inches, then there will be a in. of slack in a, b, and c. To take up this slack, it will be necessary for P to move $3 \times a$ in. $= 3a$ in. Hence, when W moves a in., P moves $3a$ in., and the velocity ratio is $\frac{W}{P} = r = \frac{3a}{a} = 3$, from which $W = 3P$.

50. It will be noted that the diameter of sheave A is less than that of sheave B; the reason for this is that it was thought desirable to proportion the two diameters so that the angular velocity (which equals the velocity of a point on the circumference divided by the radius) of the two sheaves will be equal. For instance, suppose the diameters of the two sheaves were equal, and suppose the movable block M to be raised say 1 in. There will be a slack of 1 in. in all three plies (parts) of the rope. To take up this slack, b must move downwards 2 in. and c must move upwards 3 in. In other words, when a point on the ply c moves from e to f along the arc ef, whose length is $R\theta$, θ being the angle eOf in radians, a point e' on ply a will move only $\frac{2}{3}$ of this distance, or $\frac{2}{3}R\theta$. Consequently, when the sheaves have equal diameters, and θ' is the arc moved through by the point on sheave A, $\theta' = \frac{2}{3}R\theta$. To make $\theta' = \theta$, all that is necessary is to make the radius of A equal to $\frac{2}{3}R$; that is, if the diameter of A is two-thirds that of B, both pulleys will turn through the same angle for any movement of P, and the angular velocities of the two sheaves will be equal—they will both make the same number of revolutions per minute.

51. Instead of arranging sheaves A and B so that one will be above the other, they may be placed side by side, as in Fig. 33, (a) and (b). In such cases, the sheaves are usually made of the same diameter and turn on the axle, instead of being keyed to it and turning the axle. If, however, the diameters are proportioned as just described, Art. **50**, then the sheaves may be keyed to the axle.

52. Whatever the number of sheaves in the fall block or in the movable block, the velocity ratio is equal to the number of plies

of rope that sustains the movable block. Thus, referring to Fig. 34, two different arrangements for six pulleys are shown. In both cases, there are 6 plies sustaining the movable block (and the load); consequently, the velocity ratio is 6. Another rule for cases of this kind is: the velocity ratio is equal to the total number of sheaves in the fall block and movable block. In

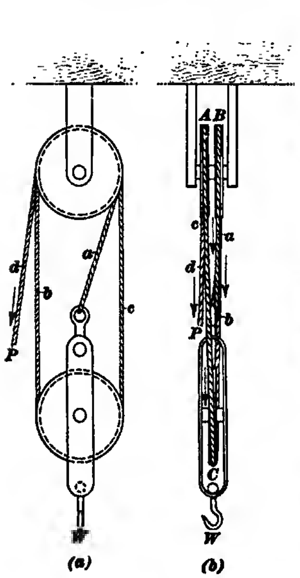

FIG. 33.

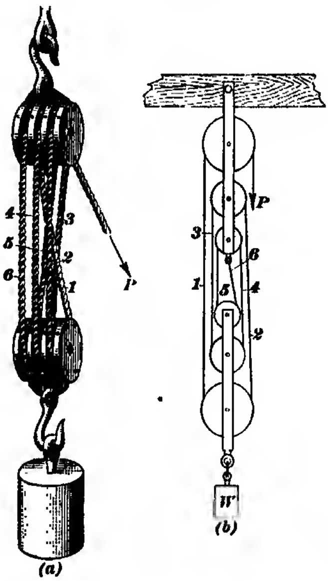

FIG. 34.

Fig. 34, the number of sheaves is 6, which is the velocity ratio; in Figs. 32 and 33, the number of sheaves is 3, the velocity ratio.

Knowing the velocity ratio r, $W = Pr$, and $P = \frac{W}{r}$.

EXAMPLE.—If the fall block of a block and tackle contains four sheaves and the movable block has three sheaves, what is the velocity ratio? What theoretical force P must be exerted on the free end to lift 600 pounds?

SOLUTION.—The velocity ratio is equal to the number of sheaves, or $4 + 3 = 7$. *Ans.*

The theoretical force is the force required to raise the weight when the weight of the movable block, friction, and all hurtful resistances are neglected, hence, $P = \frac{W}{r} = \frac{600}{7} = 85\frac{5}{7}$ lb. *Ans.*

53. The Compound Pulley.—Whenever a system of pulleys has more than one rope, it is called a **compound pulley**; thus, in Fig. 35, the pulley systems (*a*) and (*b*) both have three ropes, as indicated by *a*, *b*, and *c*. System (*a*), however, has four pulleys, while system (*b*) has three.

To find the velocity ratio of (*a*), let pulley *A* (with weight *W*) be raised say 1 in.; there will then be a slack of 1 in. in *a* and 1 in. in *a'*, and *B* must move up $1 + 1 = 2$ in. to take up this slack. There is now a slack of 2 in. in *b* and 2 in. in *b'*, and *c* must move up $2 + 2 = 4 = 2^2$ in. to take up the slack in *a*, *a'*, *b*, and *b'*. There is now a slack of 4 in. in *c* and 4 in. in *c'*, and *P* must move down $4 + 4 = 8 = 2^3$ in. to take up all the slack. In general, if *n* be the number of ropes, the distance *P* moves will be 2^n times the distance *W* moves; hence, the velocity ratio is $r = 2^n$.

If, however, n' = the number of pulleys, the number of ropes is $n' - 1$, and the velocity ratio is $r = 2^{n'-1}$.

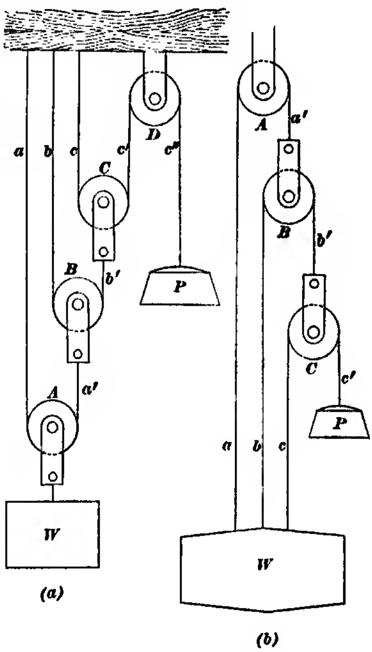

FIG. 35.

The velocity ratio of the arrangement shown in (*b*) is found in a similar manner. Thus, if *W* be raised 1 in., the slack in *a* and *a'* will be 1 in. in each ply, and *B* must move down $1 + 1 = 2$ in. to take up this slack; but there is still a slack of 1 in. in *b* due the raising of *W*, making the total downward movement of *B* $1 + 2 = 3 = 2^2 - 1$ in. There is now a slack of $1 + 3 = 4$ in. in *c* and 3 in. in *c'*, and *C* must move down $4 + 3 = 7 = 2^3 - 1$ in. to take up this slack. With this arrangement of pulleys, the

number of pulleys = the number of ropes = n, and for any number of pulleys, $r = 2^n - 1$.

EXAMPLE.—What theoretical force P is required to raise a load of 500 lb. with four pulleys arranged as in (*a*), Fig. 35? If the pulleys are arranged as in (*b*), what theoretical force is required?

SOLUTION.—For the first case letting n' = the number of pulleys, $r = 2^{n'-1} = 2^{4-1} = 2^3 = 8$; hence, the theoretical force is $P = \frac{500}{8} = 62.5$ lb. *Ans.*

For the second case, letting n = the number of pulleys, $r = 2^n - 1 = 2^4 - 1 = 15$; hence, the theoretical force is $P = \frac{500}{15} = 33\frac{1}{3}$ lb. *Ans.*

THE WHEEL AND AXLE

54. The wheel and axle consists of two pulleys rigidly attached to a common shaft, all three parts having the same axis. The arrangement is shown in Fig. 36. The pulley M, to which the

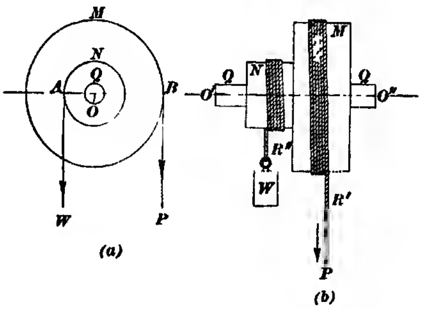

FIG. 36.

power is applied, is called the **wheel**; the pulley N, which raises the weight, is called the **axle**; both wheel and axle are keyed to the shaft Q; and all three parts, M, N, and Q have a common axis $O'O''$. Referring to (*a*), and taking O as the origin of moments, $P \times OB = W \times OA$, from which

$$\frac{W}{P} = \frac{OB}{OA} = r_v = \frac{R}{r} = \frac{2R}{2r} = \frac{D}{d}$$

in which r_v = the velocity ratio, R = radius of wheel, r = radius of axle, D = diameter of wheel, and d = diameter of axle.

55. It is not essential that the entire wheel be a part of the arrangement; in Fig. 36, the entire wheel was necessary, because

two separate ropes R' and R'' were used, and as R' unwinds from M, R'' winds up on N, the effect produced is the same as though the force P acted at the circumference of M and moved around M. If the wheel M be replaced with a crank having a handle that can be grasped with the hands and be made to turn in a circle, as in Fig. 37, the effect will be exactly the same as that produced by the arrangement of Fig. 36, insofar as lifting the weight is concerned. The apparatus shown in Fig. 37 is called a

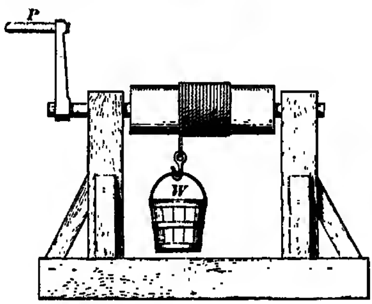

Fig. 37.

windlass. If the force is always exerted so its action line will be tangent to the circle described by a point on the handle, it will have exactly the same effect on the axle as is produced by the rope acting on the wheel. The radius of the circle described is the same as the radius of a wheel having the same diameter as the circle.

The windlass is frequently used in combination with a block and tackle to raise heavy loads. In such cases, let r' be the velocity ratio of the windlass and r'' the velocity ratio of the block and tackle; then the velocity ratio of the combination is

$$r = r' \times r''.$$

Example.—Suppose the radius of the circle described by the handle of a windlass is 15 in. and the diameter of the axle is 8 in.; suppose further that the weight end of the rope leading from the axle forms the free end of a block and tackle, in which the fall block contains 3 sheaves and the movable block contains 2 sheaves, somewhat as illustrated in Fig. 38. What is the velocity ratio of the combination? What theoretical weight will a force of 45 lb. exerted on the handle of the windlass lift?

Solution.—The diameter of the circle described by the handle H is $15 \times 2 = 30$ in. The velocity ratio of the windlass is $r' = \frac{30}{8}$. The number of sheaves contained in the block and tackle is $3 + 2 = 5$, which

is the velocity ratio of the block and tackle; see Art. 52, and $r'' = 5$. The velocity ratio of the combination is $r = r' \times r'' = \frac{30}{8} \times 5 = 18.75$. *Ans.* The theoretical weight that can be lifted is $W = Pr = 45 \times 18.75 = 843.75$ lb. *Ans.*

EXAMPLES

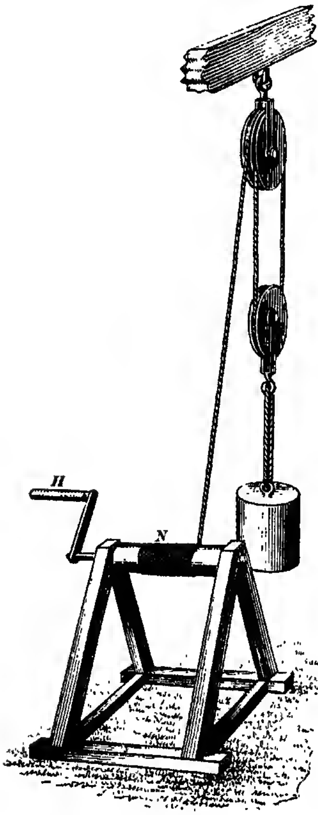

Fig. 38.

(1) In a block and tackle consisting of 3 fixed and 2 movable pulleys, (*a*) what is the velocity ratio? (*b*) Neglecting friction and other hurtful resistances, what must be the pull on the free end to lift a load of 675 lb.?

Ans. (*b*) 135 lb.

(2) What (*a*) theoretical load can be lifted with a differential pulley by application of a force of 90 lb., if the diameter of the larger sheave is 11 in. and of the smaller sheave $10\frac{1}{4}$ in.? (*b*) the velocity ratio?

Ans. $\begin{cases} (a)\ 2640 \text{ lb.} \\ (b)\ 29\frac{1}{3} \end{cases}$

(3) A compound pulley made up of four pulleys arranged as in Fig. 35(*b*), is required to raise a load of 960 lb.; what power must be applied? *Ans.* 64 lb.

(4) Referring to Fig. 38, suppose that the diameter of the axle is $8\frac{1}{2}$ in., the radius of the circle described by the handle is 16 in., and that the fall block and movable block each contain 3 sheaves; (*a*) what is the velocity ratio? (*b*) what load can be lifted by an application of a power of 36 lb.?

Ans. $\begin{cases} (a)\ 22\frac{10}{17} \\ (b)\ 813.2 - \text{lb.} \end{cases}$

MECHANICS AND HYDRAULICS

(PART 1)

EXAMINATION QUESTIONS

(1) Describe (*a*) how a force may be represented by a line. (*b*) Why is it necessary to use segments of right lines to represent forces?

(2) Suppose a railway train to be moving with a velocity of 55 ft. per sec. and that a stone is thrown in the same direction from the train in such a manner that it has an average horizontal velocity of 82 ft. per sec. If the stone strikes the earth 6½ sec. after being thrown, how far will it be from its starting point with reference to the earth? *Ans.* 890.5 ft.

(3) Two forces, $A = 64$ lb. and $B = 88$ lb., have a common point of application and act at right angles to each other; what is the value of the resultant? *Ans.* 108.8 lb.

(4) A crowbar is used to lift a load placed between the ends by *raising* the free end. (*a*) To what class does this lever belong? (*b*) If the crowbar is 54 in. long and the center of the load is 2¾ in. from the fixed end, what power is required to lift 1350 lb.? *Ans.* (*b*) 69¾ lb.

(5) Referring to the example of Art. **14** and Fig. 11, suppose the weight W had been 125 lb.; what would be the vertical force tending to lift the pulley O''? *Ans.* 175.6 lb.

(6) Referring to Fig. 14, if length of AB = 3 ft. 4 in., of BC = 4 ft. 9 in., and of AC = 4 ft. 4½ in., what is the magnitude of the forces acting in the arms? *Ans.* $\begin{cases} \text{In } AB, \text{ 381 lb.} \\ \text{In } BC, \text{ 543 lb.} \end{cases}$

(7) Referring to Question 6, what is the moment of the couple produced by the weight W? *Ans.* 1608 ft.-lb.

(8) (*a*) What difference in effects is produced by a moment

and by a couple, the arms being equal? (*b*) When is a body in complete equilibrium under the action of forces?

(9) Referring to Fig. 18, suppose the beam is 14 ft. long, of uniform cross-section throughout its length, and that it weighs 28 lb. per foot of length. If the beam is horizontal and is acted on by four vertical forces A, B, C, and D, the magnitudes of which are A = 1400 lb., B = 800 lb., C = 500 lb., and D = 1800, what are the reactions of the supports, when A is 2 ft. from the right-hand support, B is 3 ft. from A, C is 4 ft. from B, and D is 4 ft. from C, and at what point does the resultant act?

Ans. Reaction of left support = 2674 4/7 lb.
Reaction of right support = 2217 3/7 lb.
6.345 + ft. from left end.

(10) In a lever of the third class, the distance from the fulcrum to weight is 22 in. and from the fulcrum to the power is 9 in. (*a*) How far will the power move when the weight moves ¼ in.? (*b*) What is the velocity ratio?

Ans. (*a*) .1023– in.
(*b*) .4091–.

(11) Suppose that the weight and the power were to change places in the case of Question 10, (*a*) to what class would the lever then belong? (*b*) what would be the velocity ratio?

Ans. (*b*) 2 4/9 = 2.444+

(12) If the distance from the cutting point to the pin of a pair of shears for cutting sheet metal is 1¼ in., distance from pin to handles is 13¾ in., and a pressure of 26 pounds is applied to the handles by the fingers, what is (*a*) the cutting force? (*b*) the velocity ratio?

Ans. (*a*) 286 lb.
(*b*) 11

(13) In a compound lever made up of four simple first-class levers, the power arms are 12 in., 9 in., 11 in., and 8 in., the weight arms are 2 in., 1¾ in., 1⅞ in., and 1½ in.; what is (*a*) the velocity ratio? (*b*) what theoretical weight can be raised when the power is 12 lb.?

Ans. (*a*) 965.5–
(*b*) 11,586– lb.

(14) Referring to the example of Art. **39**, suppose the weight of the valve and stem to be 3 lb. 12 oz., weight of lever is 25 lb., and distance of its center of gravity from O is 22½ in. The other dimensions and weights being unchanged, at what distance from O must the ball be placed so that the steam pressure shall not exceed 75 pounds per square inch. *Ans.* 40.7 in. from O.

(15) In a block and tackle, the fall block has 3 sheaves and the movable block has two sheaves; (*a*) how far will the load move when the power moves 18 in.? (*b*) what is the velocity ratio?

Ans. $\begin{cases} (a) \ 3.6. \text{ in.} \\ (b) \ 5 \end{cases}$

(16) If the free end of a block and tackle containing 6 sheaves is attached to the drum of a windlass as in Fig. 38, (*a*) what theoretical load can be lifted when a force of 36 lb. is applied to the handle, the radius of the handle being 16 in. and diameter of drum 7 in.? (*b*) what is the velocity ratio?

Ans. $\begin{cases} (a) \ 987.4 \text{ lb.} \\ (b) \ 27\frac{3}{7} \end{cases}$

MECHANICS AND HYDRAULICS

(PART 2)

STATICS (Continued)

SIMPLE MACHINES (Continued)

THE INCLINED PLANE

56. In the three simple machines so far described, the idea of *rotation* is involved and the principle of moments can be applied to find the velocity ratios. In the case of the next three machines, which completes the list of simple machines, rotation is not a feature of their operation, and the velocity ratios must be obtained by resolution of forces into components. The pulley and wheel and axle may be considered as forms of levers; the wedge and the screw may be considered as forms of the inclined plane.

An inclined plane may be represented by a right triangle, one leg being horizontal, the other vertical, and the hypotenuse being the **inclined plane,** or **slope;** see Fig. 39, where AB in each case represents an inclined plane. Assume that there is no friction between the load W and the plane; then, if AB were horizontal, occupying the position AC, the force representing the weight of W would act vertically downwards, would be resisted by an equal force acting vertically upwards, and there would be no tendency for W to move in any direction except vertically downwards. Moreover, the perpendicular pressure against the plane would then be exactly the same as the weight of W. But, with AB in the position shown, the pressure against the plane will generally be different from the weight of W, there will be a tendency

for W to slide down the plane, and this must be resisted by the application of a force P. In accordance with the angle that action line of the force P makes with the inclined plane, there are three cases, viz.: when P acts parallel to the plane; when P acts parallel to the base AC; and when P acts in a line making an angle with the base that differs from the angle made by the plane. Each case will be considered separately.

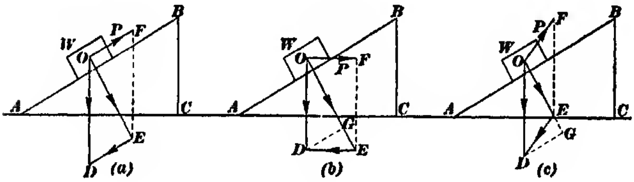

Fig. 39.

57. First Case.—Referring to Fig. 39 (*a*), let O be the center of gravity of the body, whose weight is W, and OF, parallel to AB, be the action line of P. Draw OD vertical, and make its length represent W to some convenient scale. Draw OE perpendicular to the plane and DE parallel to the plane. OD is then resolved into two components OE and ED acting in the directions indicated by the arrow heads. OE represents the force with which W presses against the plane (the perpendicular pressure), while ED represents the force urging the body down the plane and which must be counteracted by P. Completing the parallelogram $ODEF$, $OF = ED = P =$ the force required to prevent W from slipping down the plane. Considering the triangles ABC and ODE, C and E are right angles by construction, and since OD and OE are perpendicular to AC and AB respectively, $DOE = BAC$, and the triangles are similar. Therefore, $ED : OD = CB : AB$, or, since $ED = P$ and $OD = W$,

$$P : W = CB : AB$$

whence,
$$P = \frac{W \times CB}{AB} \qquad (1)$$

and
$$W = \frac{P \times AB}{CB} \qquad (2)$$

The velocity ratio is
$$r = \frac{W}{P} = \frac{AB}{CB} \qquad (3)$$

In other words, *where the power is parallel to the plane, the velocity ratio is equal to the length of the plane divided by the height of the plane;* and it is evident that the longer the plane in proportion to its height the greater will be the velocity ratio. Also, the steeper the plane the greater will be the power required to raise the weight through the height CB, until, when AB becomes vertical, the power equals the weight.

58. Second Case.—Referring to Fig. 39 (*b*), let the power act in the direction OF, parallel to the base; the length and height of the plane and the weight W of the body are the same as in (*a*). As before, draw OE perpendicular to the plane; draw DE parallel to the *base;* then OE represents the perpendicular pressure against the plane and ED represents the force which, acting parallel to the base, will move the body up the plane or, rather, keep it from moving down the plane. The perpendicular pressure against the plane is greater than in the first case; for, drawing DG parallel to AB, if OD in (*b*) is equal to OD in (*a*), GE represents this additional pressure to the same scale that OD represents W. The reason for this additional pressure is readily seen; thus, completing the parallelogram $ODEF$, $OF = P = ED$. But P ($= ED$) can be resolved into two components, one acting parallel to the plane (represented by GD) and the other perpendicular to the plane (represented by GE); and since ED is greater than GD, P in (*b*) is greater than P in (*a*).

Considering the triangles BAC and EOD, they are similar right triangles; hence, $ED : OD = CB : AC$, or, since $ED = P$ and $OD = W$,

$$P : W = CB : AC$$

whence,
$$P = \frac{W \times CB}{AC} \qquad (1)$$

and
$$W = \frac{P \times AC}{CB} \qquad (2)$$

The velocity ratio is
$$r = \frac{W}{P} = \frac{AC}{CB} \qquad (3)$$

In other words, *when the power is parallel to the base, the velocity ratio is equal to the length of the base divided by the height of the plane.* The velocity ratio is smaller in the second case than for the first case; this is caused by the fact that $P = ED$ is larger in (*b*) than in (*a*).

59. Third Case.—Suppose P to act in the direction OF, as indicated in Fig. 39 (c), OF making a greater angle with the base than AB, and W being the same as before. As before, draw OE perpendicular to the plane and DE parallel to the line of action of P; then OE represents the perpendicular pressure against the plane and $ED = P$, the force required to hold the body in position when acting in the direction OF. Producing OE and drawing DG parallel to AB, $DG = DE$ in (a). The perpendicular pressure against the plane, represented by OE is *less* than OE in (a) by the amount EG, the decrease being caused by the fact that P tends to lift the body off the plane. This is shown by resolving DE in (c) into two components DG and GE. Since DE is greater than DG, the velocity ratio in (c) is less than in (a); because when W is the same, if P is greater, the ratio $r = \frac{W}{P}$ must be less. It is therefore evident that the velocity ratio is greater for the first case than for either of the other two.

EXAMPLE 1.—A wagon is hauled up an inclined plane that is one-half mile long; when the wagon reaches the top, it is 72 feet higher than when it started. Assuming that the power moving the wagon is exerted parallel to the plane and that the wagon and its contents weight 1800 lb., what force is required, friction and other hurtful resistances being neglected?

SOLUTION.—This evidently corresponds to the first case; hence, applying formula (1), Art. 57, since one-half mile $= 5280 \div 2 = 2640$ ft.,

$$P = \frac{1800 \times 72}{2640} = 49\tfrac{1}{11} \text{ lb.} \quad \textit{Ans.}$$

This force of 49.1 lb., very nearly, is the force which, acting parallel to the plane, will just keep the body from sliding down the plane, assuming there is no friction; or, if the body is in motion up the plane and this force acts upon it, it will keep the body in motion with a uniform velocity.

EXAMPLE 2.—Suppose the wagon in the last example had been pushed up the plane, the direction of push being parallel to the plane's base; what force would be required?

SOLUTION.—This falls under the second case; hence, apply formula (1), Art. 58, first calculating the length of the base. Since the length of the plane is the hypotenuse and the height of the plane is one leg of a right triangle, the other leg, the base, is $\sqrt{2640^2 - 72^2} = 2639$ ft. Consequently,

$$P = \frac{1800 \times 72}{2639} = 49.11 \text{ lb.} \quad \textit{Ans.}$$

Observe that the results are practically identical, which is always the case when the plane is very long in comparison with the height. For all practical purposes, when the height is not greater than about $\frac{1}{10}$th of the length of the plane, the velocity ratio may be taken as the same for both cases. In specifying

the grade for railroads, sewers, rivers, roads, etc., it is usual to give it as so many feet per mile or as a certain per cent; the meaning in such cases is that for one mile of length *horizontally*, the rise is a certain number of feet, or for 100 feet horizontally, the rise is the number of feet specified by the per cent. Thus, a grade of 26 feet per mile means a vertical rise of 26 feet for a horizontal distance of 1 mile; also, a grade of 5 per cent means a vertical rise of 5 ft. for a horizontal distance of 100 ft. Consequently, according to the above, if the grade is less than 10 per cent, the velocity ratio may be taken as the ratio of the horizontal distance to the vertical distance without material error.

60. Discussion of Inclined Planes.—The object of an inclined plane is to raise a load through a given vertical height by the application of a power which is less than the force that is equivalent to the weight of the load; the work done, however, as is the case with any machine, is the same as in the case of a direct lift, because the power acts through a greater distance. Thus, considering the first case, let l = length of plane and h = height of plane; then $P = \frac{W \times h}{l}$. Multiplying both sides of this equation by l,

$$P \times l = \frac{W \times h}{l} \times l = W \times h$$

But $P \times l$ is the work done by P, and $W \times h$ is the work done on the load, which is the same as that required to lift the load vertically through the height h. In the second case, let b = length of base; then, $P = \frac{W \times h}{b}$. Multiplying both sides of this equation by b,

$$P \times b = W \times h.$$

But $P \times b$ is the work done by P in moving the load, and $W \times h$ is the work that would be done in lifting the load through the height h. Hence, as before, the power multiplied by the distance through which it moves is equal to the weight multiplied by the distance through which it moves.

61. For the third case, a different value for P will usually be obtained for every position that the load occupies on the plane. Thus, referring to Fig. 40, suppose the load to be pulled up the plane by means of a pulley arranged as shown. For the position O', P is represented by $D'E'$; for the position O'', P is

represented by $D''E''$; and for the position O''', P is represented by $D'''E'''$. It will be noted that as the load approaches the top of the plane, P becomes more nearly equal to W.

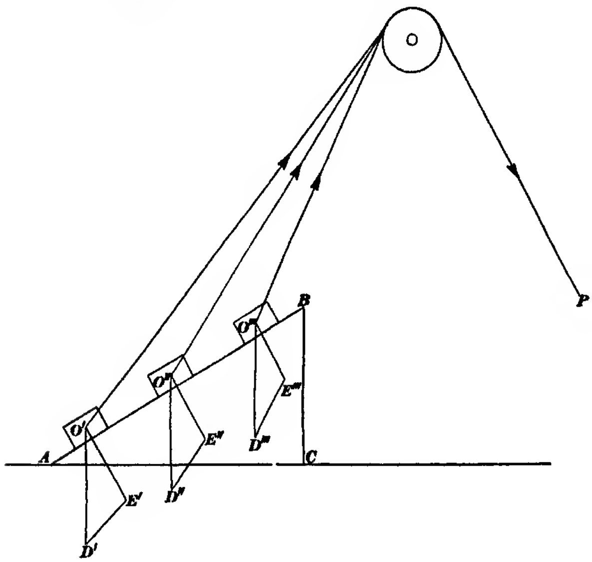

FIG. 40.

THE WEDGE

62. In the case of the inclined plane, the load is raised through a certain height as the result of a movement along the plane. The **wedge** is a form of inclined plane, but instead of the load moving along the plane, it has a movement, the direction of which is always in a right line that makes the same angle with the plane, and the plane itself moves. The shape of the wedge is that of a triangular prism, two of the sides meeting in a sharp acute angle, as indicated in Fig. 41 (*a*) and (*b*), where ABC is an end view of the wedge, the sides meeting at A; usually, the angle BAC is smaller than here represented. Let BC be the back of the wedge, and draw AD perpendicular to BC. Assume the power to be applied at D; then the wedge will be acted on by three forces, the load or resistance W acting perpendicular to the side AB, the reaction R acting perpendicular to the side AC, and the force P

acting perpendicular to the back of the wedge. Assume that these three forces concur at O; then the wedge will be in equilibrium under the action of these three forces. To find the value of these forces, let ED represent P to some convenient scale; draw DF parallel to W, and perpendicular to AB, and draw EF parallel to R, and perpendicular to AC. Placing the arrowheads as shown, they all point in the same general direction around the triangle, thus indicating that the forces are in equilibrium (see Art. **18**). DF represents the value of W and FE represents the value of R, both measured to the same scale as ED.

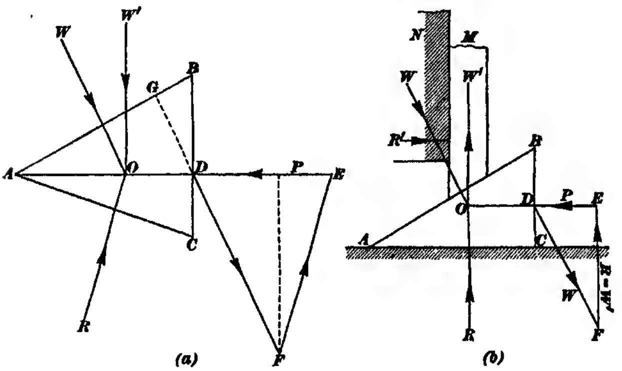

Fig. 41.

63. Considering the triangles BAC and DFE, DF is perpendicular to AB and FE is perpendicular to AC; hence, angle $DFE = BAC$. Producing FD to G, DGB is a right triangle, right-angled at G; angle $GDB = CDF$; $GBD = 90° - GDB$, and $FDE = 90° - CDF = 90° - GDB$; that is, the angles GBD and FDE are equal. Since two angles of the two triangles BAC and DFE are equal, the third angle of the two triangles must also be equal, and the two triangles are similar. Therefore,

$$P : W = BC : AB$$

and

$$P : R = BC : AC$$

From these two proportions,

$$P = \frac{W \times BC}{AB} \qquad (1)$$

$$P = \frac{R \times BC}{AC} \qquad (2)$$

If the sides AB and AC are equal, the triangle is isosceles, $W = R$, and the velocity ratio is

$$r = \frac{W}{P} = \frac{AB}{BC} = \frac{AC}{BC} \qquad (3)$$

In other words, *the velocity ratio is then equal to the length of one of the sides divided by the length of the back of the wedge.*

64. *The wedge shown in Fig. 41 (a) is called a* **double wedge;** if one side be perpendicular to the back, as in Fig 41 (*b*), the wedge is called a **single wedge** or a **simple wedge.** The formulas given in Art. **63** will apply to this case also, since the side AC in Fig. 41 (*a*) then coincides with AD.

The simple wedge may be used to raise heavy loads, as indicated in Fig. 41 (*b*). Here the load M is kept from sliding down the wedge by the reaction R' of the wall N Suppose it is desired to find the vertical force W' tending to lift M. The wedge is kept in equilibrium, as before, by the action of the power P, the force W perpendicular to AB, and the reaction R perpendicular to AC and, therefore, vertical. R acts upwards and is exactly equal to W' acting downwards. Draw ED to represent P; then draw DF parallel to W, and FE parallel to R; FE represents the effect of P in raising M. The triangles ACB and FED are similar right triangles; therefore,

$$P : W' = BC : AC$$

or

$$P = \frac{W' \times BC}{AC} \qquad (1)$$

and

$$r = \frac{W'}{P} = \frac{AC}{BC} \qquad (2)$$

Also, $P : W = BC : AB$, since $W = DF$; from which,

$$P = \frac{W \times BC}{AB}$$

which is exactly the same as formula (1), Art. **63.**

65. Power is usually applied to a wedge in the form of a blow struck with a hammer or sledge. If the angle A is quite small, so that the sides are very long compared with the back, a powerful blow will create an immense force. It is for this reason that wedges are so frequently used to split logs, stone, etc. This may be illustrated by an example.

EXAMPLE.—An iron wedge having equal sides 8 in. long, and the back of which measures ¾ in. is used to split a block of stone. If struck a blow equivalent to a power of 450 lb., what force does the wedge exert?

SOLUTION.—The required force is the weight or load W. The velocity ratio is, by formula (3), Art. **63**, $r = 8 + \frac{2}{3} = \frac{32}{3}$. Therefore,

$$W = P \times r = 450 \times \frac{32}{3} = 4800 \text{ lb. } Ans.$$

From the first proportion in Art. **63**,

$$P \times AB = W \times BC.$$

When the wedge moves in the direction DA, it must move a distance AB, Fig. 41 (a) in order to raise the load W through a height CB; hence, the above equation states once more that: the power multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves, since the power moves through the same distance that the wedge moves.

A thin wedge may also be used to move a weight a very small distance, as is frequently necessary in adjusting machinery.

THE SCREW

66. The Helix.—Referring to Fig. 42 (a), $ABCD$ represents a cylinder on which has been wound a fine thread in such a manner that the distance between any two consecutive turns is constant when measured on a line parallel to the axis mn thus, $bd = df = ac = ce = hi = ij$, etc. The curved line thus formed by the thread is called a **helix.** A little consideration will show that a point (as the point of a pencil) tracing the helix will in going once around the cylinder, advance along the cylinder a distance equal to $bd = ac = hi$, etc. The path of the point may be represented by a right line in the following manner: Lay off $a'b'$, Fig. 42 (c), equal in length to the circumference of the cylinder $= \pi d = \pi \times AB$; draw $b'd'$ perpendicular to $a'b'$, and make it equal in length to $bd = ac = hi$; join b' and d', and $a'd'$ will be the development of one turn of the helix. That this is a fact may be proved by cutting out the triangle $a'b'd'$ and rolling it around a cylinder having the diameter $d = AB$ in such a manner that $a'b'$ will be perpendicular to every element of the cylinder; it will then be found that $a'd'$ will coincide with the helix throughout one turn. The distance $bd = ac = hi$ is called the **pitch** of the helix.

67. The Screw and Nut.—If a groove be cut into a cylinder in such a manner that the inside and outside edges of the groove form helixes, the part that is left is called a **screw thread,** and the entire piece is called a **screw**; thus, in Fig. 42 (b), is shown a

screw, the curved projecting part being the screw thread, or simply, the **thread.** The diameter d is called the *diameter at top of the thread*, or *outside diameter*, and the diameter d' is called the *diameter at bottom of the thread*, or *inside diameter.*

If the same kind of a thread be cut inside a hollow cylinder whose inside diameter is the same as d', and the depth of the thread so cut is the same as on the screw, or $\frac{d-d'}{2}$, the result is called a **nut,** and the thread on the nut will fit the spaces between the threads on the screw. In Fig. 42 (*b*), N is a nut. If the nut be revolved, it will advance along the screw; or, if the nut be held stationary and the screw turned, the screw will travel through the nut. Assuming that the nut turns, a point on the helix at the *bottom* of the thread in the nut will travel a distance represented by $a'd'$ in (*c*) while the nut travels the distance $b'd'$ (= the pitch) along the axis. The effect is exactly the same as in the first case of the inclined plane, the pitch of the thread (helix) corresponding to the height of the plane and the length of the helix corresponding to the length of the plane.

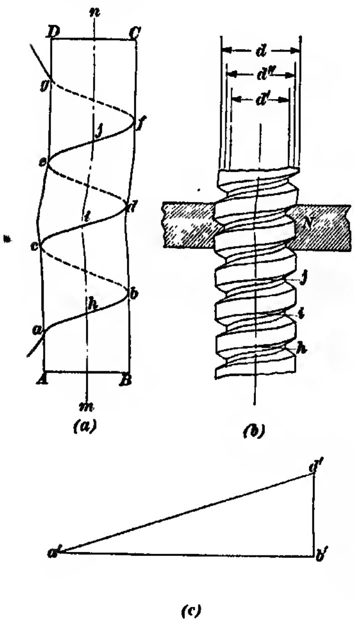

FIG. 42.

68. Classification of Screw Threads.—Screw threads are classified according to their shape by taking a longitudinal section through the axis; the shape of the section may be a triangle, a square, a rectangle, or a trapezoid; the thread shown in Fig. 42 (*b*) is a trapezoidal thread. When of triangular shape, they are usually called **V threads,** and when the tops and bottoms are sharp, they are called **sharp-V threads.** In most cases, V threads are flattened at top and bottom to make the screw stronger. In what is called the **Whitworth thread,** the V's are

rounded at top and bottom. Whatever their shape, the pitch is the distance measured on the top of the thread parallel to the axis between a point on a helix and the corresponding point at the beginning of the next turn of the helix.

A **right-hand thread** is one that moves *away* from the turning force when the screw or nut is turned clockwise, and a **left-hand thread** is one that moves *toward* the turning force when the screw or nut is turned clockwise. For instance, when turning a right-hand screw with a screw-driver, if the handle be turned clockwise, it is necessary to follow the screw with the screw-driver, in order to keep in contact with the screw.

Screws are also classified as **single-**, **double-**, **triple-**, or **quadruple-threaded** according to whether they have *one, two, three,* or *four* sets of helixes. Most multiple-threaded screws have square threads or trapezoidal threads. The pitch is measured in the same manner as for a single thread, as stated above; that is, from a point on a helix along a line parallel to the axis to where the line intersects the next turn of the *same* helix. Multiple threads are used when the pitch is large and it is not desired to cut the thread as deep as would ordinarily be required with a single thread; they do not alter in any way the relations between the power and the load; in other words, the velocity ratio is the same as for a single-threaded screw of the same pitch.

When the pitch is less than 1 inch, screws are usually identified as a certain number of threads per inch; the **number of threads per inch** is the *reciprocal* of the pitch, and vice versa. For instance, if the pitch is $\frac{1}{7}$ in., the screw has 7 threads per inch; and if a screw has 13 threads per inch, the pitch is $\frac{1}{13}$ in. Consequently, to find the pitch of a screw, lay a rule along the tops of the thread so the scale will be parallel to the axis, and then count the number of turns (usually called the number of threads) between one inch-mark and the next one; the reciprocal will be the pitch. In some cases, this number may be a fraction, in which case, it is best to count the number of threads for 2 in. or 4 in., as the case may be. Thus, many pipe threads are $11\frac{1}{2}$ to the inch, but the threaded part is seldom 2 inches long; but if 2 in. can be measured off, the number of threads in such a case would be 23, or $23 \div 2 = 11\frac{1}{2}$ threads per inch.

69. Velocity Ratio.—Referring to Fig. 42 (*b*), suppose the nut N to carry a load; it will act parallel to the axis, and when the nut makes one turn, the load will be moved along parallel to the

axis a distance equal to the pitch. The power, on the contrary, will act through a distance equal to the circumference of a circle whose diameter is the mean between the outside diameter and the diameter at the bottom of the thread; representing this by d'', $d'' = \frac{d + d'}{2}$, and the distance moved through by the power is $\pi d''$. Letting p = the pitch, W = the load, and P = the power,

$$P \times \pi d'' = W \times p$$

whence, the velocity ratio is $r = \frac{W}{P} = \frac{\pi d''}{p}$ (1)

and $W = P \times \frac{\pi d''}{p}$ (2)

70. A screw or nut can be turned only through the action of a moment or a couple, which must act through the entire circumference of a circle. In the case of a screw-driver, the couple acts on the handle; and while it is transferred to the head of the screw, where an equal couple acts, the effect is the same as though the screw-driver and screw were all one piece. Therefore, let d be the diameter of the handle and p the pitch of the screw; then, the velocity ratio is $r = \frac{\pi d}{p}$. Thus, suppose the diameter of the handle is $1\frac{1}{4}$ in. and the screw has 10 threads per inch; then a force of, say 25 lb. exerted on the handle will cause a forward pressure by the screw of $W = 25 \times \frac{\pi \times 1.25}{\frac{1}{10}} = 25 \times \pi \times 1.25 \times 10 = 981.75$ lb.

The velocity ratio is $r = \pi \times 1.25 \times 10 = 39.27 = \frac{981.75}{25}$.

If n = the number of threads per inch, $n = \frac{1}{p}$, and $p = \frac{1}{n}$; hence, substituting $\frac{1}{n}$ for p in formula (2) of Art. **69**,

$$r = \frac{\pi d''}{\frac{1}{n}} = \pi d'' n \qquad (1)$$

And if d = the diameter of the circle through which the power moves,

$$r = \pi d n \qquad (2)$$

whence, $W = P \times \pi d n$ (3)

which was the formula used above in calculating the power exerted on the screw by the screw-driver.

It will be noticed that the length of the screw-driver and the diameter of the screw have nothing whatever to do with the value of the velocity ratio, which depends entirely on the ratio of the distances moved through by the power and weight.

71. The Screw Jack.—In most cases, screws are turned by means of a handle, a wheel, or by a pulley or gear keyed to the screw. Fig. 43 shows what is called a **jackscrew** or **screw-jack.** The stand S forms the nut, and the screw is turned by means of the handle H, the load to be lifted being placed on top of the screw. Jack-screws are used to raise very heavy loads, such as lifting buildings from their foundation; in such cases, the jack-screw is placed under the load to be lifted, and the screw is turned until the load is raised to the desired height. The velocity ratio is calculated exactly the same as above. Let nn' be the axis of the screw; call the distance from nn' to the point of the handle where the power is applied r; then r is the radius of the circle described by the power, and the circumference is $2\pi r$. The velocity ratio is

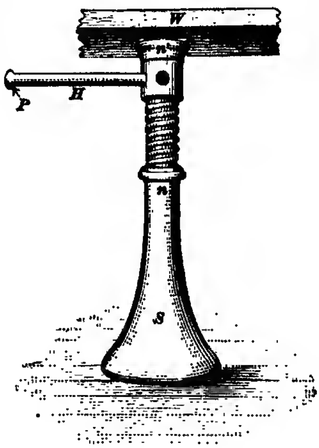

FIG. 43.

$$2\pi r n = \frac{2\pi r}{p}.$$

EXAMPLE.—The screw of a jackscrew has 4 threads per inch, the radius of the circle described by P is 30 in.; what power is required to raise a load of 4500 lb.?

SOLUTION.—The velocity ratio is $r = 2\pi \times 30 \times 4 = 240\pi$. Therefore,

$$P = \frac{W}{r} = \frac{4500}{240 \times 3.1416} = 6 \text{ lb., nearly. } \textit{Ans.}$$

The press rolls of paper machines are usually raised by a combination of lever and screw. The bearing is carried by one arm of a bell crank lever and the other end is moved by a screw which draws or pushes a nut fastened to the power arm; the screw has no motion but rotation.

72. The Endless Screw or Worm and Wheel.—Another application of the screw is shown in Fig. 44; it is called a **worm and wheel.** A worm and wheel form a part of the mechanism for raising the roll in a beater. The screw S is called the **worm** and the toothed wheel T is called the **worm wheel or wheel.** The threads on the worm engage with the teeth on the wheel, and when the worm is turned one revolution by means of the handle H, it causes a point on the circumference of the wheel to turn through an arc equal in length to the pitch of the worm. As shown in the cut, the wheel carries an axle M, which winds up a rope from which is suspended the weight W. The whole, therefore, is a combination of a worm and wheel and a wheel and axle. The worm and wheel is also called an **endless screw,** because the screw may be turned any number of times and the only effect produced is to turn the wheel, the axis of the worm being stationary.

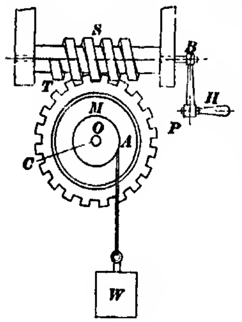

FIG. 44.

To find the velocity ratio of the combination, let $r = PB$ = radius of circle described by P, $r' = OC$ = radius of wheel (pitch circle of), and $r'' = OA$ = radius of axle; then, velocity ratio of worm $= r_w = 2\pi rn$, velocity ratio of wheel and axle $= r_a = \frac{r'}{r''}$, and velocity ratio of the combination $= r_c = 2\pi rn \times \frac{r'}{r''} = \frac{2\pi rr'n}{r''}$, in which $n = \frac{1}{p}$ = reciprocal of pitch of screw.

EXAMPLE.—In the case of an endless screw and wheel and axle, what is the velocity ratio when the radius of the handle is 14 in., radius of wheel is 12 in., radius of axle is 3 in., and the worm has 5 threads per inch? What theoretical weight W can be lifted when P = 15 lb.?

SOLUTION.—The velocity ratio is $r_c = \frac{2 \times 3.1416 \times 14 \times 12 \times 5}{3} = 560\pi$. *Ans.*

The theoretical weight W that can be lifted by an application of 15 lb. to the handle is

$$W = Pr_c = 15 \times 560 \times 3.1416 = 26{,}389 \text{ lb.} \quad \textit{Ans.}$$

It will be observed that the velocity ratio of the worm and wheel is very great; consequently, the distance moved by W is

exceedingly small compared with that moved by the power. For this reason, the worm and wheel is much used in dividing a given distance into very small parts; it is also used as a reducing motion, where a high velocity is changed to a low one, as in the mechanism for rotating digesters.

A combination of the worm, worm wheel and screw is found in the mechanism for raising and lowering the roll of a beater. The bearing is carried near the center of a lever of the second class; the free end carries a fixed nut, as N, Fig. 42 (*b*), in which a screw, having no vertical movement, turns; this raises or lowers the nut and the end of the lever. The head of the screw is a worm wheel as M, Fig. 44, which is turned by the worm S and handle H or a hand wheel. An enormous velocity ratio is thus obtained.

73. The Toggle Joint.—The six simple machines so far described constitute the foundation for all machines, and any machine, no matter how complicated, makes use of one or more of these simple machines.

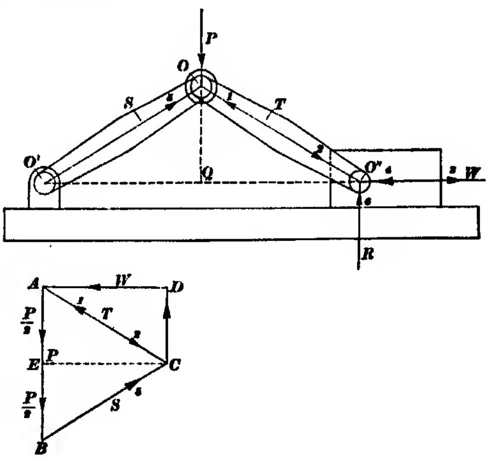

FIG. 45.

There is, however, one other device that is occasionally used, especially in pulp or baling presses, which does not properly come under the head of one of the six simple machines; it is called a **toggle joint** or a **knee joint**, and is shown in Fig. 45. It consists of two bars or arms of equal length and having a common joint O. The other end of arm S is jointed at O' and is

fixed, while the other end of arm T is jointed at O'' and is free to move along the line joining O' and O''. The power is applied at O in a direction perpendicular to $O'O''$; as the joint O moves down, the joint O'' moves out, increasing the distance between O' and O'' and exerting a pressure against the bearing of the joint O'', the horizontal component of which corresponds to the load W.

To find the velocity ratio, first consider the forces acting on joint O; these are the force (power) P and the reactions S and T of the arms, which are equal when the arms are equal. The directions of the reactions are indicated by the arrow heads 1 and 5. Draw AB to represent P to some convenient scale; then draw AC and BC, parallel respectively to T and S; they intersect at C; whence, $BC = S$ and $CA = T$. The joint O'' is also acted on by three forces, the force T (in the direction of the arrowhead 2), the reaction R and the reaction W, the two latter acting in the directions indicated by the arrowheads 4 and 6. Draw CD and AD, parallel respectively to R and W; then, CD represents the reaction R and DA represents the reaction W, both to the same scale that AB represents P.

The triangle $O'OO''$ is isosceles and ACB is also isosceles. Draw CE perpendicular to AB; it will be parallel to $O'O''$, and $AE = EB = \frac{P}{2}$. The triangles OQO'' and AEC are similar right triangles; consequently,

$$T : \frac{P}{2} = OO'' : OQ$$

Let $OQ = h$ and $OO'' = L$ then,

$$T : \frac{P}{2} = L : h, \text{ or } T = \frac{PL}{2h}$$

The triangles OQO'' and CDA are also similar right triangles; consequently,

$$W : T = O''Q : L, \text{ or } W = \frac{T \times O''Q}{L}$$

Let $O'O'' = D$; then, $O''Q = \frac{D}{2}$. Substituting in the expression for W the values of T and $O''Q$, and reducing,

$$W = \frac{PD}{4h} \qquad (1)$$

and

$$r = \frac{W}{P} = \frac{D}{4h} \qquad (2)$$

The smaller h is in comparison with D, the distance between the joints O' and O'', the greater is the velocity ratio; and when h is very small, the force exerted on joint O'' becomes enormous. For example, suppose $D = 28$ in. and $h = \frac{1}{16}$ in.; then $r = \frac{28}{4 \times \frac{1}{16}} = 112$, and $W = 112P$.

EXAMPLES

(1) The length of a smooth inclined plane is 125 ft. and the height is 23 ft.; (*a*) what theoretical force acting parallel to the plane is required to keep a body weighing 2500 lb. from sliding down the plane? (*b*) What work would be done by this force in pulling the body 56 ft. up the plane?

Ans. (*a*) 460 lb. (*b*) 25,760 ft.-lb.

(2) In the preceding example, suppose the force had acted parallel to the base; (*a*) what work would be done in pulling the body 56 ft. up the plane? (*b*) What is the magnitude of the force? (*c*) why is the work done in the two cases equal?

Ans. (*a*) 25,760 ft.-lb. (*b*) 468– lb.

(3) A corner of a building is to be raised by driving a wedge between the sill and the foundation. The wedge is 18 in. long, the back measures 1⅞ in., and a pressure of 652 lb. is applied to it; (*a*) what load will the wedge raise? (*b*) when the wedge has moved 7¾ in., how high has the corner of the building been lifted?

Ans. (*a*) 6259 lb. (*b*) .81 – in.

(4) The jaws of a vise are forced toward each other by means of a screw that has 6 threads per inch; if the distance between the axis of the screw and the point on the handle where the force is applied is 12¼ in., (*a*) what pressure will the jaws exert when the force applied to the handle is 66 lb.? (*b*) what is the velocity ratio?

Ans. (*a*) 30,480 lb. (*b*) 461.8+

(5) Suppose a screw to be attached to the back of a wedge in such manner that when the screw moves, the wedge moves and lifts vertically a load resting on the wedge, as the body M in Fig. 41 (*b*). If the screw has 18 threads per inch, its axis is horizontal, and the slope of the wedge is equivalent to 12 in. horizontal to ¼ in. vertical, how far will the body resting on the wedge move when the screw makes $\frac{1}{30}$th of a turn? *Ans.* $\frac{1}{43200}$th in.

(6) In a toggle joint, the distance between the fixed joint and the movable joint that presses against the load is 38 in., the distance between the line joining these two joints and the middle joint is ⅜ in.; (*a*) what pressure will be exerted when a force of 84 lb. is applied to the middle joint? (*b*) what is the velocity ratio?

Ans. (*a*) 2128 lb. (*b*) 25⅓

BELT PULLEYS AND GEARS

74. Velocity and Speed Ratios of Belt Pulleys.—Belt pulleys are connected and driven by belts. What is called the **face** of the pulley is the part touched by the belt. The face is either flat or else it is a little higher in the middle than at the outside; in other words, the diameter in the middle is greater than at the outside edges. This difference in diameter is termed the **crowning** or **crown,** and its object is to keep the belt from running off the pulley, since a belt always tends to run to the highest point of the pulley face. When measuring the diameter of a pulley, always take the diameter at the middle of the face, i.e., at the top of the crown.

The principal use of belt pulleys is to transmit power; they are seldom used to raise loads, as was the case with the pulleys previously described. The word *power* here has a meaning different from that previously given to it (which was synonymous with force), and is equivalent to rate of doing work—a certain number of foot-pounds of work per second or per minute. (See **Art. 165.**) The problems relating to belt pulleys may be divided into two classes: (1) those relating to the velocity or speed ratio; (2) those relating to the power that can be transmitted by a belt of given dimensions, speed, and material. Only the first class of problems will be considered here.

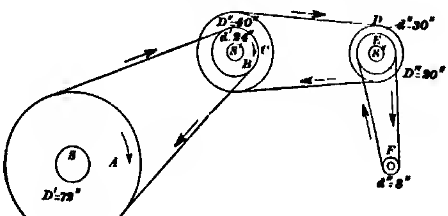

Fig. 46.

75. Referring to Fig. 46, suppose the pulley B to be driven by a belt that, in turn, is driven by the pulley A. Pulley A is then called the **driver** and pulley B is called the **driven pulley** or **driven.** When pulley A is caused to turn in the direction of the arrow, the belt is caused to move in the direction of the arrows by reason of the friction between the belt and the pulley. The friction of the belt on pulley B causes that pulley to turn also

and in the *same* direction as pulley A. If, however, the belt is crossed, the two pulleys will turn in opposite directions. The velocity of the belt will be the same as the velocity of a point on the circumference of pulley A, assuming that there is no *slip* (i.e., sliding of belt on the pulley). As a point on the belt passes around pulley B, it keeps in contact with a point directly underneath it on the pulley as long as the belt is in contact with the pulley; hence, the velocity of a point on the circumference of B is the same as on the circumference of A. In other words, the lineal velocity of the belt and the peripheral velocity of the two pulleys are all equal. The velocity of the belt, then, can be found as soon as the diameter of either pulley and the number of revolutions it makes per minute (r.p.m.) are known. Thus, let d = diameter of pulley in inches; its circumference is πd inches $= \frac{\pi d}{12}$ feet $= \frac{3.1416 \times d}{12} = .2618d$ ft. Let N = number of revolutions per minute (r.p.m.) made by the pulley, and v = the velocity of the belt in feet per minute; then,

$$v = .2618dN$$

For example, if the diameter of one of the pulleys is 56 in. and it makes 180 r.p.m., the velocity of the belt is

$$v = .2618 \times 56 \times 180 = 2639 \text{ ft. per min., very nearly.}$$

76. Since the peripheral velocity of driver and driven is the same, it is evident that if one pulley is larger than the other, it will make a smaller number of r.p.m. than the other. Thus, let D = diameter of larger pulley and N the r.p.m. it makes; let d and n be the same quantities for the smaller pulley; then, $.2618DN = .2618dn$, from which

$$DN = dn \qquad (1)$$

The speed ratio is $s = \frac{D}{d} = \frac{n}{N}$ (2)

Note that the last equation in formula (2) gives the proportion

$$D : d = n : N$$

that is, *the revolutions per minute vary inversely as the diameters.*

77. Referring again to Fig. 46, pulleys B and C are keyed to the same shaft S'; pulleys D and E are keyed to the same shaft S''; hence, pulleys B and C each make the same number of r.p.m., and pulleys D and E each make the same number of r.p.m. A, C, and E are drivers and B, D, and F are driven pulleys

(also called **followers**). Representing the diameters of A, C, and E, the drivers, by D', D'', D''', and the diameters of B, D, and F, the followers, by d', d'', and d''', the speed ratio of pulleys A and B is $\frac{D'}{d'} = s'$, of C and D is $\frac{D''}{d''} = s''$, and of E and F is $\frac{D'''}{d'''} = s'''$. The speed ratio of the entire combination is always equal to the product of the speed ratios of all the separate machines that make up the combination, or, in this case,

$$s_c = s's''s''' = \frac{D'}{d'} \times \frac{D''}{d''} \times \frac{D'''}{d'''} = \frac{D'D''D'''}{d'd''d'''} \qquad (1)$$

In other words, the speed ratio of any combination of pulleys is equal to the product of the diameters of all the drivers divided by the product of the diameters of all the followers. If the number of revolutions per minute made by the first driver is known and is represented by N, the number n made by the last follower will be

$$n = s_c N \qquad (2)$$

and if the number of revolutions per minute made by the last follower is known, the number N made by the first driver is

$$N = \frac{n}{s_c} \qquad (3)$$

If the number of revolutions per minute made by the first driver and last follower are known, the speed ratio of the combination is

$$s_c = \frac{n}{N} \qquad (4)$$

EXAMPLE.—Referring to Fig. 46, suppose the pulleys to have the following diameters: A = 72 in., B = 24 in., C = 40 in., D = 30 in., E = 20 in., and F = 8 in.; if pulley A makes 150 r.p.m., what is the speed of F? what is the speed ratio of the combination?

SOLUTION.—The speed ratio of the combination is, by formula (1),

$$s_c = \frac{72 \times 40 \times 20}{24 \times 30 \times 8} = 10. \quad \textit{Ans.}$$

The number of revolutions per minute made by F is, by formula (2),

$$n = 10 \times 150 = 1500 \text{ r.p.m.} \quad \textit{Ans.}$$

The reason for using the term **speed ratio** instead of velocity ratio is that the velocity ratio is the ratio of the distance that the power moves to the distance that the weight moves. In the case of two pulleys, connected by a belt, whatever their diameters, the peripheral velocities are the same, and the power and

weight move through the same distance; hence, the velocity ratio is always 1. But when two pulleys are keyed to the same shaft and have different diameters, the velocity ratio is equal to the diameter of pulley receiving the power divided by the diameter of pulley transmitting the power that is,

$$r = \frac{d}{D}$$

Therefore, the *velocity ratio* of the combination in Fig. 46 is found as follows: the velocity ratio of pulleys B and C is $r' = \frac{24}{40}$. The velocity ratio of pulleys D and E is $r'' = \frac{30}{20}$; for A and B, for C and D, and for E and F, the velocity ratio is 1; hence the velocity ratio of the entire combination is $1 \times \frac{24}{40} \times \frac{30}{20} \times 1 = .9$. In other words, if the belt connecting A and B exerts an effective pull of 1 lb., the belt connecting E and F will exert an effective pull of .9 lb. From this it will be seen that the speed ratio is a very different quantity from the velocity ratio. The speed ratio relates to revolutions per minute, while velocity ratio relates to peripheral velocities, and determines the ratio of the velocity of the power to the velocity of the load.

78. Suppose that the revolutions per minute of the first driver were known, say a pulley on the main shaft that makes 110 r.p.m., and that it were desired to drive a small emery wheel at 2200 r.p.m. There are two countershafts the diameter of the pulley on the emery-wheel shaft must not be smaller than 6 in., and the diameters of the other pulleys must not exceed 36 in. It is required to find a set of pulleys that will produce the desired result. The first step is to find the speed ratio of the combination; this is, by formula (4) of Art. 77, $s_c = \frac{2200}{110} = 20$. The arrangement of pulleys and shafts is shown in Fig. 47, M being the main shaft, C' and C'' the countershafts, and E the emery-wheel shaft. It is now necessary to find three numbers which, when multiplied together, will give a product of 20; these numbers will be the speed ratios of the parts of the combination. It is desirable, though not necessary, that the three numbers be of approximately the same value; if they were exactly the same, they would be equal to $\sqrt[3]{20} = 2.71+$. Taking one of the numbers as 2.5, $20 \div 2.5 = 8$; calling one of the other two numbers 3, $8 \div 3 = 2\frac{2}{3}$; hence, the three speed ratios may be taken as

(also called **followers**). Representing the diameters of A, C, and E, the drivers, by D', D'', D''', and the diameters of B, D, and F, the followers, by d', d'', and d''', the speed ratio of pulleys A and B is $\frac{D'}{d'} = s'$, of C and D is $\frac{D''}{d''} = s''$, and of E and F is $\frac{D'''}{d'''} = s'''$. The speed ratio of the entire combination is always equal to the product of the speed ratios of all the separate machines that make up the combination, or, in this case,

$$s_c = s's''s''' = \frac{D'}{d'} \times \frac{D''}{d''} \times \frac{D'''}{d'''} = \frac{D'D''D'''}{d'd''d'''} \qquad (1)$$

In other words, the speed ratio of any combination of pulleys is equal to the product of the diameters of all the drivers divided by the product of the diameters of all the followers. If the number of revolutions per minute made by the first driver is known and is represented by N, the number n made by the last follower will be

$$n = s_c N \qquad (2)$$

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If the number of revolutions per minute made by the first driver and last follower are known, the speed ratio of the combination is

$$s_c = \frac{n}{N} \qquad (4)$$

EXAMPLE.—Referring to Fig. 46, suppose the pulleys to have the following diameters: $A = 72$ in., $B = 24$ in., $C = 40$ in., $D = 30$ in., $E = 20$ in., and $F = 8$ in.; if pulley A makes 150 r.p.m., what is the speed of F? what is the speed ratio of the combination?

SOLUTION.—The speed ratio of the combination is, by formula (1),

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$$n = 10 \times 150 = 1500 \text{ r.p.m.} \quad \textit{Ans.}$$

The reason for using the term **speed ratio** instead of velocity ratio is that the velocity ratio is the ratio of the distance that the power moves to the distance that the weight moves. In the case of two pulleys, connected by a belt, whatever their diameters, the peripheral velocities are the same, and the power and

imaginary, in that they are not shown on the gears, but they must be shown on working drawings giving dimensions for making the gears.

80. Pitch of Gears.—What is called the **circular pitch** of a gear is the distance from the edge (or center) of one tooth to the corresponding edge (or center) of the next tooth, *measured on the pitch circle;* it is the length of the circular arc aa' or bb',

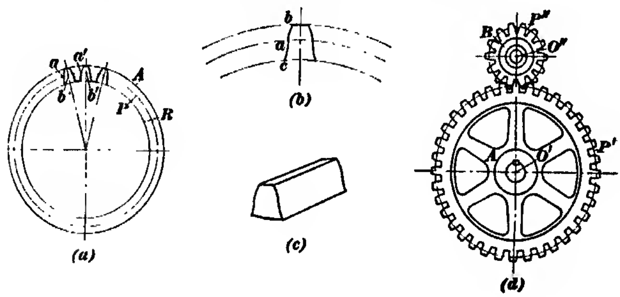

FIG. 48.

Fig. 48 (*a*). Let d = diameter of pitch circle, n = number of teeth in the gear, and p_c = the circular pitch; then, since the teeth must be equally spaced around the gear,

$$p_c = \frac{\pi d}{n} \qquad (1)$$

What is called the **diametral pitch** is the number of teeth in the wheel divided by the diameter of the wheel, the diameter being expressed in inches. Let p_d = diametral pitch, d = diameter of gear in inches, and n = number of teeth; then,

$$p_d = \frac{n}{d} \qquad (2)$$

Note that circular pitch is a length, while diametral pitch is a ratio; the diameters of gears are always expressed in inches when the English system of measures is used. Thus, a gear having 72 teeth and a diameter of 12 in. will have a circular pitch of $p_c = \frac{\pi \times 12}{72} = .5236$ in., and the diametral pitch will be $p_d = \frac{72}{12} = 6$. The latter result may be expressed as 6 teeth per inch of diameter, but a gear having this diametral pitch would usually be called a *6-pitch gear.*

The relation between the diametral pitch and the circular

pitch is easily found. Thus, from formula (2), $\frac{1}{p_d} = \frac{d}{n}$; substituting this value of $\frac{d}{n}$ in formula (1),

$$p_c = \frac{\pi}{p_d}, \qquad (3)$$

from which

$$p_c d_d = \pi \qquad (4)$$

For instance, in the last paragraph, the diametral pitch of the gear was 6 and the circular pitch was .5236; then, $.5236 \times 6 = 3.1416 = \pi$.

When making a drawing of a gear, it is necessary to use the circular pitch and to lay off this distance *on* the pitch circle; hence, if the diametral pitch is known, the circular pitch can be found by dividing π by the diametral pitch. Thus, the circular pitch of an 8-pitch gear will be $\frac{3.1416}{8} = .3927$ in.

81. Shape of Teeth.—The sides of a gear tooth are curved surfaces as indicated in Fig. 48 (*c*). A cross section through the tooth perpendicular to the axis of the gear will usually have an outline similar to that shown at (*a*) or (*b*). This outline, called the **tooth profile,** has been given different shapes by different designers, but in most cases, it belongs to one of two systems of gear teeth, the **cycloidal system** and the **involute system.** In the cycloidal system, the profile is made up of two curves; the upper part *ab*, Fig. 48 (*b*), is a segment of an *epicycloid,* and the lower part *ac* is a segment of a *hypocycloid.* These are frequently called **double-curved teeth.** In the involute system, the entire profile is a single curve, which is a segment of an *involute* of a circle. These are frequently called **single-curved teeth.** Lack of space prevents further discussion of these shapes.

82. Velocity Ratio of Gears.—When two gears having the same pitch and the same kind of profiles are so placed that their axes are parallel and a tooth of one gear fits the space between two teeth of the other gear, they are said to be *in mesh.* If one gear is caused to turn, each of its teeth presses in turn against the teeth of the other gear and causes it to turn also. If the gears are properly placed, the two pitch circles will be tangent to each other, as shown in Fig. 48(*d*). The lineal velocities of the two pitch circles will be equal, one gear will turn in the *opposite* direction from that of the other, and the result will be exactly the same as though the gears were pulleys driven by a crossed

belt, the diameters of the pulleys being the same as the diameters of the pitch circles of the gears. The velocity ratio of two gears in mesh is always 1, because the peripheral velocities are alike and the pressure exerted by the teeth in contact is the same on both gears. The case is exactly the same as that of a compound lever with the fulcrum in the center of both levers; the weight lifted will then be exactly equal to the power exerted. If, however, there are two gears keyed to the same shaft and they have different diameters, then the velocity ratio will be equal to the diameter of the pitch circle of the gear receiving the power (a follower) divided by the diameter of the pitch circle of the gear transmitting the power (a driver) representing these diameters by D and d respectively,

$$r = \frac{d}{D} \qquad (1)$$

If the pitch diameters of the gears are not known, it is usually difficult to determine them accurately; but it is easy to count the number of teeth in a gear. Since the number of teeth is directly proportional to the diameter, let N = number of teeth in the driver and n = number of teeth in the follower; then

$$r = \frac{n}{N} \qquad (2)$$

The speed ratios are found in exactly the same manner as in the case of belt pulleys, substituting the number of teeth instead of the diameters in the formulas of Art. **77**. Thus, formula (1) becomes

$$s_r = \frac{N'N''N'''}{n'n''n'''} \qquad (3)$$

83. Idlers.—If three gears are in mesh, so that gear A meshes with gear B, and gear B meshes with gear C, the only effect produced by gear B is to change the *direction of rotation* of gear C; it has no effect on the speed ratio or on the velocity ratio, and for this reason is called an **idle gear** or **idler.** In the case of four gears A, B, C, and D in mesh, B and C will be idlers, and D will turn in the same direction as though it meshed with gear A. The reason for using two idlers is to obviate the use of two large gears, which would be necessary if the distance between the shafts remains the same.

84. Gear Trains.—Referring to Fig. 49, the circles C, D, E, and F, represent the pitch circles of gears, and circles A, B, G and H represent belt pulleys. Pulley B and gear C are keyed to the

same shaft, as also are gears D and E, and gear F and pulley G. A set of gears connected in this manner form what is called a **train of gears**, a **gear train**, or simply a **train**. Since a gear is equivalent to a pulley having the same diameter as the pitch circle of the gear, if it is desired to find the speed ratio of the combination, the product of the diameters and number of teeth of the drivers divided by the product of the diameters and number of teeth of the followers will equal the speed ratio. Thus, suppose the diameters of the drivers A and G are 60 in. and 15,

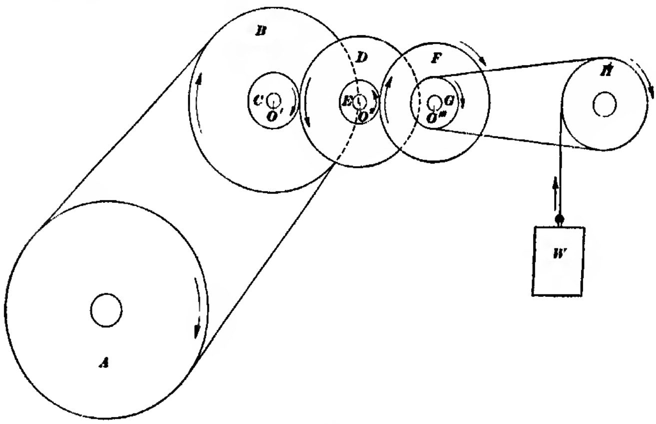

FIG. 49.

of the followers B and H are 48 in. and 24 in., number of teeth in the drivers C and E is 16 and 12, and in the followers D and F is 36 and 30; then,

$$s_c = \frac{60 \times 16 \times 12 \times 15}{48 \times 36 \times 30 \times 24} = \frac{5}{36}$$

If, therefore, pulley A make 180 r.p.m., pulley H will make $180 \times \frac{5}{36} = 25$ r.p.m.

To find the velocity ratio of the combination, pulley B and gears D and F are followers, gears C and E and pulley G are drivers, and the velocity ratio of the combination is

$$r_c = \frac{48 \times 36 \times 30}{16 \times 12 \times 15} = 18$$

Hence, if the effective pull of the belt connecting A and B is 200 lb., the load W that can be lifted is $200 \times 18 = 3600$ lb.

85. Kinds of Gears.—The gears so far described are called **spur gears**; the axes of these gears are always parallel. When

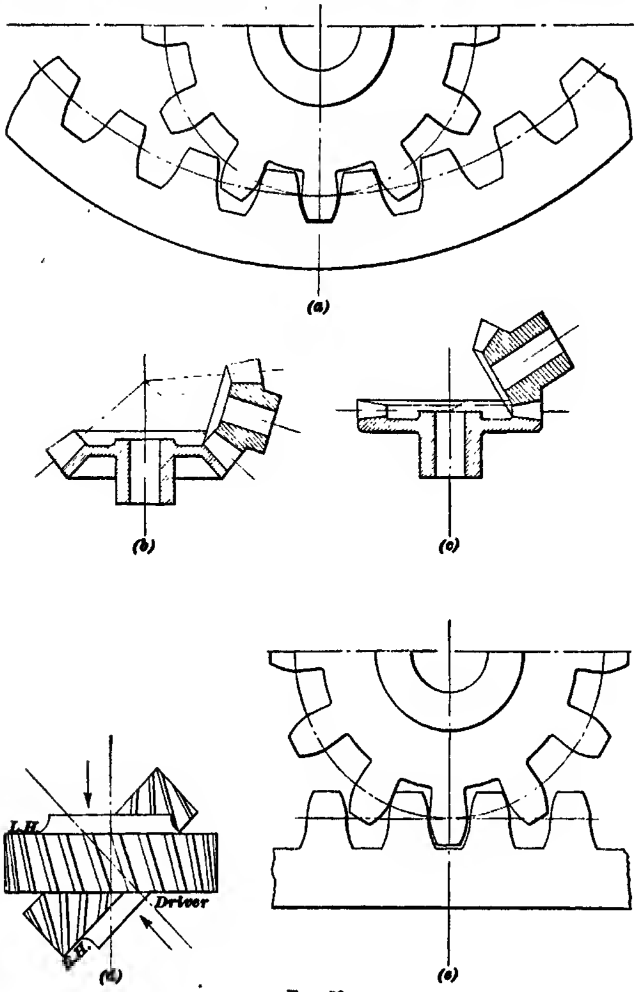

FIG. 50.

the axes lie in the same plane and intersect, as shown at (*b*), Fig. 50, the gears are called **bevel gears.** When one gear turns

inside the other, as shown at (a), Fig. 50, the **gears are called internal, or annular, gears.** When one gear **has a flat face,** as in (c), it is called a **crown gear.** The wheel in the case of a worm and wheel is called a **worm gear.** In all these cases, except the last, the speed ratio may be found by taking the ratio of the number of teeth in the gears that mesh. When the axes do not lie in the same plane and do not intersect as shown at (*d*), Fig. 50, the gears are called **helical gears,** though frequently, but erroneously, they are called **spiral gears.** The teeth of helical gears have helical surfaces; that is, the edges of certain sections taken through the teeth will be helices instead of right lines. Helical gears may be (and are) used to connect shafts whose axes are parallel, and also those which intersect, being used in place of spur and bevel gears; a worm gear is a special case of a helical gear. The speed ratio of helical gears depends upon the design, which varies, and rules for determining it must be omitted here. A straight bar with teeth cut in it to mesh with a spur gear is called a **rack**; see (*e*), Fig. 50. The spur gear that meshes with the rack is called a **pinion.** The smaller of any two gears in mesh is also commonly called the pinion gear or pinion.

FRICTION

86. Kinds of Friction.—Up to this point, it has been assumed that equilibrium was produced by the action of *active* forces, and that the slighest increase or decrease in any one of the forces would result in causing the body to move, since the equilibrium would then be destroyed. In actual practice, however, this is not the case, since a *passive* force, called **friction** is always present and always acts on any body in motion. It must be overcome before any motion can result. A simple experiment will show some of the effects and laws of friction.

Referring to Fig. 51, an iron block of weight W is shown resting on top of a wooden table, the table being flat and level (horizontal). The block has the shape of a prism, the three dimensions *ab*, *ac*, and *ad* being different. A string is attached to the block, passes over a small pulley, and a weight w is attached to the other end of the string. The weight w may be a pail into which sand or water can be poured until its weight is just sufficient to cause the block to move. If there were no friction, the weight w would be extremely small; theoretically, it would be 0. On trying

the experiment, however, it will be found that w is always a measurable quantity, and its magnitude represents the force of friction.

It will likewise be found that if the weight of W be increased, say by placing another and equal weight on top of the block, the weight w must also be increased the same amount; for instance, if W be doubled, w will also be doubled, and the force of friction will be twice as great as before. Since the pressure exerted by the block is normal (perpendicular) to the surface of the table top, one of the laws of friction is thus made evident: *friction is directly proportional to the normal p essure exerted by a body sliding on another body.*

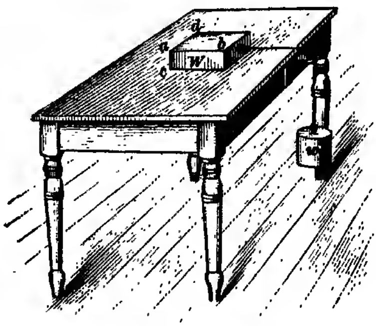

Fig. 51.

Friction of the kind just mentioned is called **sliding friction**, and is always created when one body *slides* on another. When a body *rolls* on another body, as when a wheel rolls on a flat surface, or when a ball rolls on a flat surface or in a bearing, another kind of friction is created, called **rolling friction.** Again, when water or other liquid or fluid flows through a pipe or channel, it meets with a resistance, called **fluid friction.**

87. Cause of Friction.—Friction is caused by the fact that every surface, no matter how smooth it apparently may be, is really a succession of little humps and depressions, as may be seen when examined under a microscope. Consequently, when one surface moves over another, the result is somewhat like drawing a heavy wagon over a rough and rocky road. It is important to note that friction does not exist except when there is motion; the force of friction, therefore, never tends to produce motion, but always tends to prevent or destroy it. Friction always acts *contrary* to the force that produces the motion. Since a force cannot act on a body without producing some effect on it, and since the force of friction never tends to produce motion, what does it do to the body on which it acts? the answer to this question is *heat!* Again,

since the friction exists as long as the body moves, the result is the same as though a force acted through a distance, that is, work is done; and this work is changed into heat energy. Ordinarily, the heat thus created will be dissipated into the surrounding atmosphere; but, if the velocity of the moving body is high and the pressure is considerable, the bodies in contact will become heated, especially in the case of journals and their bearings, the result being that the journals become quite hot, which causes them to expand, thus increasing the normal pressure. This increases their temperature, causes them to expand still more, and so on, the final result being that the bearings may melt or the journals may stick, thus stopping the machine.

88. Coefficient of Friction.—Let P = the normal pressure and F = the force of friction; then, the ratio of F to P is called the **coefficient of friction,** which is usually represented by the Greek letter μ (pronounced *mu*). From this definition,

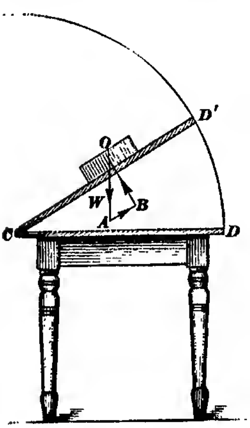

FIG. 52.

$$\mu = \frac{F}{P} \qquad (1)$$

Referring to Art. 86 and Fig. 51, $P = W$ and $F = w$; hence, in the case there described,

$$\mu = \frac{w}{W}$$

If, then, the coefficient of friction and the normal pressure are known, the force of friction can be found, since, by formula (1),

$$F = \mu P \qquad (2)$$

89. Experimental Determination of the Coefficient of Friction.—The coefficient of friction may be determined in the manner previously described; but it is not easy, because there is friction between the string and the pulley and between the pulley journals and their bearings. These last two factors may be eliminated in the following manner: Referring to Fig. 52, suppose the iron block to rest on a horizontal plane surface, which is the upper surface of a board CD or other material that is to be tested and which is hinged at C. By raising the end at D, it will swing through the arc DD', and a point will be reached such that the

slightest increase in the angle $D'CD$ will cause the block to move down the plane. This angle is called the **angle of friction,** and experiment shows that for the same materials, with the contact surfaces in the same condition, this angle is constant; that is, it has the same value regardless of the normal pressure. Draw OA to represent the weight of the block, which acts vertically downwards; then BO, drawn perpendicular to the surface, represents the reaction on the block, and AB, parallel to the surface, represents the force that keeps the block from moving. These three forces produce equilibrium; OB, equal and opposite to BO, is the normal pressure, and AB is the force of friction, both to the same scale that OA represents the weight. Let P = the normal pressure and F = the force of friction; then,

$$F : P = AB : OB$$

or $$F = P \times \frac{AB}{OB}$$

But, $$F = P \times \mu$$

hence, $$\mu = \frac{AB}{OB}$$

The triangle OBA is a right triangle, right-angled at B, and the angle $AOB = D'CD$, the angle of inclination of the surface to the horizontal. In trigonometry, the ratio of the side opposite an acute angle of a right triangle to the other short side is called the **tangent** of the angle; hence, the tangent of the angle O (expressed as tan O) is $\tan O = \frac{AB}{OB}$, and

$$\mu = \tan O = \tan D'CD$$

Therefore, if a table giving the values of the tangents of angles is at hand, and the angle of inclination has been found by experiment, the tangent of this angle will be the coefficient of friction. Thus, for a cast-iron block sliding on an oak surface, it will be found that the angle $D'CD$ is about 26°; the tangent of 26° is .4877; hence, the coefficient of friction of cast iron on oak is .49, to two significant figures, which is close enough for all practical purposes.

90. The coefficient of friction is frequently, perhaps usually, expressed as a per cent. In the case just mentioned, the coefficient of friction of cast iron on oak would then be 49 per cent. Referring to Art. **88**, it was shown that $\mu = \frac{w}{W}$; from which,

$$w = \mu W$$

since the friction exists as long as the body moves, the result is the same as though a force acted through a distance, that is, work is done; and this work is changed into heat energy. Ordinarily, the heat thus created will be dissipated into the surrounding atmosphere; but, if the velocity of the moving body is high and the pressure is considerable, the bodies in contact will become heated, especially in the case of journals and their bearings, the result being that the journals become quite hot, which causes them to expand, thus increasing the normal pressure. This increases their temperature, causes them to expand still more, and so on, the final result being that the bearings may melt or the journals may stick, thus stopping the machine.

88. Coefficient of Friction.—Let P = the normal pressure and F = the force of friction; then, the ratio of F to P is called the **coefficient of friction,** which is usually represented by the Greek letter μ (pronounced *mu*). From this definition,

$$\mu = \frac{F}{P} \qquad (1)$$

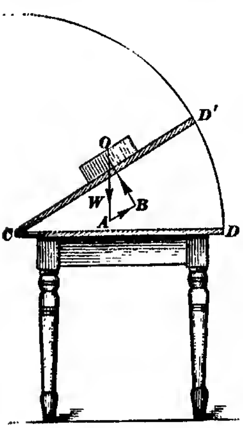

FIG. 52.

Referring to Art. 86 and Fig. 51, $P = W$ and $F = w$; hence, in the case there described,

$$\mu = \frac{w}{W}$$

If, then, the coefficient of friction and the normal pressure are known, the force of friction can be found, since, by formula (1),

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that is, the friction of cast iron on brass, for example, is not exactly the same as the friction of brass on cast iron.

6. Friction is greater between rough surfaces than between smooth surfaces. Consequently, friction may be diminished by polishing the surfaces in contact; also, by placing between them a lubricant, such as oil, grease, graphite, etc.

92. Journal Friction.—Journal friction is a special case of sliding friction, the journal sliding around its bearing. Usually, the journal presses against only one half of the bearing, as indicated in Fig. 53, the other half being merely a cover. The total load on the bearing may be represented by P, and it is distributed over the surface $ABCD$, being greatest at B and 0 at A and C. The average pressure on the bearing per unit of area, called the *bearing pressure*, is equal to P divided by the projected area of the journal JJ'. Thus, let d = diameter of journal and l = length of journal; then, the projected area = ld, and

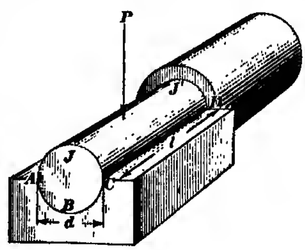

FIG. 53.

$$p_b = \frac{P}{ld}$$

in which (when l and d are measured in inches) p_b is the bearing pressure per square inch.

Assuming that the bearings are well and properly lubricated, the value of p_b must not exceed about 800 lb. per sq. in.; otherwise, the lubricant will be forced out, the bearings will heat, and the lining, which is generally of some soft material (brass, Babbitt metal, phosphor bronze, etc.) will melt or will expand so much as to cause the journal to stick.

93. Rolling Friction.—Rolling friction is very much less than sliding friction; for this reason, roller bearings and ball bearings are used when feasible whenever it is desired to reduce the friction as much as possible. As with sliding friction, rolling friction is directly proportional to the normal pressure. Let W = the weight of the wheel and any load that it may carry, μ_r = coefficient of rolling friction, r = radius of wheel, roller or ball, and P = force required to overcome the rolling friction; then.

$$\text{rolling friction} = \mu_r W = P$$

The force P is assumed to act at the center of the wheel, and the wheel turns about the point of contact A, Fig. 54, as a center. For a very slight movement, therefore, A may be taken as the origin of moments, and the moment of P about A is Pr. In order to produce equilibrium, it is necessary for this turning moment to be counterbalanced by an equal and opposite moment, which may be considered to be represented by $\mu_r W$, W being the force and μ_r the arm of the moment. Placing these two moments equal to each other,

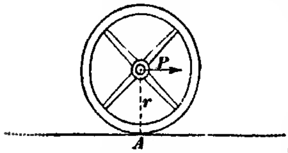

FIG. 54.

$$Pr = \mu_r W$$

or

$$P = \frac{\mu_r W}{r}$$

This formula shows that the greater the radius of the wheel, the smaller will be the value of P, the force required to overcome the friction.

The coefficients of rolling friction vary from .001 to about .005. It is said to be noted that while sliding friction can be reduced by means of lubricants, this is not so with rolling friction; but, nevertheless, the harder and smoother the surfaces in contact the less will be the rolling friction.

EFFICIENCY

94. Theoretical Power Required to Lift Load.—It will be evident from what has preceded that the actual force necessary to raise a load by means of a machine is greater than that required to produce equilibrium when all hurtful resistances are neglected, because whenever there is movement there is friction, and friction may be considered as a force that is acting in opposition to the power; hence, a greater power is required to raise a load than would be required if there were no friction. The power required when friction and other hurtful resistances are neglected is called the **theoretical power**; representing it by P and the actual power by P', the ratio of the theoretical power to the actual power is called the **efficiency** of the machine. The efficiency is almost invariably represented by the Greek letter η (pronounced ayta): hence,

$$\eta = \frac{P}{P'}$$

For instance, suppose in the case of a block and tackle, the theoretical force, as found by calculation, required to raise a certain load is 32 lb., while the actual force required to raise the same load is 40 lb.; then, the efficiency is

$$\eta = \frac{32}{40} = .8, \text{ or } 80\%$$

As in the case of the coefficient of friction, efficiencies are generally expressed as a per cent.

95. Hurtful Resistances.—A **hurtful resistance** is any force that tends to oppose the motion or impede the action of a machine and which is not considered in finding the velocity ratio. For example, referring to the gear and pulley train of Fig. 49, Art. **84**, pulleys A and B are not considered when calculating the velocity ratio; but when finding the efficiency of the entire combination, they must be considered, because they increase the number of hurtful resistances. If all the hurtful resistances are here considered, they must include the journal friction of pulleys A and H and of the shafts carrying the three sets of gears, the force required to bend the belts around the pulleys, the effect of centrifugal force on the belts, the friction of the belts on the pulleys in case of the belts slipping, the friction of the gear teeth, the bending of the rope around pulley H, and one or two others that are never considered in practice, because their effects are so small that they are neglible or cannot be measured. It is evidently a very difficult matter to measure accurately all the hurtful resistances. In the case of Fig. 49, the best plan to pursue would be to ascertain what pull on the belt connecting pulleys A and B is required to raise the weight W; call this pull P'. The theoretical pull is $\frac{W}{r}$, where r = the velocity ratio of the combination; then, the efficiency may be taken as $\eta' = \frac{P}{P'}$. The actual efficiency η will probably differ slightly from η', but the difference is so slight that it may be entirely neglected in practice.

96. Efficiency of any Combination of Machines.—The efficiency of any combination of machines is equal to the product of the efficiencies of each machine making up the combination; the efficiency of any set of combinations is equal to the product of the efficiencies of each separate combination. For example, suppose that a steam engine drives a main shaft, several countershafts, and, finally, a rotary drying furnace. If the efficiency of

the engine (running without connection to the main shaft) is $\eta' = 85\%$, of the shafting, pulleys, etc., is $\eta'' = 92\%$, and of the furnace with its gears, etc., is $\eta''' = 90\%$, the efficiency of the entire combination is

$$\eta = \eta'\eta''\eta''' = .85 \times .92 \times .90 = .7038 = 70.38\%$$

97. Another Method of Computing Efficiency.—Instead of computing the efficiency by finding the ratio of the theoretical power to the actual power required to raise a load, it is frequently more convenient to find the ratio of the works; that is, suppose the power to act through a distance s' and that this causes the load to be raised (or the resistance to be overcome) through a distance s''; the work done on the machine is Ps'; the work done by the machine is Ws'', W representing the load or resistance overcome; then,

$$\eta = \frac{Ws''}{Ps'} \qquad (1)$$

The value of the efficiency obtained by this method is exactly the same as by the previous method. For, let r = the velocity ratio; then, $s' = s''r$; P is the actual power, and corresponds to P' in the formula of Art. **94**; $\frac{W}{r} = P$ of Art. **94**; hence,

$$\eta = \frac{Ws''}{P's''r} = \frac{W}{P'r} = \frac{\frac{W}{r}}{P'} = \frac{P}{P'}$$

Therefore, if the work supplied to the machine be denoted by L', and the work done by the machine by L,

$$\eta = \frac{L}{L'} \qquad (2)$$

The value of the efficiency obtained by formulas (1) and (2) and the formula of Art. **94** is called the **mechanical efficiency.**

EXAMPLES

(1) The flywheel of an engine is 84 in. in diameter and makes 160 r.p.m.; it connects by belt with a 56-inch pulley on the main shaft; (*a*) how many revolutions per minute does the main shaft make? A 36-inch pulley on the main shaft drives a 28-inch pulley on the countershaft; (*b*) how many r.p.m. does the countershaft make? A 36-inch pulley on the countershaft drives a 12-inch pulley on a band saw; (*c*) how many r.p.m. does the 12-inch

pulley make? (*d*) what is the velocity ratio of the combination? (*e*) the speed ratio?

Ans. (*a*) 240 r.p.m.; (*b*) $308\frac{4}{7}$ r.p.m.; (*c*) $925\frac{5}{7}$ r.p.m.; (*d*) 1.21 –; (*e*) 5.786 –

(2) A main shaft makes 180 r.p.m. and a pulley on the spindle of a lathe makes 30 r.p.m.; if the pulley is 18 in. in diameter, find diameters of a set of pulleys for the main shaft and countershaft. What is the (*a*) speed ratio? (*b*) the velocity ratio of the combination? *Ans.* (*a*) $\frac{1}{6}$.

(3) What is (*a*) the addendum circle? (*b*) the root circle? (*c*) the pitch circle?

(4) The diametral pitch of a spur gear is 5, (*a*) what is the circular pitch? (*b*) If the gear has 40 teeth, what is its diameter?

Ans. (*a*) .6283 in.; (*b*) 8 in.

(5) What is (*a*) the cause of friction? (*b*) what should be the greatest bearing pressure per unit of projected area?

(6) In a gear train, the number of teeth in the drivers D′, D″, D‴ is 16, 36, and 30 respectively; the number of teeth in the followers *F*′, *F*″, F‴ is 32, 90, and 18 respectively; what is (*a*) the speed ratio of the combination? (*b*) the velocity ratio?

Ans. (*a*) $\frac{1}{3}$; (*b*) $2\frac{2}{3}$

(7) In the last Question, suppose gear F‴ is keyed to the leadscrew of a lathe and gear D′ is keyed to the lathe spindle; the leadscrew works in a nut attached to the carriage, and when the leadscrew turns, the carriage moves; how far will the carriage move when gear D′ makes one turn? The leadscrew has 6 threads per inch. *Ans.* $\frac{1}{18}$ in.

(8) If the efficiencies of the various mechanisms that make up a machine are 92%, 87%, $66\frac{2}{3}$%, and 96%, what is the efficiency of the machine? *Ans.* 51.23 – %.

CENTER OF GRAVITY

CENTER OF GRAVITY OF LINES

98. Definition.—Suppose *ABCD*, Fig. 55, to be a thin, flat, iron plate having the shape of a rectangle and lying in a horizontal plane. Suppose further that this plate be divided into a very large number of little squares, all equal; then, the weight of each square may be represented by w. If the number of squares is n, the weight of all the squares will be nw, and this must equal W, the weight of the plate. The weight w of one of these squares represents a vertical force that is exerted on the square

by the action of gravity; these forces are all parallel and equal, and are indicated by the little arrows. Each little arrow, which completely represents a force, acts at the geometrical center of the square whose weight it represents. There are, therefore, n parallel forces acting on the plate; the resultant of these parallel forces is $nw = W = R$, and it now remains to be shown how to find the position of this resultant.

Bisect the rectangle by drawing pq parallel to AB; also bisect it by drawing mn parallel to BC; these two lines are axes of symmetry and intersect in O the geometrical center of the rectangle. Moreover, O is also the point through which the line of action of the resultant R must pass, in this case, because, if the plate be assumed to be balanced on a knife edge extending along pq, it is evident that the forces on the right of pq will exactly balance those on the left of pq, the case being exactly the same as that of a lever with equal arms; hence, the resultant must pass through some point on pq. For the same reason, it must also pass through some point on mn; it must, therefore, pass through their point of intersection O. If, then, the plate be suspended from the point O, either by placing a pivot directly under O or by attaching a string to the plate directly over O, the plate will balance; that is, it will, when stationary and when so placed, lie in a horizontal plane. The slightest increase in weight anywhere, no matter how small or where situated, will cause the plate to tip, the extra weight causing that part on which it lies to tip *downwards*.

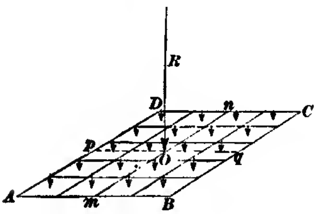

Fig. 55.

The point O is called the **center of gravity** of the plate; its nature is such that if any right line be drawn through O in the plane of the plate, the moment of the part of the plate on one side of the line is equal to the moment of the part on the other side of the line. It is for this reason that the intersection of any two such lines determines the center of gravity of an area.

99. The Right Line.—A right line may be considered as the axis of a straight wire. If such a wire be balanced on a knife edge, the point on the axis directly over the knife edge will be the mid-

dle point of the axis, the case being exactly the same as that of a lever with equal arms; hence, the center of gravity of a right line is a point on the line midway between its ends, that is, the center of the line.

100. The Broken Line.—To find the center of gravity of a broken line, as *ABCDEF*, Fig. 56, the easiest method is the following: bisect each of the lines *AB*, *BC*, etc. in *O*, *P*, *Q*, *R*, and *S*, and these points will be centers of gravity of the lines composing the broken line. Through one of these points, as *O*, draw a

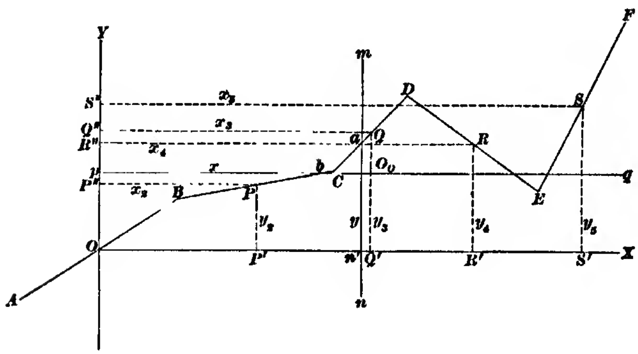

FIG. 56.

horizontal line *OX*, called the **axis of X,** and a vertical line *OY*, called the **axis of Y.** These lines are also called the **axis of abscissas** and the **axis of ordinates,** respectively. From *O*, *P*, *Q*, etc. draw perpendiculars to *OX* and *OY*, and denote the lengths of these perpendiculars by y_1, y_2, y_3, etc. and by x_1, x_2, x_3, etc. Let l_1, l_2, l_3, etc. denote the lengths of *AB*, *BC*, *CD*, etc. Let y denote the distance of the center of gravity of the entire line from *OX*, and let x denote the distance of the center of gravity of the entire line from *OY*; then,

$$y = \frac{l_1y_1 + l_2y_2 + l_3y_3 + l_4y_4 + \text{etc.}}{l_1 + l_2 + l_3 + l_4 + \text{etc.}} \quad (1)$$

and
$$x = \frac{l_1x_1 + l_2x_2 + l_3x_3 + l_4x_4 + \text{etc.}}{l_1 + l_2 + l_3 + l_4 + \text{etc.}} \quad (2)$$

For example, suppose that l_1, l_2, l_3, etc. equal 1.76 in., 1.55 in., 1.08 in., 1.37 in., 1.60 in., that y_1, y_2, y_3, etc. equal 0 in., .52 in.,

1.04 in., .96 in., 1.31 in., and that x_1, x_2, x_3, etc. equal 0 in. 1.50 in., 2.67 in., 3.65 in., 4.53 in.; then,

$$y = \frac{1.76\times 0 + 1.55\times .52 + 1.08\times 1.04 + 1.37\times .96 + 1.60\times 1.31}{1.76+1.55+1.08+1.37+1.60} = .726 \text{ in.}$$

$$x = \frac{1.76\times 0 + 1.55\times 1.50 + 1.08\times 2.67 + 1.37\times 3.65 + 1.60\times 4.53}{1.76+1.55+1.08+1.37+1.60} = 2.372 \text{ in.}$$

NOTE.—It will be observed that the distances x_1 and y_1 are equal to 0 in Fig. 56, because the axes of X and Y pass through the center of gravity of the segment AB.

To locate the center of gravity O_0 on the drawing, lay off on OY $Op = y = .726$ in., draw pq parallel to OX, and lay off $pO_0 = x = 2.372$ in. Or, lay off on OX, $On' = 2.372$ in., draw $n'm$ parallel to OY, and lay off $n'O_0 = y = .726$ in.

101. Formulas (1) and (2) of the last article are so important that a rather full discussion of them is advisable. While a line, no matter how long, has no weight, it is assumed that each of the short lines that forms a part of the broken line has a certain weight that is proportional to its length. The entire line thus tends to turn about the line OX as an axis, called the **axis of moments.** The moment of any one of these lines about OX as an axis is therefore equal to its length multiplied by the distance of its center of gravity from OX; and the sum of these moments is the numerator in formula (1). It is now assumed that the moment of the entire line, which is equal to its entire length multiplied by the distance of its center of gravity from OX is equal to the sum of the moments of the individual parts of the line, and experiment shows this to be true. Consequently, letting y = distance of center of gravity of entire line from OX, and L = length of this line, $L = l_1 + l_2 + l_3 + l_4 + l_5 +$ etc.; whence, $L \times y = l_1y_1 + l_2y_2 + l_3y_3\ l_4y_4 + l_5y_5 +$ etc., and

$$y = \frac{l_1y_1 + l_2y_2 + l_3y_3 + l_4y_4 + l_5y_5 + \text{etc.}}{L}$$

which is the same as formula (1).

The line also tends to turn about the line OY as an axis, and by proceeding in exactly the same manner as above, exchanging the distances x, x_1, x_2, etc. for y, y_1, y_2, etc., it will be found that

$$x = \frac{l_1x_1 + l_2x_2 + l_3x_3 + l_4x_4 + l_5x_5 + \text{etc.}}{L}$$

which is the same as formula (2).

102. It will be noted that the center of gravity of the line does not lie on the line, but at a point O_0. If the broken line be assumed to be the center line of a round, fine wire, of such stiffness that it will not bend under its weight, and that it be connected to the point O_0 by fine wires aO_0 and bO_0 that have no weight, but will support the wire L, this wire will be in equilibrium when supported at O_0; hence, O_0 is the center of gravity of the wire L, and, consequently, of the broken line that forms its center line. The line, however, will balance on a knife edge laid along pq or mn, and their intersection is the point O_0.

There are many cases where the center of gravity lies entirely outside of the line, area, or body. In all such cases, the center of gravity may be considered as the point in which the entire length, area, or mass of the body or system of bodies may be concentrated to produce equilibrium.

103. When drawing the axes of moments OX and OY, it is advisable, when practicable, to draw them through a center of gravity of one of the lines; then the distance from this point to the axis of moments is 0, and one of the terms in the numerator of the formula will disappear, thus making the calculation easier. These axes may be drawn anywhere, and it is not necessary that they be at right angles to each other, provided the lines drawn through the centers of gravity of the individual lines are *parallel* to the axes; but it is customary and easier to draw one horizontal and the other vertical, as shown in Fig. 56. Should the centers lie on both sides of either or both axes, the moments on one side of an axis must be considered as positive and those on the other side as negative, since they tend to turn the line in opposite directions. This may be avoided by selecting the position of the axes so that the centers of gravity of the individual parts will all lie on the same side of either axis.

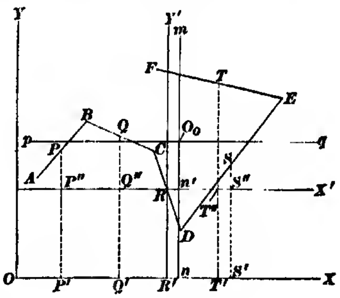

Fig. 57.

It may also be observed that it is not necessary to draw the lines PP'', QQ'', etc., because the lengths of these lines are equal to

the distances OP', OQ', etc., which may be measured instead of the former lines.

EXAMPLE.—In Fig. 57, let OX and OY be the axes of moments, and let the lengths of AB, BC, etc. be .93 in., .93 in., 1.02 in., 2.03 in., and 1.60 in.; let P, Q, etc. be the centers of gravity of the individual lines, and PP' = 1.46 in., QQ' = 1.67 in., RR' = 1.04 in., SS' = 1.33 in., and TT' = 2.30 in.; let OP' = .67 in., OQ' = 1.31 in., OR' = 1.94 in., OS' = 2.72 in., and OT' = 2.53 in. Find the position of the center of gravity O_0 of the broken line $ABCDEF$. Also, draw parallel axes through R, and find the position of O_0 with reference to these axes, and show that it has the same position with reference to the broken line as in the first case.

SOLUTION.—The total length of the broken line is .93 + .93 + 1.02 + 2.03 + 1.60 = 6.51 in. Then,

$$y = \frac{.93\times1.46+.93\times1.67+1.02\times1.04+2.03\times1.33+1.60\times2.30}{6.51} = 1.590 \text{ in.}$$

$$x = \frac{.93\times.67+.93\times1.31+1.02\times1.94+2.03\times2.72+1.60\times2.53}{6.51} = 2.057 \text{ in.}$$

Lay off On = 2.057 in., draw nm parallel to OY, and lay off nO_0 = 1.59 in.; then, O_0 is the center of gravity of the broken line $ABCDEF$.

Through R, draw RX' and RY' parallel to OX and OY; then the distances PP'', QQ'', etc. are equal to $PP' - P''P' = PP' - RR'$, $QQ'' = QQ' - RR'$, etc. Similarly, $RP'' = OP' - OR' = -R'P'$, $RQ'' = OQ' - OR' = -R'Q'$ $RS'' = OS' - OR' = R'S'$, and $RT'' = OT' - OR' = R'T'$. The work of performing these subtractions is most conveniently arranged as follows:

PP'' = 1.46 − 1.04 =	.42 in.	RP'' = .67 − 1.94 =	− 1.27 in.
QQ'' = 1.67 − 1.04 =	.63 in.	RQ'' = 1.31 − 1.94 =	− .63 in.
SS'' = 1.33 − 1.04 =	.29 in.	RS'' = 2.72 − 1.94 =	.78 in.
TT'' = 2.30 − 1.04 =	1.26 in.	RT'' = 2.53 − 1.94 =	.59 in.

The distance of R from the point of intersection of the axes is 0. Substituting these values in the formulas,

$$y' = \frac{.93\times.42+.93\times.63+1.02\times0+2.03\times.29+1.60\times1.26}{6.51} = .550 \text{ in.}$$

$$x' = \frac{.93\times-1.27+.93\times-.63+1.02\times0+2.03\times.78+1.60\times.59}{6.51} = .117 \text{ in.}$$

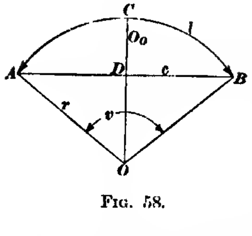

FIG. 58.

Lay off $Rn' = x'$ = .117 in.; it falls on mn, because $Rn' = R'n$, and $OR' + R'n$ = 1.94 + .117 = 2.057 in. = $On = x$; hence, on nm, lay off $n'O_0$ = .550 in.; then nO_0 = $nn' + n'O_0$ = $RR' + n'O_0$ = 1.04 + .550 = 1.590 = y. From this it is seen that either calculation gives the same position for the center of gravity O_0. It is to be noted that distances measured to the *right* of RY' are positive or +, while those measured to the *left* are negative or −; those measured from RX' *upward* are positive or +, while those measured *downward* are negative or −.

104. The Circular Arc.—In Fig. 58, let ACB be a circular arc whose length = l, chord = c, radius = r, and whose center is O.

Draw OD bisecting the chord; it will also bisect the arc and will be an axis of symmetry of the arc. The center of gravity must therefore lie on the bisecting radius OC; its distance from the center O is $\frac{rc}{l}$, or

$$OO_0 = \frac{rc}{l} \qquad (1)$$

Denoting the angle AOB by v, $l = rv$, when v is in radians, and

$$OO_0 = \frac{rc}{rv} = \frac{c}{v} \qquad (2)$$

If v be measured or expressed in degrees, $v = \frac{\pi v^\circ}{180}$, and

$$OO_0 = \frac{180c}{\pi v^\circ} = \frac{57.296c}{v^\circ} \qquad (3)$$

For a semicircle, $c = 2r$ and $v^\circ = 180°$; therefore,

$$OO_0 = \frac{180 \times 2r}{\pi \times 180} = .63662r \qquad (4)$$

EXAMPLE.—If the chord of an arc is 10.74 in. long and the angle at the center is 127° how far is the center of gravity from the center of the arc?

SOLUTION.—Applying formula (3),

$$OO_0 = \frac{57.296 \times 10.74}{127} = 4.845 \text{ in. } Ans.$$

104. Regular Curved Lines.—For a closed curve of regular outline as a circle or ellipse, or any plane curve having two or more axes of symmetry, the center of gravity will be at the intersection of the axes of symmetry. This same statement applies to any plane figure, as a regular polygon, having two or more axes of symmetry.

105. Irregular Curved Lines.—There is no general method for finding the exact position of the center of gravity of an irregular curved line, a part or all of which curved. The approximate method is difficult of application, and since the center of gravity of such a line is very seldom required, the method is omitted.

CENTER OF GRAVITY OF PLANE AREAS

106. Symmetrical Areas.—The center of gravity of any plane area that has two axes of symmetry lies at the point of intersection of those axes; thus, the area of the figure shown in Fig. 59

has two axes of symmetry *mn* and *pq*, and the center of gravity lies at O_0, the point of intersection of these axes. The figure really has any number of axes of symmetry, since any line drawn though the point O_0 will be an axis of symmetry with respect to the point O_0. From this, it is evident that, since any regular polygon may be inscribed in a circle, the center of gravity of any regular polygon also lies at the center of the circumscribing circle, because a circle has any number of axes of symmetry, all of which intersect at the center.

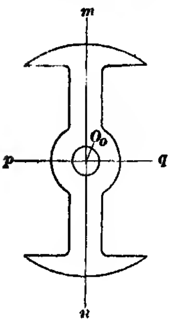

FIG. 59.

In general, any figure that has two axes of symmetry has an infinite number of them with respect to a point, the point being the center of gravity; by two axes of symmetry is usually meant two axes at right angles to each other.

107. Rectangles and Regular Polygons.—Since an axis of symmetry bisects the figure, the distances from an axis to corresponding symmetrical points on either side of the axis are equal (by definition of symmetry). Consequently, if a line be drawn through the center of gravity of a rectangle parallel to the two long sides, the distance from this line to either long side is one-half the short side, since the line is an axis of symmetry; similarly, if a line be drawn through the center of gravity parallel to the two short sides, the distance of this line from the short sides will be one-half the long side. Thus, if *b* be a long side and *d* a short side, the distance of the center of gravity from either long side will be $\frac{d}{2}$, and the distance from either short side will be $\frac{b}{2}$.

For a regular polygon, the distance from the center of gravity to any side is equal to the apothem (the distance from the center of the circumscribed circle to the middle point of a side.)

108. The Triangle.—If a line be drawn from any vertex of a triangle to the middle point of the side opposite (thus bisecting that side), the line is called a **medial line.** Every triangle has three medial lines, as *Aa*, *Bb*, *Cc*, Fig. 60; and it is proved in geometry that these three medial lines intersect in a common

point O_0. It is also proved in geometry that $O_0a = \frac{Aa}{3}$, that $O_0b = \frac{Bb}{3}$, and $O_0c = \frac{Cc}{3}$. But, O_0 is the center of gravity of the triangle; hence, the center of gravity of any triangle may be found by drawing a medial line, as Bb, and then measuring back a distance bO_0 on this line equal to one-third of its length. Or, draw any two medial lines; their point of intersection will be the center of gravity. Thus, the point of intersection of Bb and Aa is O_0, the center of gravity of the triangle.

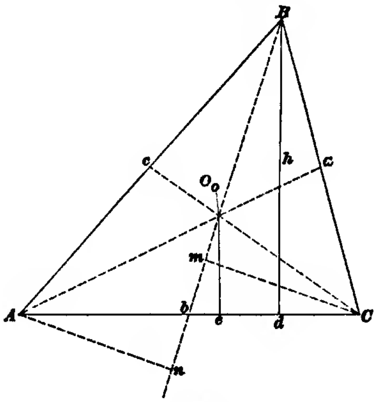

FIG. 60.

Let $Bd = h$, the altitude of the triangle, and let O_0e be the normal distance from the center of gravity to the base AC; then $O_0e = \frac{h}{3}$. The same result will be obtained if either of the other two sides be taken as the base.

If b = length of base AC, area of triangle $= A = \frac{bh}{2}$; from which, $h = \frac{2A}{b}$. Since $O_0e = \frac{h}{3}$,

$$O_0e = \frac{2A}{3b}$$

a formula that may be used to calculate the normal distance of the center of gravity from any side when the length of the side and the area of the triangle are known. For example, if the area of a triangle is 11.4 sq. in. and the length of one of the

has two axes of symmetry mn and pq, and the center of gravity lies at O_o, the point of intersection of these axes. The figure really has any number of axes of symmetry, since any line drawn though the point O_0 will be an axis of symmetry with respect to the point O_0. From this, it is evident that, since any regular polygon may be inscribed in a circle, the center of gravity of any regular polygon also lies at the center of the circumscribing circle, because a circle has any number of axes of symmetry, all of which intersect at the center.

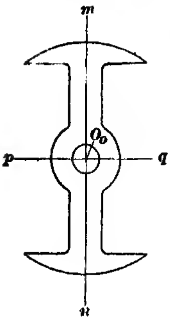

FIG. 59.

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107. Rectangles and Regular Polygons.—Since an axis of symmetry bisects the figure, the distances from an axis to corresponding symmetrical points on either side of the axis are equal (by definition of symmetry). Consequently, if a line be drawn through the center of gravity of a rectangle parallel to the two long sides, the distance from this line to either long side is one-half the short side, since the line is an axis of symmetry; similarly, if a line be drawn through the center of gravity parallel to the two short sides, the distance of this line from the short sides will be one-half the long side. Thus, if b be a long side and d a short side, the distance of the center of gravity from either long side will be $\frac{d}{2}$, and the distance from either short side will be $\frac{b}{2}$.

For a regular polygon, the distance from the center of gravity to any side is equal to the apothem (the distance from the center of the circumscribed circle to the middle point of a side.)

108. The Triangle.—If a line be drawn from any vertex of a triangle to the middle point of the side opposite (thus bisecting that side), the line is called a **medial line.** Every triangle has three medial lines, as Aa, Bb, Cc, Fig. 60; and it is proved in geometry that these three medial lines intersect in a common

= length of longer side AD, l'' = length of shorter side BC, and h = altitude Bd = normal distance between the parallel sides; then,

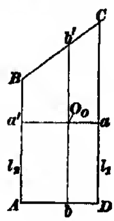
$$O_0c = \frac{(2l'' + l')h}{3(l' + l'')} \qquad (1)$$

EXAMPLE.—The lengths of the parallel sides of a trapezoid are 6.3 in. and 9.7 in.; if the altitude is 5.2 in., what is the distance of the center of gravity from the longer side?

SOLUTION.—Substituting in the formula the values given,

$$O_0c = \frac{(2 \times 6.3 + 9.7)5.2}{3(9.7 + 6.3)} = 2.416 \text{ in.} \quad Ans.$$

A special case, and one that occurs with considerable frequency, is when one of the sides of the trapezoid is perpendicular to the two parallel sides, as in Fig. 62. Here AD is perpendicular to the parallel sides AB and CD. This figure may be called a *semi-rectangular trapezoid*. The distance of the center of gravity O_0 from CD is given by the preceding formula; the distance from AD, the perpendicular side, may be found by the following formula, in which l_1 = the longer and l_2 = the shorter of the two parallel sides:



$$O_0b = \frac{l_1^2 + l_1l_2 + l_2^2}{3(l_1 + l_2)} \qquad (2)$$

FIG. 62.

EXAMPLE.—If the length of shorter side of a semi-rectangular trapezoid is $6\frac{1}{4}$ in., of the longer side $8\frac{1}{2}$ in., and the altitude (perpendicular distance between the parallel sides), is $5\frac{3}{8}$ in., what is the distance of the center of gravity from the longer side? also from the perpendicular side?

SOLUTION.—Applying formula (1),

$$O_0a = \frac{(2 \times 6.25 + 8.5)5.375}{3(8.5 + 6.25)} = 2.551 - \text{in.} \quad Ans.$$

Applying formula (2),

$$O_0b = \frac{8.5^2 + 8.5 \times 6.25 + 6.25^2}{3(8.5 + 6.25)} = 3.716 + \text{in.} \quad Ans.$$

To locate the center of gravity, draw $b'b$ parallel to CD and at a distance from it equal to 2.55 in.; draw $a'a$ perpendicular to CD (and parallel to AD) and at a distance from AD of 3.72 in.; the point of intersection of these two lines will be the center of gravity O_0.

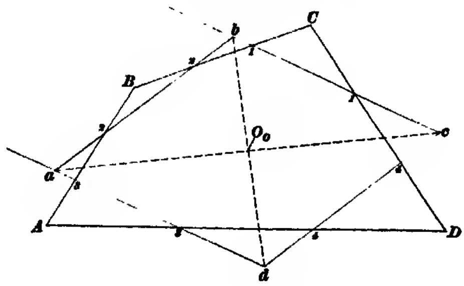
If desired, formula (2) may be written as follows, a form that is somewhat easier to calculate,

$$O_0b = \tfrac{1}{3}\left(l_1 + l_2 - \frac{l_1l_2}{l_1 + l_2}\right) \qquad (3)$$

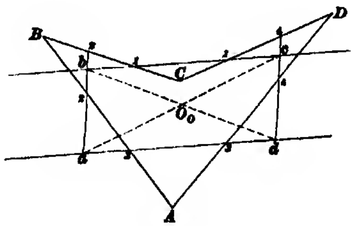
Substituting in this formula the values given in the last example,

$$O_0b = \tfrac{1}{3}\left(8.5 + 6.25 - \frac{8.5 \times 6.25}{8.5 + 6.25}\right) = 3.716+ \text{ in.}$$

111. Any Quadrilateral.—Divide each of the four sides of the quadrilateral, see (*a*) and (*b*), Fig. 63, in three equal parts, thus locating the points 1, 1 on either side of the vertex *C*, 2, 2 on either side of the vertex *B*, etc. Through these four pairs of points, draw lines which, by their intersections, form a parallelo-



(*a*)



(*b*)

Fig. 63.

gram *abcd*. The intersection of the diagonals of the parallelogram locates the point O_0, which is the center of gravity of the quadrilateral. This construction may be applied to the trapezoid, if desired, instead of the one given in Art. **110.** There is no formula for calculating the distance of O_0, the center of gravity, from one of the sides, in terms of the sides. If it is desired to calculate the position of the center of gravity, it must be done by the method of Art. **113.**

112. The Sector and Segment of a Circle.—Let AOB, Fig. 64, be any sector of a circle and ACB a segment, both sector and segment having the same central angle v. Both have one common axis of symmetry, the radius OC perpendicular to the chord AB; hence, the center of gravity of either must lie on OC. For the sector, let v = central angle in degrees, θ = the same angle in radians, r = radius of arc, $c = AB$ = chord of arc, l = length of arc, A = area of sector; then, the distance of O_0', the center of gravity of the sector, from the center of the arc is

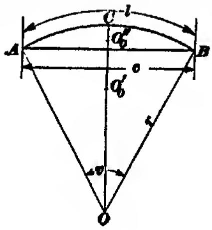


Fig. 64.

$$OO_0' = \frac{2c}{3\theta} = \frac{38.197c}{v} = \frac{2rc}{3l} = \frac{r^2c}{3A} \qquad (1)$$

For the segment, the distance of O_0'', the center of gravity, from the center of the arc is

$$OO_0'' = \frac{c^3}{6rl - 3c\sqrt{4r^2 - c^2}}$$
$$= \frac{c^3}{.10472r^2v - 3c\sqrt{4r^2 - c^2}} = \frac{c^3}{12A} \qquad (2)$$

In formula (1), A = area of sector; in formula (2), A = area of segment.

Example.—On a certain drawing, the central angle (measured with a protractor) was found to be about 54°, the radius is 15½ in., and the chord was found to be $14\frac{5}{64}$ in.; find the distances from the center of the arc of the centers of gravity of the sector and segment.

Solution.—Using formula (1) to find the center of gravity of the sector,

$$OO_0' = \frac{38.197 \times 14\frac{5}{64}}{54} = 9.96 \text{ in.} \quad Ans.$$

Using formula (2) to find the center of gravity of the segment,

$$OO_0'' = \frac{(14\frac{5}{64})^3}{.10472 \times 15.5^2 \times 54 - 3 \times 14\frac{5}{64}\sqrt{4 \times 15.5^2 - (14\frac{5}{64})^2}}$$
$$= 14.52 \text{ in.} \quad Ans.$$

For a semicircle, formulas (1) and (2) reduce to

$$OO_0' = OO_0'' = \frac{2c}{3\theta} = \frac{2 \times 2r}{3\pi} = \frac{4r}{3\pi} = .42441r \qquad (3)$$

113. Any Plane Area.—If the figure has an axis of symmetry, the center of gravity lies on that axis; if it has two axes of symmetry, the center of gravity lies at their point of intersection. If the figure has such a shape that the center of gravity cannot be

found by the methods previously given, but can be subdivided into triangles, rectangles, etc. whose centers of gravity can be found, the center of gravity of the entire figure can be found by the method of moments, as explained in Art. **100**, substituting

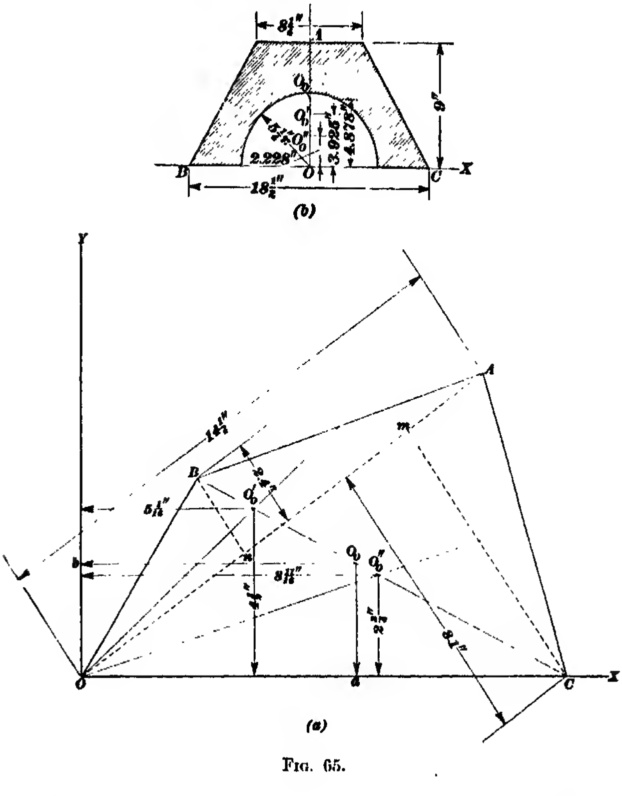


FIG. 65.

areas for lengths in formulas (1) and (2). The process is best illustrated by examples.

Referring to (*a*), Fig. 65, which represents a trapezium, divide it into two triangles by drawing one of the diagonals, say OA. Take OA as the base of both triangles, and draw and measure the altitudes Cm and Bn. Draw medial lines and locate the centers of gravity O_0' and O_0'' of the triangles. Taking the side OC as the axis of X, draw a perpendicular to OC at O for the axis of Y,

and measure the distances of O_0' and O_0'' from the axes; these distances, together with the length of the base OA and the altitudes, are all marked on figure.

$$\text{Area of triangle } OBA = \frac{14.25 \times 2.4}{2} = 17.1 \text{ sq. in.}$$

$$\text{Area of triangle } OCA = \frac{14.25 \times 8.1}{2} = 57.71 \text{ sq. in.}$$

$$\text{Area of trapezium} = 17.1 + 57.71 = 74.81 \text{ sq. in.}$$

$$\text{Then, } y = O_0a = \frac{17.1 \times 4.5 + 57.71 \times 2.75}{74.81} = 3.15 \text{ in.} \quad \textit{Ans.}$$

$$\text{and} \quad x = O_0b = \frac{17.1 \times 5\tfrac{1}{16} + 57.71 \times 8\tfrac{11}{16}}{74.81} = 7.86 \text{ in.} \quad \textit{Ans.}$$

As another example, take the area shown in (*b*), Fig. 65. This area is a trapezoid from which a semicircular segment has been cut out, as shown. The figure is symmetrical about the radius OA, which is taken as the axis of Y, the base BC being taken as the axis of X. The area of the figure = area of trapezoid − area of semicircle. Taking the moments of these areas as they stand, letting A = area of figure, A' = area of trapezoid, A'' = area of semicircle, O_0O = distance of c.g. (center of gravity) of A from BC, the axis of X; $O_0'O$ = distance of c.g. of A' from BC; and $O_0''O$ = distance of c.g. of A'' from BC,

$$A \times O_0O = A' \times O_0'O - A'' \times O_0''O$$

$$\text{from which,} \quad O_0O = \frac{A' \times O_0'O - A'' \times O_0''O}{A}$$

$$\text{Area of trapezoid} = \frac{8.25 + 18.5}{2} \times 9 = 120.375 \text{ sq. in.} = A'$$

$$\text{Area of semicircle} = \tfrac{1}{2}\,\pi \times 5.25^2 = 43.295 \text{ sq. in.} = A''$$

$$\text{Area of figure} = 120.375 - 43.295 = 77.08 \text{ sq. in.} = A$$

$$O_0'O = \frac{(2 \times 8.25 + 18.5)9}{3(8.25 + 18.5)} = 3.925 \text{ in.}$$

$$O_0''O = .42441 \times 5.25 = 2.228 \text{ in.} \quad \text{(See Art. } \mathbf{112}.)$$

$$\text{Then,} \quad O_0O = \frac{120.375 \times 3.925 - 43.295 \times 2.228}{77.08} = 4.878 \text{ in.}$$

Ans.

Note that whenever an area is subtracted, its moment is also subtracted.

EXAMPLE.—Fig. 66 is a working drawing of a plane sectional area; find the position of the center of gravity.

SOLUTION.—Let A = area of figure, B = area of trapezoid *abcd*, C = area of trapezoid *efgh*, D = area of parallelogram *hi*, E = area of parallelogram

gj, F = area of rectangle $seqr$, G = area of trapezoid $knml$, and H = area of quadrant epq; then, $A = B - C - D - E + F - G + H = \frac{9.25 + 15.5}{2} \times 10.5 - \frac{8.875 + 11.125}{2} \times 4 - 3.25 \times 1.75 - 3.25 \times 1.75 + 6.5 \times 11.25 - \frac{9.5 + 12}{2} \times 3.5 + \frac{1}{4}\pi \times 6.5^2 = 129.94 - 40 - 5.69 - 5.69 + 73.13 - 37.63 + 33.18 = 147.24$ sq. in. Taking the position of the moment axes OX and OY as shown, OX being sr produced, find the distances of the centers of gravity of these areas from the axes as follows: Distance of c.g. of B from

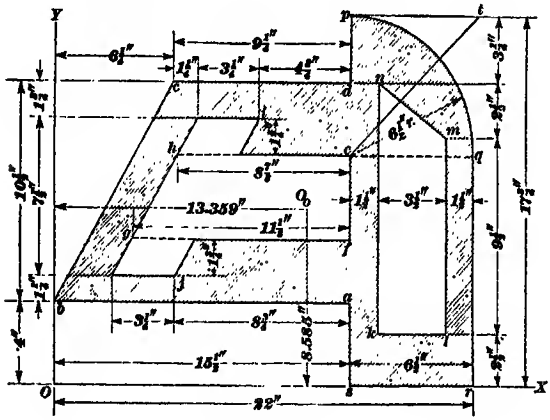


FIG. 66.

$OX = 4 + \frac{(2 \times 9.25 + 15.5)10.5}{3(9.25 + 15.5)} = 8.808$ in.; c.g. of B from $OY = 15.5$ $-$ c.g. of B from $sp = 15.5 - \frac{1}{3}\left(9.25 + 15.5 - \frac{9.25 \times 15.5}{24.75}\right) = 9.181$ in.; c.g. of C from $OX = 7 + \frac{(2 \times 8.875 + 11.125)4}{3(8.875 + 11.125)} = 8.925$ in.; c.g. of C from $OY = 15.5 - \frac{1}{3}\left(8.875 + 11.125 - \frac{8.875 \times 11.125}{20}\right) = 10.479$ in.; c.g. of D from $OX = 11 + \frac{1.75}{2} = 11.875$; c.g. of D from $OY = 15.5$ $- \frac{8.875 + 4.75}{2} = 8.688$ in.; c.g. of E from $OX = 4 + 1.25 + \frac{1.75}{2} = 6.125$ in.; c.g. of E from $OY = 15.5 - \frac{8.75 + 11.125}{2} = 5.563$ in.; c.g. of F from $OX = \frac{11.25}{2} = 5.625$ in.; c.g. of F from $OY = 15.5 + \frac{6.5}{2} = 18.75$ in.; c.g. of G from $OX = 2.5 + \frac{1}{3}\left(9.5 + 12 - \frac{9.5 \times 12}{21.5}\right) = 7.899$ in.; c.g. of G from $OY = 17 + \frac{(2 \times 9.5 + 12)3.5}{3(9.5 + 12)} = 18.682$ in.; c.g. of H from e,

measured along et, which bisects arc pq and makes an angle of 45° with eq and ep, is (by formula 1, Art. 112) $\frac{38.197 \times 6.5\sqrt{2}}{90} = 3.901$ in., since the chord of a quadrant is $r\sqrt{2}$; the distance of this point from ep or eq is $3.901 \times \sqrt{\frac{1}{2}} = 2.759$ in.; hence, c.g. of H from $OX = 11.25 + 2.759 = 14.009$; and distance of c.g. from $OY = 15.5 + 2.759 = 18.259$ in.

Consequently, the distance of the center of gravity of the figure from OX is

$$\frac{129.94 \times 8.808 - 40 \times 8.925 - 5.69 \times 11.875 - 5.69 \times 6.125 + 73.13 \times 5.625 - 37.63 \times 7.899 + 33.18 \times 14.009}{147.24}$$

$= 8.585$ in. *Ans.*

The distance from OY is

$$\frac{129.94 \times 9.181 - 40 \times 10.479 - 5.69 \times 8.688 - 5.69 \times 5.563 + 73.13 \times 18.75 - 37.63 \times 18.682 + 33.18 \times 18.259}{147.24}$$

$= 13.357$ in. *Ans.*

CENTER OF GRAVITY OF SOLIDS

114. Simple Solids.—As stated in *Elementary Applied Mathematics*, if a solid have three planes of symmetry, one of which is at right angles (perpendicular) to the other two, the point of intersection of the three planes is the center of gravity of the solid. Hence, the center of gravity of a right prism whose bases are regular polygons is at the middle point of the axis; this is also true for a right cylinder whose bases are circles or ellipses. For an oblique prism (or oblique cylinder) whose bases are any plane figure, find the center of gravity of both bases, join them by a right line, and the center of gravity will lie at the middle point of the line. Thus, referring to (a), Fig. 67, $O'O''$ is the line joining the centers of gravity of the two bases of the prism. A plane parallel to the two bases and midway between them will be a plane of symmetry and must contain the c.g. of the prism; this plane intersects $O'O''$ in O_0, the c.g. of the prism.

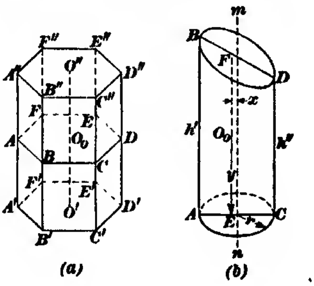


FIG. 67.

If a right cylinder be cut by a plane making an angle with the base, called a **truncated cylinder,** as in (b), Fig. 67, the center of gravity will no longer lie in the axis mn. Let $h' = AB$, the longest element of the truncated cylinder, $h'' = CD$, the shortest element (which will be diametrically opposite AB), and $r =$ the

gj, F = area of rectangle *seqr*, G = area of trapezoid *knml*, and H = area of quadrant *epq*; then, $A = B - C - D - E + F - G + H = \frac{9.25 + 15.5}{2} \times 10.5 - \frac{8.875 + 11.125}{2} \times 4 - 3.25 \times 1.75 - 3.25 \times 1.75 + 6.5 \times 11.25 - \frac{9.5 + 12}{2} \times 3.5 + \frac{1}{4}\pi \times 6.5^2 = 129.94 - 40 - 5.69 - 5.69 + 73.13 - 37.63 + 33.18 = 147.24$ sq. in. Taking the position of the moment axes OX and OY as shown, OX being *sr* produced, find the distances of the centers of gravity of these areas from the axes as follows: Distance of c.g. of B from

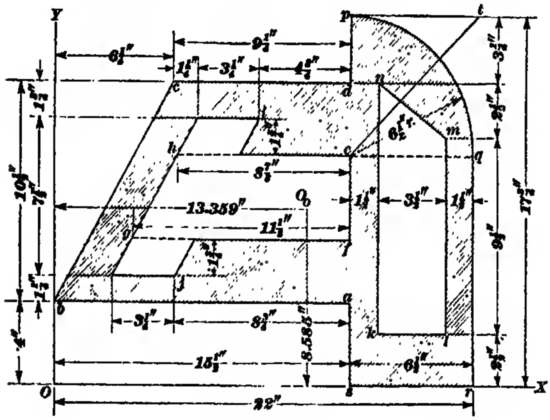


FIG. 66.

$OX = 4 + \frac{(2 \times 9.25 + 15.5)10.5}{3(9.25 + 15.5)} = 8.808$ in.; c.g. of B from $OY = 15.5$ − c.g. of B from $sp = 15.5 - \frac{1}{3}\left(9.25 + 15.5 - \frac{9.25 \times 15.5}{24.75}\right) = 9.181$ in.; c.g. of C from $OX = 7 + \frac{(2 \times 8.875 + 11.125)4}{3(8.875 + 11.125)} = 8.925$ in.; c.g. of C from $OY = 15.5 - \frac{1}{3}\left(8.875 + 11.125 - \frac{8.875 \times 11.125}{20}\right) = 10.479$ in.; c.g. of D from $OX = 11 + \frac{1.75}{2} = 11.875$; c.g. of D from $OY = 15.5 - \frac{8.875 + 4.75}{2} = 8.688$ in.; c.g. of E from $OX = 4 + 1.25 + \frac{1.75}{2} = 6.125$ in.; c.g. of E from $OY = 15.5 - \frac{8.75 + 11.125}{2} = 5.563$ in.; c.g. of F from $OX = \frac{11.25}{2} = 5.625$ in.; c.g. of F from $OY = 15.5 + \frac{6.5}{2} = 18.75$ in.; c.g. of G from $OX = 2.5 + \frac{1}{3}\left(9.5 + 12 - \frac{9.5 \times 12}{21.5}\right) = 7.899$ in.; c.g. of G from $OY = 17 + \frac{(2 \times 9.5 + 12)3.5}{3(9.5 + 12)} = 18.682$ in.; c.g. of H from e,

SE or $S'E'$ (if E does not fall outside the base), lay off EF (or $E'F'$) $= \frac{SE}{4}$, and pass a plane MN through F parallel to the base; it intersects SO' in O_0, the center of gravity of the prism (or cone). The line $F'FO_0$ will lie in this plane.

116. The Frustum.—Referring to Fig. 68, let the area of the upper base of a frustum of a pyramid or cone be A'', of the lower base A', and the altitude h; then, the distance y of the c.g. from the lower base ($= QG$ in the figure) is

$$y = \left(\frac{A' + 2\sqrt{A'A''} + 3A''}{A' + \sqrt{A'A''} + A''}\right)\frac{h}{4} \qquad (1)$$

If the frustum is that of a right cone with circular bases, let R = radius of lower base and r = radius of upper base; then

$$y = \left(\frac{R^2 + 2Rr + 3r^2}{R^2 + Rr + r^2}\right)\frac{h}{4} \qquad (2)$$

Example.—How far from the lower base is the center of gravity of a frustum of a cone of revolution if the radius of the lower base is 19¼ in., of the upper base 12½ in., and the altitude is 14 in.?

Solution.—Substituting in formula (2),

$$y = \left(\frac{19.25^2 + 2 \times 19.25 \times 12.5 + 3 \times 12.5^2}{19.25^2 + 19.25 \times 12.5 + 12.5^2}\right)\frac{14}{4} = 6.023\text{–in.} \quad \textit{Ans.}$$

117. Rectangular Prismoid.—Fig. 69 represents a prismoid whose bases are rectangles. Let $AB = b'$, $AD = d'$, $A'B' = b''$, and $A'D' = d''$. If O' and O'' are the centers of gravity of the bases, the c.g. of the prismoid lies in $O'O''$ at a distance y from the lower base. Letting h = the altitude of the prismoid,

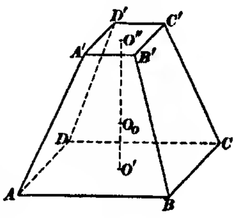


Fig. 69.

$$y = \left(\frac{b'd' + 3b''d'' + b'd'' + b''d'}{2b'd' + 2b''d'' + b'd'' + b''d'}\right)\frac{h}{2} \qquad (1)$$

Example.—Suppose the dimensions of a rectangular prismoid are as follows: $b' = 34.5$ in., $d' = 21.25$ in., $b'' = d'' = 16.5$ in., and $h = 40$ in. What is the distance of the c.g. from the lower base?

Solution.—Substituting in the formula the values given,

$$y = \frac{34.5 \times 21.25 + 3 \times 16.5^2 + 34.5 \times 16.5 + 16.5 \times 21.25}{2 \times 34.5 \times 21.25 + 2 \times 16.5^2 + 34.5 \times 16.5 + 16.5 \times 21.25}$$
$$= 16.854 \text{ in.} \quad \textit{Ans.}$$

If the upper base be a line only and parallel to AB and CD of the lower base, the side $A'D' = B'C' = 0$, and the prismoid becomes a wedge, in which case, formula (1) reduces to

$$y = \left(\frac{b' + b''}{2b' + b''}\right)\frac{h}{2} \qquad (2)$$

and if $b' = b''$,

$$y = \frac{h}{3} \qquad (3)$$

117. Solid of Revolution.—In general, the position of the center of gravity of a solid of revolution can be found only approximately, the best practical method being to apply Simpson's rule as indicated in the following: This rule was applied to areas in *Elementary Applied Mathematics*, but it can be applied equally well to volumes by dividing the given volume into any even number of parallel slices, making them all of the same thickness, by passing parallel planes through the solid at equal distances apart; the areas of the sections thus formed are substituted in the formula instead of the ordinates y_0, y_1, y_2, etc. Thus, referring to Fig. 70, suppose the outline to represent the projection of a solid on the plane of the paper. The lines AA', BB', CC', etc. represent the projections of plane sections at equal distances apart. Let h = altitude $O'O''$; then; if n = number of slices (n must be an even number), the thickness of each slice is $\frac{h}{n}$, which is equal to h in the formula of Simpson's rule. Referring to the figure, let area of bottom section = A_0, of the next section A_1, of the next A_2, etc.; then the volume of the solid is

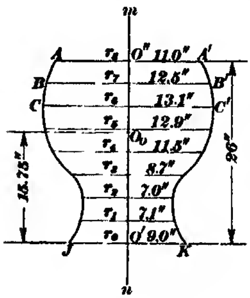


FIG. 70.

$$V = \frac{h}{3n}[A_0 + A_n + 4(A_1 + A_3 + A_5 + \cdots + A_{n-1}) + 2(A_2 + A_4 + A_6 + \cdots A_{n-2})] \quad (1)$$

This formula may be applied to any solid, provided the slices are taken sufficiently thin, but for most purposes 8 or 10 will be sufficient.

If the solid be one of revolution, $O'O''$ will be the axis of revolution, the sections will all be circles having radii r_0, r_1, r_2, r_4, etc.

and the areas will be πr_0^2, πr_1^2, πr_2^2, etc. Substituting these values for A_0, A_1, A_2, etc. in the above formula, it reduces to

$$V = \frac{\pi h}{3n}[r_0^2 + r_n^2 + 4(r_1^2 + r_3^2 + r_5^2 + \cdots\cdots + r_{n-1}^2)$$
$$+ 2(r_2^2 + r_4^2 + r_6^2 + \cdots\cdots + r_{n-2}^2)] \qquad (2)$$

The dots in formulas (1) and (2) indicate missing terms; thus, when $n = 12$, $r_{n-1}^2 = r_{11}^2$, $r_{n-2}^r = r_{10}^2$, and the missing terms are r_7^2, r_8^2, and r_9^2.

EXAMPLE.—Referring to Fig. 70, the dimensions marked on the horizontal lines are the lengths of the radii; find the volume of the solid.

SOLUTION.—The number of slices is 8; hence, $n = 8, n - 1 = 7, n - 2 = 6$, and formula (2) becomes, for this case,

$$V = \frac{\pi h}{3 \times 8}[r_0^2 + r_8^2 + 4(r_1^2 + r_3^2 + r_5^2 + r_7^2) + 2(r_2^2 + r_4^2 + r_6^2)]$$

Substituting the dimensions given in the figure,

$$V = \frac{3.1416 \times 26}{3 \times 8}[9^2 + 11^2 + 4(7.1^2 + 8.7^2 + 12.9^2 + 12.5^2)$$
$$+ 2(7^2 + 11.5^2 + 13.1^2)] = 9198.6 \text{ cu. in.} \quad \textit{Ans.}$$

To find $O'O_0 = y$, Fig. 70, the distance of the center of gravity of the solid from the plane JK, use either of the two following formulas, in which the letters represent the same quantities as in the two formulas of the preceding article:

$$y = \frac{h}{n}\left[\frac{nA_n + 4(A_1 + 3A_3 + 5A_5 + \cdots\cdots + \overline{n-1}A_{n-1}) + 2(2A_2 + 4A_4 + 6A_6 + \cdots\cdots + \overline{n-2}A_{n-2})}{A_0 + A_n + 4(A_1 + A_3 + A_5 + \cdots\cdots + A_{n-1}) + 2(A_2 + A_4 + A_6 + \cdots\cdots + A_{n-2})}\right]$$

$$y = \frac{h}{n}\left[\frac{nr_n^2 + 4(r_1^2 + 3r_3^2 + 5r_5^2 + \cdots\cdots + \overline{n-1}r_{n-1}^2) + 2(2r_2^2 + 4r_4^2 + 6r_6^2 + \cdots\cdots + \overline{n-2}r_{n-2}^2)}{r_0^2 + r_n^2 + 4(r_1^2 + r_3^2 + r_5^2 + \cdots\cdots + r_{n-1}^2) + 2(r_2^2 + r_4^2 + r_6^2 + \cdots\cdots + r_{n-2}^2)}\right]$$

EXAMPLE.—Using the dimensions given, find the center of gravity of the solid shown in Fig. 70.

SOLUTION.—Since the solid is one of revolution, the c.g. must lie on mn, the axis of revolution. Substituting in formula (2) the values indicated in the figure,

$$y = \frac{26}{8}\left[\frac{8 \times 11^2 + 4(7.1^2 + 3 \times 8.7^2 + 5 \times 12.9^2 + 7 \times 12.5^2) + 2(2 \times 7^2 + 4 \times 11.5^2 + 6 \times 13.1^2)}{9^2 + 11^2 + 4(7.1^2 + 8.7^2 + 12.9^2 + 12.5^2) + 2(2 \times 7^2 + 11.5^2 + 13.1^2)}\right] = 15.746 \text{ in.} = O'O_0. \quad \textit{Ans.}$$

118. Center of Gravity of a System of Bodies.—Referring to Fig. 71, let A and B be any two bodies whose centers of gravity are known; for convenience, suppose them to be spheres, in

which case, the centers of gravity will be at the centers of the spheres. Suppose further that they are connected by a line AB having no weight; then, if the system were balanced on a knife edge, the reaction of the knife edge will be $A + B$ and the moments of the balls about the knife edge will be equal and opposite, exactly the same as in the case of a lever of the first class. Let x = the distance from the c.g. of the *larger* weight to the balancing point. For equilibrium, taking A as the origin of moments and l as the distance between the centers of gravity of the two bodies, $B \times l = (A + B)x$, from which

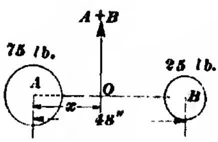


Fig. 71.

$$x = \frac{Bl}{A + B}$$

The point x is called the *center of gravity of the system;* in other words, if the two bodies were replaced by a single body of a weight equal to their combined weight and whose center of gravity was located at O ($AO = x$), it would have the same effect as that of the two bodies.

120. If there are more than two bodies, find the center of gravity of two of them; measure the distance from this point to the c.g. of one of the other bodies, and repeat the calculation, using the combined weight of the two bodies and the weight of the third body. Proceed in this manner until all the bodies have been used. If the weights are not known, the volumes of the bodies may be used instead, providing the bodies are of the same density.

Referring to Fig. 72, a system of four bodies is shown, and the weights and distances between their centers of gravity is indicated. To find the distance of the center of gravity of the system from the c.g. of the largest body, proceed as follows: Considering the two smallest bodies first, $x = \frac{30 \times 29}{30 + 160} = 4.6 -$ in. $= O'B$. Measuring the distance $O'C$, it is found to be about 37 in. Considering the weight of both bodies to be concentrated at O', their combined weight is $30 + 160 = 190$ lb., and the distance of the c.g. of the system from C is $\frac{190 \times 37}{190 + 340} = 13.3$ in. $= O''C$. Measuring the distance $O''D$, it is found to be about 43 in. Considering the weight of the three bodies to be concentrated

at O″, their combined weight is $190 + 340 = 530$ lb., and the distance of the c.g. of the entire system from D is $\frac{340 \times 43}{530 + 950}$ $= 9.88$ in. If greater accuracy were required, the measurements would be made more carefully and accurately.

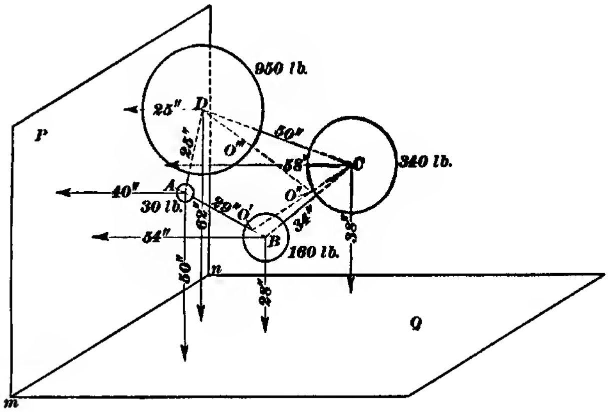


FIG. 72.

121. It is frequently desired to find the distance of the center of gravity of the system from a plane, as P or Q or both, Fig. 72. Suppose the distances of the four bodies from these planes to be as indicated, the distances between the centers of gravity of the bodies not being given in this case. Let y represent the distance of the c.g. of the system from plane Q. Then, taking the moments of the weights relative to this plane

$$(A + B + C + D) \times y = A \times y_1 + B \times y_2 + C \times y_3 + D \times y_4$$

in which y_1, y_2, etc. are the distances of A, B, etc. from plane Q. Hence,

$$y = \frac{A \times y_1 + B \times y_2 + C \times y_3 + D \times y_4}{A + B + C + D} \qquad (1)$$

Similarly, letting x_1, x_2, etc, represent the distances of A, B, etc. from the plane P, the distance x of the c.g. of the system from this plane is

$$x = \frac{A \times x_1 + B \times x_2 + C \times x_3 + D \times x_4}{A + B + C + D} \qquad (2)$$

Substituting in these formulas the values indicated in Fig.

$$y = \frac{30 \times 50 + 160 \times 28 + 340 \times 38 + 950 \times 62}{30 + 160 + 340 + 950} = 52.568$$

and

$$x = \frac{30 \times 10 + 160 \times 54 + 340 \times 58 + 950 \times 25}{1480} = 36.020 \text{ in}$$

The following example illustrates a case that frequently aris in practice, and while not a direct application of the principle o this article, it will be appropriate.

EXAMPLE.—*Referring to Fig. 73, which respresents a safety-valve lever, it is required to find what steam pressure in pounds per square inch will just* balance the downward forces exerted by the ball, the weight of the lever, and the weight of the valve and stem, the positions being as indicated. The lever is of steel, having a uniform thickness of $\frac{7}{8}$ in. and a specific weight of .2836 lb. per cu. in.

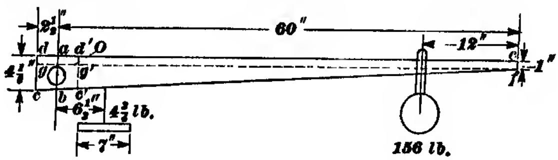


FIG. 73.

SOLUTION.—Since great refinement is not necessary in this case, draw a line $c'd'$ parallel to cd and assume that the part $abc'd'$ is equal in area and weight to $abcd$ when $ad' = ad$. This is not absolutely true, but is sufficiently exact for all practical purposes, and it greatly lessens the work of calculation. Now find the weight of that part of the lever to the right of $c'd'$. To do this, it is first necessary to find the length of $c'd'$, which is readily obtained as follows: If a line be drawn from f parallel to ed, the area $edcf$ will be divided into a rectangle and a triangle, the base of the latter being $4.125 - 1 = 3.125$ in. Since the triangles fgc and $fg'c'$ are similar, $cg : c'g' = fg : fg'$, or

$$3.125 : x = (60 + 2.5) : (60 - 2.5)$$

from which, $x = 2.875$ in. $= c'g'$, and $c'd' = 2.875 + 1 = 3.875$ in. The area of the trapezoid $c'd'cf = \frac{3.875 + 1}{2} \times 57.5 = 140.16$ sq. in. The volume of the lever to the right of $c'd' = 140.16 \times \frac{7}{8}$, and the weight is $140.16 \times .875 \times .2836 = 34.8-$lb. Since the lever is of uniform thickness. the center of gravity will lie in the middle plane, and the c.g. of that part to the right of $c'd'$ will be opposite the c.g. of the trapezoid $c'd'ef$, at a distance from $c'd' = \frac{2 \times 1 + 3.875}{3(1 + 3.875)} \times 57.5 = 23.1$ in. The weight of the ball and the rod from which it is suspended is 156 lb., and this acts downwards through the center of gravity of the ball. The weight of the valve and stem

acts on the steam only and has no effect on the lever; hence, the force P required to balance the effect of the downward forces acting on the lever is determined by the equation (taking O as the center of moments) $P \times 6.5 - 34.8 \times (23.1 + 2.5) - 156 \times 48 = 0$, from which $P = 1289$ lb. To this must be added the weight of the valve and stem, making the total upward pressure that must be exerted by the steam to balance the downward forces on the lever $1289 + 4.75 = 1293.75$. The area (of valve) pressed against by the steam is $7^2 \times .7854 = 38.4846$, say 38.48 sq. in. Therefore, the pressure per square inch is $1293.75 \div 38.48 = 33.6$ lb. per sq. in. *Ans.*

STABILITY

122. Static Equilibrium.—A body is said to be in **static equilibrium** when it is at rest and has no tendency to change its position. If a force act on such a body for an instant only (such a force is called an **impulse**), and the result of this impulse is to displace the center of gravity of the body very slightly, the body will either continue to move in the direction of the acting force or it will tend to return to its former position. In the first case, the body is said to be in **unstable equilibrium**, and in the second case, it is said to be in **stable equilibrium.**

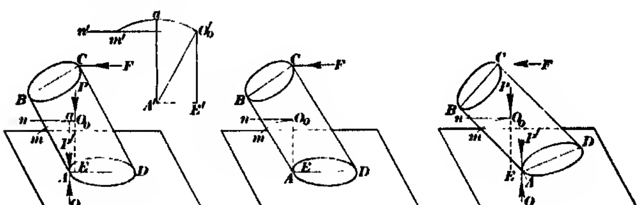


FIG. 74.

Referring to Fig. 74(*a*), *ABCD* is a truncated cylinder; suppose the c.g. to be located at O_0, and that the weight of the body be represented by the line P, which acts through the center of gravity. An impulse F acting at C tends to turn the cylinder about the point A, the c.g. moving along the arc O_0m. As O_0 moves along the arc, the distance between this point and the horizontal line AD *increases* until the point a, which is the intersection of the arc by a vertical line through A, is reached; this is shown more clearly by the view at the side, where O'_0m' represents the arc slightly exaggerated, and O'_0n' is a horizontal line through O'_0.

If, therefore, the effect of the impulse is to move the body only a part of the distance O_0a, the body will fall back to its original position as soon as the impulse has ceased to act; hence, the body is in stable equilibrium. Referring to Fig. 74(*b*), the vertical line through O_0 passes through the point A; if an impulse F act at C, the slightest movement of the center of gravity O_0 along the arc O_0m causes the distance between O_0 and AD to *decrease*, and gravity will cause the body to fall; hence, for this case, the body is in unstable equilibrium. Here it will be noted that the points A and E coincide. For the case shown at (*c*), the point E falls entirely without the base; any movement of O_0 toward the left decreases the distance between it and AD, and the body will not even stand in the position shown, but will fall as soon as it is released; it is therefore in unstable equilibrium.

123. The matter may be viewed in another way. Referring to (*a*), let P be a force acting through the center of gravity O_0, the length of P representing the weight of the cylinder. Take A as the origin of moments; then, according to Art. **30**, the force P is equivalent to an equal force P' acting through A and to the couple P,Q $(Q = P)$ whose arm is AE. This couple tends to produce right-hand rotation, which is resisted by the re-action of the base; hence, if the cylinder is subjected to the action of gravity only, it will stand and will be in stable equilibrium. In (*b*) there is no couple, since the arm $(= AE)$ is 0; the body will stand, but the slightest force acting in the direction of F will cause it to fall; it is therefore in unstable equilibrium. In (*c*), the couple tends to produce left-hand rotation, and as there is nothing to resist it, the body will turn about the point A and fall.

From the foregoing, it is evident that if the vertical through the center of gravity falls within the base, the body will stand and be in stable equilibrium; if the vertical through the center of gravity cuts the edge of the base, the body will stand, but the slightest blow will cause it to fall, and it is in unstable equilibrium; if the vertical through the center of gravity falls without the base, the body will not stand, and is in unstable equilibrium. Further, if for any movement of the body, the center of gravity is *raised*, the body is in stable equilibrium; but if it *falls*, it is in unstable equilibrium.

124. If for any movement of the body, the c.g. moves in a plane parallel to the plane of the base, it is said to be in **neutral equi-**

librium; for instance a sphere rolling on a plane surface, a right cone rolling on a plane surface. Such a body has no tendency to fall or to return to its former position; it stays where "put," and it is in neither stable nor unstable equilibrium. Any body rotating about its center of gravity is in static neutral equilibrium.

From the foregoing, it will be evident why leaning towers do not fall.

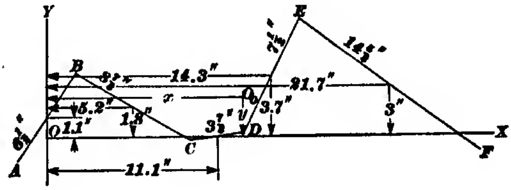


FIG. 75.

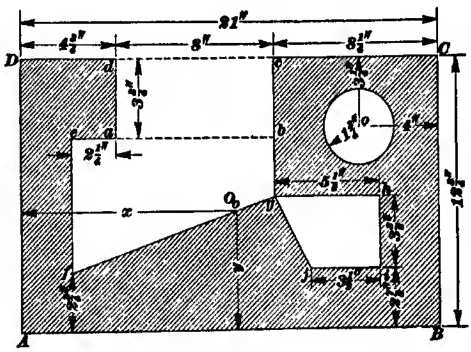


FIG. 76.

EXAMPLES

(1) Referring to Fig. 75, find the center of gravity of the broken line *ABCDEF* with reference to the axes *OX* and *OY*. Note that *OX* passes through the c.g. of *CD* and *OY* passes through the c.g. of *AB*.

Ans. $\begin{cases} x = 12.4 \text{ in.} \\ y = 2.3 \text{ in.} \end{cases}$

(2) The chord of a circular arc is $18\frac{5}{16}$ in. and the angle at the center is 106° 15′; where is the center of gravity of the arc? *Ans.* $9\frac{7}{8}$ in. from center.

(3) The area of a triangle is 17.6 sq. in., and the length of one of the sides is $8\frac{3}{4}$ in. How far from that side is the center of gravity? *Ans.* 1.341 in.

If, therefore, the effect of the impulse is to move the body only a part of the distance O_0a, the body will fall back to its original position as soon as the impulse has ceased to act; hence, the body is in stable equilibrium. Referring to Fig. 74(*b*), the vertical line through O_0 passes through the point A; if an impulse F act at C, the slightest movement of the center of gravity O_0 along the arc O_0m causes the distance between O_0 and AD to *decrease*, and gravity will cause the body to fall; hence, for this case, the body is in unstable equilibrium. Here it will be noted that the points A and E coincide. For the case shown at (*c*), the point E falls entirely without the base; any movement of O_0 toward the left decreases the distance between it and AD, and the body will not even stand in the position shown, but will fall as soon as it is released; it is therefore in unstable equilibrium.

123. The matter may be viewed in another way. Referring to (*a*), let P be a force acting through the center of gravity O_0, the length of P representing the weight of the cylinder. Take A as the origin of moments; then, according to Art. **30**, the force P is equivalent to an equal force P' acting through A and to the couple P,Q ($Q = P$) whose arm is AE. This couple tends to produce right-hand rotation, which is resisted by the re-action of the base; hence, if the cylinder is subjected to the action of gravity only, it will stand and will be in stable equilibrium. In (*b*) there is no couple, since the arm ($= AE$) is 0; the body will stand, but the slightest force acting in the direction of F will cause it to fall; it is therefore in unstable equilibrium. In (*c*), the couple tends to produce left-hand rotation, and as there is nothing to resist it, the body will turn about the point A and fall.

From the foregoing, it is evident that if the vertical through the center of gravity falls within the base, the body will stand and be in stable equilibrium; if the vertical through the center of gravity cuts the edge of the base, the body will stand, but the slightest blow will cause it to fall, and it is in unstable equilibrium; if the vertical through the center of gravity falls without the base, the body will not stand, and is in unstable equilibrium. Further, if for any movement of the body, the center of gravity is *raised*, the body is in stable equilibrium; but if it *falls*, it is in unstable equilibrium.

124. If for any movement of the body, the c.g. moves in a plane parallel to the plane of the base, it is said to be in **neutral equi-**

MECHANICS AND HYDRAULICS

(PART 2)

EXAMINATION QUESTIONS

(1) If the length of the base of an inclined plane is 46 ft. and the height of the plane is 38 ft., what theoretical force acting parallel to the base is required to keep a body weighing 4600 lb. from sliding down the plane? *Ans.* 3800 lb.

(2) Referring to Question 1, (*a*) what work must be expended in moving the body from the bottom to the top of the plane? (*b*) If the coefficient of friction is .21, what is the force of friction, it being considered as acting parallel to the plane?

Ans. (*a*) 174,800 ft.-lb.; (*b*) 1253 lb.

(3) The velocity ratio of a certain machine is 3.6. By experiment, it is found that an application of a power of 170 lb. will overcome a resistance of only 492 lb.; what is the efficiency of the machine? *Ans.* 80.4%.

(4) The diameter of the handle of a screwdriver is 1 in., its length is 11 in., the diameter of the screw is $\frac{1}{2}$ in., and the number of threads per inch is 6; (*a*) what theoretical pressure will be applied to the end of the screw when a turning force of 46 pounds is imparted to the handle of the screwdriver? (*b*) what will be the actual pressure if the efficiency is 56%? (*c*) what is the velocity ratio?

Ans. (*a*) 867 lb.; (*b*) 485.5 lb.; (*c*) 18.85 –

(5) In a gear and pulley train, the diameters of the driving pulleys are 66 in. and 18 in., the diameters of the follower pulleys are 42 in. and 24 in., the number of teeth in the driving gears are 92, 72, and 68, and the number of teeth in the followers are 46, 48, and 64; (*a*) what is the speed ratio? (*b*) the velocity ratio?

(*c*) what is the number of revolutions per minute made by the first driving pulley if the second follower pulley makes 526 r.p.m.? The order of pulleys and gears is to be taken as here given; that is, the first driving gear is keyed to the same shaft as the first follower pulley, the first follower gear and second driving gear are keyed to the same shaft, etc.

Ans. (*a*) 3.757 −; (*b*) .732 +; (*c*) 140 r.p.m.

(6) When pulling a load up an inclined plane, why must the power be greater (*a*) when it acts parallel to the base than when it acts parallel to the plane? (*b*) if it requires a greater power in one case than in the other, why is the work done in pulling a body a given distance along the plane the same in both cases, in accordance with the definition that work equals force multiplied by the distance through which it acts?

(7) In a toggle joint, the distance between the line joining the centers of the outside joints and the center of the middle joint is $1\frac{5}{8}$ in., and the distance between the centers of the outside joints is $54\frac{1}{2}$ in. (*a*) What power must be applied to the middle joint to cause the movable joiut to exert a pressure of 1350 lb.? (*b*) what is the velocity ratio?

Ans. (*a*) 161 lb.; (*b*) 8.385 −

(8) If the diametral pitch of a spur gear is $2\frac{1}{2}$, what (*a*) is the circular pitch? (*b*) If the diameter of the gear is 28 in., how many teeth has it?

Ans. (*a*) 1.2566 in.; (*b*) 70 teeth

(9) When the diametral pitch is used, it is customary to make the addendum of a tooth equal to the reciprocal of the pitch; what must be the diameter of the blank from which the gear is cut when the diametral pitch is $2\frac{1}{2}$ and the number of teeth is 80? The diameter of the blank is evidently the same as the diameter of the circle described by a point on the outside of a tooth. *Ans.* 32.8 in.

(10) If the length of a wedge is 15 in. and the thickness at the head is $2\frac{1}{2}$ in., (*a*) what force must be applied to the head to make the sides exert a pressure of 630 lb.? (*b*) what is the velocity ratio?

Ans. (*a*) 105 lb.; (*b*) 6

(11) What is (*a*) a left-hand screw? (*b*) if you hold a screw in your hand, how can you tell whether it is right- or left-handed?

(12) Explain the difference between the speed ratio and the velocity ratio of a train of pulleys and gears?

(13) The addendum for gears cut according to the cycloidal system is quite commonly taken as $.3p_c$, that is $\frac{3}{10}$ths of the circular pitch. If the pitch circle of a spur gear is to be $12\frac{1}{4}$ in. in diameter and the gear is to have 40 teeth, what (*a*) will be the circular pitch? (*b*) what should be the diameter of the blank?

Ans. $\begin{cases} (a) \text{ .9621 in.} \\ (b) \text{ 12.8273 in.} \end{cases}$

(14) The velocity ratio of a certain machine is 2.25 and its efficiency is 91%; what power must be applied to raise a load of 336 lb.? *Ans* 164.1 lb.

(15) From the dimensions given in Fig. 1, calculate to three decimal places the lengths of the line segments that form the broken line *ABCDEFG*, locate their centers of gravity with respect to *OA* and *OH* as axes of moments, and calculate x' and y', the distances of the center of gravity of the broken line, from *OH* and *OA*.

Ans. $\begin{cases} x' = 6.753 \text{ in.} \\ y' = 17.238 \text{ in.} \end{cases}$

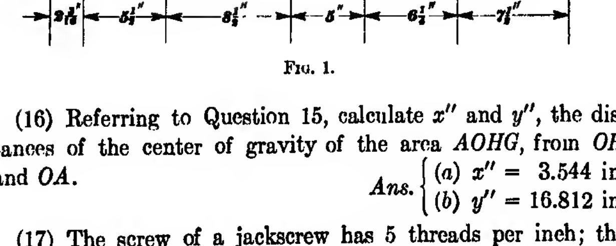


Fig. 1.

(16) Referring to Question 15, calculate x'' and y'', the distances of the center of gravity of the area *AOHG*, from *OH* and *OA*.

Ans. $\begin{cases} (a)\ x'' = 3.544 \text{ in.} \\ (b)\ y'' = 16.812 \text{ in.} \end{cases}$

(17) The screw of a jackscrew has 5 threads per inch; the length of the handle from the axis of the screw to the joint where the power is applied is 16 in.; (*a*) what load can be raised by a power of 75 lb. if the efficiency is 46%? (*b*) what is the velocity ratio?

Ans. $\begin{cases} (a) \text{ 17,342 lb.} \\ (b) \text{ 502.66} - \end{cases}$

(18) The worm of a worm and wheel has 4 threads per inch, the diameter of the pitch circle of the wheel is 13 in., the diameter of the axle keyed to the same shaft as the wheel is $5\frac{1}{4}$ in. The worm is turned by applying a couple to a wheel rigidly connected to the worm and 10 in. in diameter. If one of the equal forces constituting the couple is 18 lb., (*a*) what theoretical load can be raised? (*b*) if the load actually raised is only 4560 lb., what is the efficiency? (*c*) what is the velocity ratio of the combination?

Ans. $\begin{cases} (a)\ 10{,}693 \text{ lb.} \\ (b)\ 42.65\% \\ (c)\ 297 \end{cases}$

(19) In Fig. 2, *AB* and *CD* are arcs of circles and *OE* bisects them. From the dimensions given, find the distance y, O_0 being the center of gravity of the area *ABCD*.

Ans. $y = 1.65$ in.

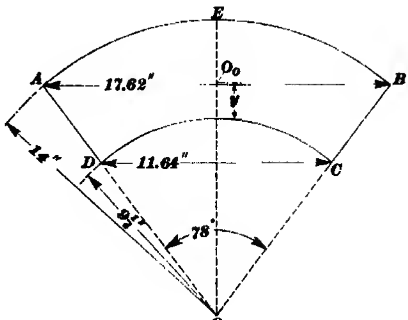


Fig. 2.

(20) The diameter of the lower base of a right conical frustum is 12.6 in., of the upper base 7.4. in., and the altitude is 6.8 in.; how far is the center of gravity from the lower base?

Ans. 2.824– in.

MECHANICS AND HYDRAULICS

(PART 3)

DYNAMICS

MOTION AND VELOCITY

125. Dynamics is that branch of mechanics that relates to bodies in motion but not in equilibrium; that is, the motion is variable, the velocity increasing or decreasing as the result of forces acting on the bodies. Dynamics is also called **kinetics,** though the two terms have a slightly different meaning.

GRAPHICAL REPRESENTATION OF MOTION

126. Uniform Velocity.—Velocity may be represented by a line, in the same manner as force. While velocity is measured by a compound unit, as one foot per second, one mile per hour, etc., the time element is always unity—one second, one hour, etc.—with the result that the velocity is the number of feet, miles, etc. traveled in the unit of time, and may be represented by a right line whose length is the distance (space) moved in the unit of time. By selecting the proper scale, the line may be made of any desired length. Placing an arrowhead on the line will indicate the direction in which the body is moving. Thus, in Fig. 77, suppose a body is moving along the line OY with a velocity of 18 ft. per sec. If a scale of 12 ft. = 1 in. be selected, a line OO' $= 18 \div 12 = 1.5$ in. will represent 18 ft. per sec., the arrowhead indicates that the body is moving in the direction from O to O' (the direction of motion or of velocity), and O is the point from which it starts. Therefore, OO' completely represents the motion of a body that starts from O and moves with a uniform velocity of 18 ft. per sec. in the direction OO'.

127. Uniform velocity may also be represented by a geometrical figure—a rectangle. For, referring to Fig. 77, draw OX and OY at right angles to each other. Lay off on OX a series of equal spaces, $O1$, 12, 23, etc., each space representing one unit of time, say one second. Through each of the points, draw lines parallel to OY, and lay off on these lines distances $1a$, $2b$, $3c$, etc. to represent the velocity at the time indicated by the division mark on OX; thus, at the end of one second, the velocity is $1a$, at the end of two seconds, the velocity is $2b$, etc. Pass a line through the points O', a, b, etc.; then, the length of a line drawn from any point on OX parallel to OY and included between OX and $O'f$ will be the velocity at the instant indicated by the point on OX; for instance, at the point 4, which indicates 4 sec. after starting, the velocity is $4d$.

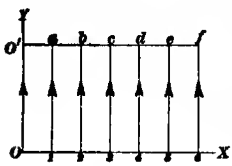


FIG. 77.

When the velocity is uniform, all the lines OO', $1a$, $2b$, etc. have the same length; hence, $O'f$ is a right line parallel to OX, since every point in $O'f$ is at the same distance from OX, which is one of the definitions of parallel lines. This is the kind of line drawn by a tachometer (speed indicator) on a paper machine running at a constant speed. The area of the rectangle is $O6 \times OO'$ = time $\times$ velocity, or

$$A = t \times v$$

when A = area, t = time, and v = velocity. Letting s = distance (space) traveled in time t, $v = \frac{s}{t}$ for uniform velocity. Substituting this value of v in the above equation,

$$A = t \times \frac{s}{t} = s$$

that is, the *area* of the rectangle represents the distance (space) traveled in time t. To make this last statement true, however, it is necessary that the distances representing t and v be measured in the same units. Thus, if t is in minutes and v is in feet per minute, and the scale adopted is 12 ft. = 1 in., then, $\frac{1}{12}$ in. = 1 ft., and $\frac{1}{12}$ in. must also be taken as representing 1 min. When this is done, the area in square inches multiplied by $12^2 = 144$ (the *square of the scale*, as it is called) will give the distance or space traveled in the time t. As an example, suppose the velocity of paper on a paper machine is 440 ft. per min. and that the

scale is 100 ft. = 1 in. At the end of, say 9 min., the distance to be laid off on OX would be $9 \div 100 = .09$ in., the distance along $OY = 440 \div 100 = 4.4$ in., and the area of the rectangle $= .09 \times 4.4 = .396$ sq. in. Multiplying this by the square of the scale, $.396 \times 100^2 = 3960$ ft. $= s$, the distance traveled by a point on the paper in 9 min. = the length of the sheet of paper made in 9 min. That this result is correct is easily seen, since space = velocity × time $(s = vt) = 440 \times 9 = 3960$ ft.

The area of paper made in time t is obtained by multiplying s, the length of sheet made in time t, by the width of the sheet.

128. Variable Velocity.—When the velocity is different at different time intervals, it is said to be **variable.** For example, suppose a railway train that is running at 36 miles per hour to be brought to a stop. The speed (velocity) will decrease from 36 mi. per hr. (52.8 ft. per sec.) to 0; assuming that it takes, say, 10 sec. to stop, the velocity of the train passes through every conceivable value between 52.8 and 0 ft. per sec. during this interval of 10 sec.; the velocity probably has a different value also at every instant of this interval. At some particular instant, the velocity is, say, 28 ft. per sec.; this means if the velocity were to become uniform at that instant, the train would then travel 28 ft. in one second. *In all cases of variable velocity, the velocity at any instant is the velocity the body would have if the velocity became uniform at that instant.*

129. Variable velocity may also be represented by a geometrical figure; thus, referring to Fig. 78 (a), draw OX and OY, as

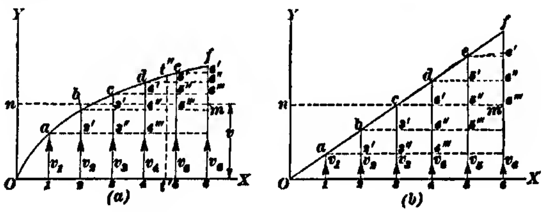


Fig. 78.

before, and suppose, for convenience, that the velocity at the start is 0. As in the case of uniform velocity, lay off the times along OX, making $O1 = 12 = 23$, etc., and draw $1a$, $2b$, $3c$, etc., making their lengths equal the velocities at the instants indicated by 1, 2, 3, etc. By taking the time intervals sufficiently small, a

curve may be drawn through the points O, a, b, etc., as shown. The velocity at any instant may be found by laying off Ot' equal to the time interval from the starting point O, drawing $t't''$ parallel to OY, and measuring the length of $t't''$; this length will be the velocity at the instant indicated by t'.

Let v_a be the *average* velocity between O and, say, 6; then, $t \times v_a = s =$ the space traveled by the body in the time $t = O6$. But $v_a t$ is the area of the rectangle $O6mn$, and this must equal the area $Of6$ under the curve, because v_a is the mean of all the lines $1a$, $2b$, $3c$, etc. and both figures have the same base $O6$. To find the value of v_a, find the area of $Of6$ by Simpson's rule (see *Elementary Applied Mathematics*) and divide it by the length $O6$, the quotient will be the value of v_a. Since $A = v_a t$, and $v_a = \frac{s}{t}$,

$$A = \frac{s}{t} \times t = s,$$

that is, the area under the curve represents the space traveled by the body, the same scale being used to lay off the times and velocities.

130. Suppose the increase (or decrease) in velocity is constant; then the curve will become a right line, as shown by the line Of in Fig. 78 (b). Here the increases in velocity are represented by $1a = 2'b = 3'c =$ etc., thus making the triangles $Oa1$, $ab2'$, $bc3'$, etc. equal, with the result that $O6f$ is a right triangle. The line mn, which denotes by its distance from OX the average (mean) velocity, coincides in this case with $c6'''$, because $3c = \frac{6f}{2}$ and the area of the triangle is $A = \frac{O6 \times 6f}{2} = O6 \times \frac{6f}{2}$.

131. Acceleration.—As stated in *Physics*, acceleration is *rate* of change in velocity. If the change in velocity is uniform, the rate of change (acceleration) is also uniform. Thus, referring to Fig. 78 (b), the change in velocity during the first second is $1a$, an increase in this case. During the second second, the gain in velocity is $2'b$; during the third second, the gain is $3'c$; etc. Since this change is uniform, these gains divided by the time (1 second) are also uniform, and $1a = 2'b = 3'c =$ etc. Consequently, the acceleration is uniform and is represented by $1a = 2'b = 3'c =$ etc. Let $a =$ acceleration in feet per second per second, or feet per second^2 (see *Physics*), $v =$ velocity in feet per second, $s =$ distance (space) passed through in feet, and $t =$ the time in

seconds; then, since $1a$ = the velocity and also the acceleration, the velocity at the end of 1 sec. equals the acceleration during that second. At the end of 6 sec., the velocity is $6 \times a$; at the end of t sec., the velocity is $t \times a$; hence,

$$v = at \qquad (1)$$

Thus, in the figure, $f6' = 6'6'' = 6''6''' =$ etc.; whence, $6f = v = 6 \times 6'f = t \times a = at$. The area of the triangle is $\frac{1}{2} \times O6 \times 6f = \frac{1}{2} \times t \times v$, and since the area represents the space passed through by the body in the time t, $\frac{1}{2} \times t \times v = s$,

and
$$s = \tfrac{1}{2}vt \qquad (2)$$

Substituting in (2) the value of v given in (1), $s = \frac{1}{2} \times at \times t$, or

$$s = \tfrac{1}{2}at^2 \qquad (3)$$

In formulas (1), (2), and (3), the acceleration is uniform, which is usually the case in practice. Cases in which the acceleration is not uniform seldom occur and will not be considered here. It may be mentioned, however, that the line Of will not be straight in such cases, but will be a curve as shown in Fig. 78 (a); here the accelerations $1a$, $2'b$, $3'c$, etc. are not equal, and the three formulas just given do not apply.

132. If in any case of variable velocity, the velocity is *decreasing* instead of increasing, the acceleration is said to be **negative**; in Fig. 78 (b), for example, the velocity might decrease from $6f$ to 0, but if the acceleration were the same numerically, the space passed over would be the same as before. Thus, during the first second, the velocity decreases by the amount $f6'$, the acceleration; during the second second, the velocity decreases $e5' = 6'6''$, etc. until at the end of 6 seconds, it becomes 0.

Negative acceleration is sometimes called **retardation.**

133. Initial Velocity.—If a body is already in motion, the velocity that it has at the instant that the time begins to be considered is called the **initial velocity.** Referring to Fig. 79, suppose the initial velocity $= v_0$ and that the body has a constant acceleration a, which increases the velocity of the body. Lay off OO' equal to v_0; draw $O'X'$ parallel to OX; lay off $O1$, 12, 23, etc., to represent 1 sec., 2 sec., 3 sec., etc., draw $1a$, $2b$, $3c$, etc. parallel to OY and intersecting $O'X'$ in $1'$, $2'$, $3'$, etc.; lay off $1'a = a$, $2''b = 2a$, $3'''c = 3a$, etc. (since the acceleration a is uniform), and then $1a$, $2b$, $3c$, etc. will be the velocities at the end of 1 sec., 2 sec., 3 sec., etc. The area $OO'f6$ represents the space

passed through in 6 sec. and $6f$ represents the velocity at the end of 6 sec., which is called the **final velocity.** The figure $OO'f6$ is a trapezoid, and its area is $\frac{OO' + 6f}{2} \times O6$. Let $OO' = v_0$, $6f = v$, $O6 = t$; then, s, the space passed through in time t is

$$s = \frac{v_0 + v}{2} \times t \qquad (1)$$

That is, if a body have an initial velocity and a constant (uniform) acceleration, the space passed through in a time t is equal to one-half the sum of the initial and final velocities multiplied by the time.

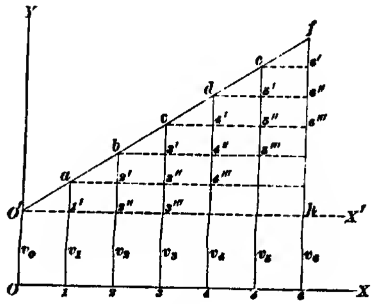


FIG. 79.

Referring again to Fig. 79, $6f = 6h + hf = OO' + hf = v_0 + at$, since if $O'h = t$, $hf = at$. Therefore, the final velocity is

$$v = v_0 + at \qquad (2)$$

The space s is also equal to $OO'h6 + O'hf = v_0t + \frac{1}{2}at^2$; that is,

$$s = v_0t + \tfrac{1}{2}at^2 \qquad (3)$$

If the velocity is decreasing, a will be negative, and the two formulas become

$$v = v_0 - at \qquad (4)$$

and

$$s = v_0t - \tfrac{1}{2}at^2 \qquad (5)$$

If the initial velocity, the acceleration, and the distance passed through are known and it is desired to find the final velocity, combine formulas (1) and (2) as follows:

From formula (2), $t = \frac{v - v_0}{a}$; substituting this value of t in formula (1), $s = \frac{v + v_0}{2} \times \frac{v - v_0}{a} = \frac{v^2 - v_0^2}{2a}$; from which

$$v = \sqrt{v_0^2 + 2as} \qquad (6)$$

If a is negative, formula (6) becomes

$$v = \sqrt{v_0^2 - 2as} \qquad (7)$$

Having found v, the time may be found by applying formula (1) or formula (2).

EXAMPLE 1.—If a body have an acceleration of 8.2 ft. per sec.², what will be its velocity at the end of 12.6 sec.? How far will it travel in that time? The body is assumed to start from a state of rest.

SOLUTION.—By formula (1), Art. **131**.

$$v = 8.2 \times 12.6 = 103.32 \text{ ft. per sec.} \quad \textit{Ans.}$$

By formula (3), Art. **131**,

$$s = \tfrac{1}{2} \times 8.2 \times 12.6^2 = 650.9 + \text{ft.} \quad \textit{Ans.}$$

Or, by formula (2), same article,

$$s = \tfrac{1}{2} \times 103.32 \times 12.6 = 650.9 + \text{ft., as before.}$$

EXAMPLE 2.—A body having an initial velocity of 125 ft. per sec. is brought to rest, the constant acceleration being 6.8 ft. per sec.² What will be its velocity at the end of 10 sec.? In what time will the velocity be 0? How far will it have traveled when the velocity is 5 ft. per sec.?

SOLUTION.—By formula (4) above, the velocity at the end of 10 sec. is

$$v = 125 - 6.8 \times 10 = 57 \text{ ft. per sec.} \quad \textit{Ans.}$$

To find the time required to bring the body to rest, use the same formula, making $v = 0$; then,

$$0 = 125 - 6.8 \times t$$

from which,

$$t = \frac{125}{6.8} = 18.38 \text{ sec.} \quad \textit{Ans.}$$

The same result would be obtained by using formula (1), Art. **131**.

To find the space traveled when the velocity is 5 ft. per sec., first find the time required for the velocity to decrease to 5 ft. per sec. By formula (4) above, since $v = 5$ and $v_0 = 125$,

$$5 = 125 - 6.8 \times t, \text{ or } t = \frac{120}{6.8} = 17\tfrac{11}{17} \text{ sec.}$$

Then, by formula (1) above,

$$s = \frac{125 + 5}{2} \times 17\tfrac{11}{17} = 1147 \text{ ft., very nearly.} \quad \textit{Ans.}$$

The last result might have been obtained by applying formula (5), since

$$s = 125 \times 17\tfrac{11}{17} - \tfrac{1}{2} \times 6.8 \times (17\tfrac{11}{17})^2 = 1147 \text{ ft., as before.}$$

EXAMPLE 3.—A certain elevator hoists a load 450 ft. in 10 sec. Suppose it takes 2 sec. to accelerate the load to full speed; that the speed is then uniform until the elevator begins to stop; that it takes $1\frac{1}{2}$ sec. to stop; and that the acceleration is uniform in both cases. What (*a*) is the acceleration when getting up to full speed? (*b*) the acceleration when stopping? (*c*) the uniform velocity? (*d*) the distance passed through until uniform velocity is reached? (*e*) the distance passed through in stopping? (*f*) the distance passed through while the velocity is uniform?

SOLUTION.—The conditions are shown by the diagram (not drawn to scale) in Fig. 80. The length of the line OC represents 10 sec., and is divided into

10 equal parts. Let $O2 = 2$ sec. = the time t_1 during which the load is accelerated; $AB = 10 - (2 + 1.5) = 6.5$ sec. = time t_2 during which the velocity is uniform; $DC = 1.5$ sec. = time t_3 during which the elevator is being stopped. Let s_1 = distance passed through in time t_1; s_2 = distance passed through in time t_2; and s_3 = distance passed through in time t_3. Then, $s = s_1 + s_2 + s_3 = 450$ ft., the total distance. Let a be the acceleration during time t_1; the calculation then proceeds as follows:

(a) $s_1 = \frac{1}{2}at_1^2 = \frac{1}{2}a \times 2^2 = 2a$. The uniform velocity between A and B is the velocity at the end of time t_1; denoting this by v, $v = at_1 = a \times 2$

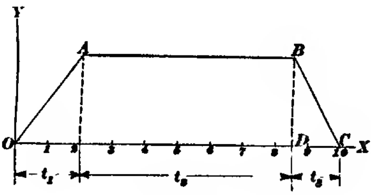


FIG. 80.

$= 2a$; hence, $s_2 = vt_2 = 2a \times 6.5 = 13a$. The acceleration a' between B and C is $a' = \frac{v}{t_3}$ (see formula 1, Art. 131), or $a' = \frac{2a}{1.5} = \frac{4a}{3}$; whence, $s_3 = \frac{1}{2}a' t_3^2 = \frac{1}{2} \times \frac{4a}{3} \times 1.5^2 = \frac{3a}{2}$. Therefore,

$$s = s_1 + s_2 + s_3 = 2a + 13a + \frac{3a}{2} = 450$$

from which $a = 27\frac{3}{11}$ ft. per sec². *Ans.*

(b) Since $27\frac{3}{11} = \frac{300}{11}$, $a' = \frac{4}{3} \times \frac{300}{11} = \frac{400}{11} = 36\frac{4}{11}$ ft. per sec.² *Ans.*

(c) The uniform velocity between A and B is the velocity at the end of the time t_1, which was found above to be $2a$; hence,

$$v = 2a = 2 \times \frac{300}{11} = \frac{600}{11} = 54\frac{6}{11} \text{ ft. per sec.} \quad \textit{Ans.}$$

(d) As found above, $s_1 = 2a = 2 \times 27\frac{3}{11} = 54\frac{6}{11}$ ft. *Ans.*

(e) As found above, $s_3 = \frac{3a}{2} = \frac{3}{2} \times \frac{300}{11} = \frac{450}{11} = 40\frac{10}{11}$ ft. *Ans.*

(f) As found above, $s_2 = 13a = 13 \times \frac{300}{11} = \frac{3900}{11} = 354\frac{6}{11}$ ft. *Ans.*

Note that $s_1 + s_2 + s_3 = 54\frac{6}{11} + 354\frac{6}{11} + 40\frac{10}{11} = 450$ ft., as it should.

Note further that the area of the figure $OABC$ (a trapezoid) is $\frac{AB + OC}{2} \times BD = \frac{6.5 + 10}{2} \times \frac{600}{11} = 450$, the total distance passed through. By using fractions in the calculation instead of decimals, more exact results can be obtained, which is especially desirable when checking the result.

EXAMPLE 4.—A body has an initial velocity of 40 ft. per sec.; it is uniformly accelerated for 6 sec., at the end of which time, its velocity is 120 ft. per sec. What is (a) the uniform acceleration? (b) what is the distance passed through during the 4th second?

Solution.—Applying formula (2) above, $v = 12$, $v_0 = 40$, $t = 6$; hence,

(a) $120 = 40 + a \times 6$, or $a = \frac{120 - 40}{6} = 13\frac{1}{3}$ ft. per sec.² *Ans.*

(b) The velocity at the end of 3 sec. is $v = 40 + 13\frac{1}{3} \times 3 = 80$ ft. per sec. The velocity at the end of 4 sec. is $v' = 40 + 13\frac{1}{3} \times 4 = 93\frac{1}{3}$ ft. per sec. By formula (1) above, the space passed through in 3 sec. is $s = \frac{40 + 80}{2} \times 3 = 180$ ft.; the space passed through in 4 sec. is $\frac{40 + 93\frac{1}{3}}{2} \times 4 = 266\frac{2}{3}$ ft. Therefore, the distance passed through in the fourth second is $266\frac{2}{3} - 180 = 86\frac{2}{3}$ ft. *Ans.*

Example 5.—A body (as an automobile) having a velocity of 80 ft. per sec. is brought to rest in 200 ft. Assuming the acceleration to be uniform, (a) in what time will the body be stopped? (b) what is the acceleration?

Solution.—By formula (2), Art. **131**, $s = \frac{1}{2}vt$, from which $t = \frac{2s}{v}$; therefore, $t = \frac{2 \times 200}{80} = 5$ sec. *Ans.*

(b) By formula (1), Art. **131**, $v = at$, from which $a = \frac{v}{t} = \frac{80}{5} = 16$ ft. per sec.² *Ans.*

Example 6.—If a body have a uniform acceleration of 28 ft. per sec.² in what time will it pass through a distance of 450 ft., starting from rest?

Solution.—By formula (3), Art. **131**, $s = \frac{1}{2}at^2$, from which $t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 450}{28}} = 5.67-$ sec. *Ans.*

134. In order to tell which of the foregoing formulas to use in any particular case, first consider what quantities are given and which one is required; then select the formula that contains these quantities. Referring to the last example, the acceleration a and the space s are given, and the time t is required; the only formula containing these three quantities and no others is formula (3) of Art. **131.** Again, referring to example 4, the initial velocity is given, the final velocity is calculated, the time is known, and the space is required; the only formula containing v_0, v, t, and s, and no other quantities, is formula (1) of Art. **133,** which is therefore used.

These formulas expressing the relation between time, space, velocity, and acceleration are very important; it is therefore well to study Arts. **129–133** very carefully. It will also be an advantage to memorize the formulas.

EXAMPLES

(1) A body is moving at the rate of 36 ft. per sec. It is then acted upon by a steady force for $12\frac{1}{2}$ sec. that gives it a uniform acceleration; when the force ceases to act, the velocity is 154 ft. per sec. (*a*) What was the acceleration? (*b*) what was the distance passed through while the body was being accelerated?

Ans. (*a*) 9.44 ft. per sec.²; (*b*) 1187.5 ft.

(2) While moving through a distance of 1000 ft., the velocity of a body decreases uniformly from 248 ft. per sec. to 52 ft. per sec. (*a*) What was the negative acceleration? (*b*) How many seconds did it take for the body to pass through 1000 ft.? (*c*) what was the distance passed through during the 6th second?

Ans. (*a*) -29.4 ft. per sec.²; (*b*) $6\frac{2}{3}$ sec.; (*c*) 86.3 ft.

(3) Referring to example 3, Art. **133**, suppose that the time t_1 required to accelerate the load to the mean velocity had been $2\frac{1}{2}$ sec. and the time required for stopping had been 2 sec.; what is (*a*) the acceleration when getting up to full speed? (*b*) the acceleration when stopping? (*c*) the uniform velocity? (*d*) the distance passed through until maximum velocity is attained? (*e*) the distance passed through in stopping? (*f*) the distance passed through while velocity is uniform?

Ans. (*a*) 23.226 ft. per sec.²; (*b*) 29.03 ft. per sec.²; (*c*) 58.06 ft. per sec.; (*d*) 72.58 ft.; (*e*) 58.06 ft.; (*f*) 319.36 ft.

(4) A body starting from rest is acted upon for 8 sec. by a steady force that gives the body an acceleration of 48 ft. per sec.² At the end of the 8th second, the force is reduced and the acceleration is also reduced, the acceleration after 8 sec. being 32 ft. per sec.² How far does the body move in 12 sec.? Note that the velocity at the end of 8 sec. becomes the initial velocity at the instant that the acceleration becomes 32 ft. per sec.²

Ans. 3328 ft.

(5) The velocity of a body increases uniformly from 80 ft. per sec. to 320 ft. per sec. If the acceleration is 20 ft. per sec.², (*a*) through what distance does the body pass? (*b*) what time was required?

Ans. (*a*) 2400 ft.; (*b*) 12 sec.

(6) A paper machine is running at the rate of 5 ft. per sec. The weight of the paper is to be changed so the velocity is increased uniformly to 360 ft. per min. in 80 seconds. (*a*) What is the acceleration? (*b*) How many feet of paper will be of variable weight due to the speed change?

Ans. (*a*) .0125 ft. per sec.²; (*b*) 440 ft.

COMPOSITION AND RESOLUTION OF VELOCITIES

135. Resolution of Velocities.—It was stated in *Physics* that all motion is *relative;* consequently, velocity is also relative. For example, suppose a body is moving northeast with a uniform velocity of 48 ft. per sec.; it has a certain velocity north and also east, and since the northeast direction makes an angle of 45° with both the east and north directions, the relative velocity north is $48 \times \sqrt{\frac{1}{2}} = 48 \times \frac{1}{2}\sqrt{2} = 33.94$ ft. per sec.; the relative velocity east is also 33.94 ft. per sec. In other words, the original velocity in a northeast direction has been resolved into two component velocities, one north and the other east. The case is exactly similar to the resolution of forces.

136. Composition of Velocities.—Suppose a ship is moving in the direction AB, Fig. 81 (*a*), with a uniform velocity of 30 ft. per sec. and that when a man standing on the deck is directly over

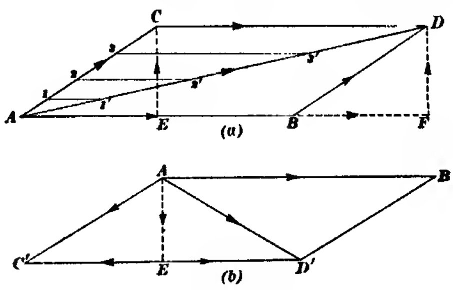


Fig. 81.

the point A on a rock underneath the ship, he starts to run across the deck in the direction AC, with a velocity of 20 ft. per sec. At the end of, say 4 sec., the man will have run $20 \times 4 = 80$ ft.; if the ship had not been moving, he would have reached the point C, and AC would equal 80 ft., the distance from A. But, in 4 sec., the ship will have moved $30 \times 4 = 120$ ft., and relative to the point A, the man will be at D, when $CD = AB = 120$ ft. Also, his direction relative to the point A will be along the diagonal AD of the parallelogram $ABDC$. At the end of 1 sec., he would be at 1′; at the end of 2 sec., at 2′; at the end of 3 sec., at 3′, etc., 1′1, 2′2, and 3′3 all being parallel to AB. Relative to the fixed point on the rock, the distance moved by the man is represented by AD, measured to the same scale as AB and AC;

and since the time was 4 sec., his velocity relative to the fixed point is $AD \div 4 = s \div t = v$, in which s = the distance (space) AD and t = the time in seconds. Consequently, if AB and AC represent velocities to some scale, AD also represents a velocity, and to the same scale, in both magnitude and direction. AD is called the **resultant velocity** or the **resultant** of the velocities AC and AB.

137. It will be noted that in the case just given, the direction in which the man runs has a component parallel to the direction of the ship; that is, drawing CE perpendicular to AB, the component AE equals the distance BF, DF being perpendicular to AB. But BF is the additional velocity in the direction AB over the velocity of the ship. In other words, the velocity AD may be resolved into a component velocity AF and another component velocity FD normal to AB; in which case, $AF = AB + BF$.

If the man had started at A and had run in exactly the opposite direction, as indicated by AC' in Fig. 81 (b), the velocity represented by AC' would have a component EC' opposed to the velocity of the ship. At the end of 4 sec., the man would be at D' relative to the point A on the rock, and AD' would represent the resultant velocity. AD' may be resolved into the component velocities AE and ED'; and the velocity of the man relative to the point on the rock is equal to $C'D'$ $(= AB) - EC' = ED'$. In the first case, the velocity of the man relative to A and in the direction AB is AF, while in the second case, it is ED'.

The same rules govern the composition and resolution of velocities that apply to forces; either the triangle or parallelogram of velocities may be used, but it is necessary that the velocities be uniform.

138. Absolute Velocity.—As regards bodies belonging to the earth, the velocity of a body relative to a fixed point on the earth is called the **absolute velocity** of the body. As was pointed out in *Physics*, there is really no such thing as absolute motion; but, except in astronomical calculations, all movements with which man is concerned pertain to the earth or to bodies on or within the earth, and it is therefore convenient and proper to define absolute velocity as above. In the last article, the absolute velocity of the man was his velocity relative to the fixed point on the rock. In Art. **135**, the absolute velocity of the body northeast was 48 ft. per sec., and the absolute velocity north (and also

east) was 33.94 ft. per sec. A statement of this kind always implies that the velocity is relative to a fixed point on the earth's surface, the earth being supposed to be stationary, unless otherwise especially stated.

139. Combination of Uniform and Variable Velocity.—A body may have a uniform motion and at the same time have a variable motion in another direction; the most common example of this is the case of a falling body when to the body is given a motion making an angle with a vertical line. Thus, referring to Fig. 82, suppose a body is moving in the direction OC with a velocity of 100 ft. per sec. and that when it reaches the point O, it begins to fall under the action of gravity; it will not fall straight down, but will continue to move in the general direction of OC while, falling, in accordance with the first law of motion. The downward velocity will be constantly accelerated, with the result that the path is a curve $Oab. \ . \ . \ h$, which can be shown by higher mathematics to be a curved line called a parabola.

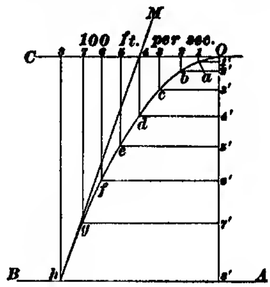


FIG. 82.

The constant acceleration is $g = a = 32.16$ ft. per sec.², and in 8 sec. the body will fall through a distance $= s = \frac{1}{2}at^2 = \frac{1}{2} \times 32.16 \times 8^2 = 1029$ ft. $= O8'$ in Fig. 82, neglecting the resistance of the air. At the same time, the body will have moved in the direction OC a distance $= O8 = s' = vt = 100 \times 8 = 800$ ft. The resultant of these two motions is the path $Oab. \ . \ . \ h$, and the velocity at any point along this path will be equal to the length of the path from O to that point divided by the time it takes to reach that point.

Suppose a railway train is moving at the rate of 70 ft. per sec. and a baseball is thrown from it in a horizontal direction at right angles to the direction of the train, with a velocity of 90 ft. per sec. The ball will continue in the general direction of the train with a uniform velocity of 70 ft. per sec.; it will also have a uniform velocity of 90 ft. per sec. at right angles to the other uniform velocity; and it will fall toward the ground with a constant acceleration of 32.16 ft. per sec.² The resultant of these three velocities will be a parabola whose plane makes an angle

of 52° 7′ 30″ with the direction of motion of the train. The resultant of the two uniform velocities will be a right line, and a vertical plane containing this line will also contain the path of the body, which will be a parabola. When a man steps from a moving train, he has the same absolute velocity as the train; on coming in contact with the earth, he is brought to a sudden stop, which has the same effect on his body as though he were impelled forwards with a push, the strength of which will depend upon the velocity of the train. By running in the same direction as the train, he brings himself gradually to rest and diminishes the effect of the push; or, if he jumps backwards, i.e., in a direction opposite to that of the train, he will diminish, perhaps destroy entirely, his absolute velocity in the direction of the train. Even when the train is moving quite slowly, the result may be a bad fall if proper precautions are not taken.

FALLING BODIES

140. Formulas for Falling Bodies.—A body descending from a higher level to a lower one under the influence of gravity is called a **falling body;** if there is nothing to oppose the motion, as when a body falls in a vacuum, the body is said to fall **freely** or it is called a **freely-falling** body. When it falls in air, it meets with a resistance that increases as the velocity increases, but this resistance is not uniform. There is also a resistance due to the buoyant action of the air, which for short heights of fall may be considered as uniform, but not for great heights, because the density of the air decreases with the altitude. However, in practice, when the velocity acquired is not very great (height of fall is comparatively small) and the density of the body is great enough to make the buoyant effect of little account, bodies are treated as freely falling when meeting with no other resistances. For instance, a stone dropped from the top of a high building will be considered as a freely falling body in what follows.

141. A body is caused to fall by reason of a constant force acting on its mass; the force is called **gravitation,** and it produces a constant acceleration, which is always denoted in works on mechanics by g. As was pointed out in *Physics,* the value of g varies with the latitude and altitude (distance above sea level) of the place where the body falls. The international Standard

for g is 980.665 cm. per sec.^{2} = 32.1741 ft. per sec.^{2}, which corresponds very closely to latitude 45° at sea level. For latitude of New York, the value of g is very *closely 32.16 ft. per sec.^{2}, and* this value will be used in all future calculations, unless otherwise specially stated. The constant $\sqrt{2g}$ occurs very frequently, in calculations pertaining to falling bodies; unless otherwise specially stated, its numerical value will be taken as 8.02.

142. While the formulas of Arts. **131–133** may be used for solving any problem pertaining to freely falling bodies, it is customary to represent the height of fall by h and the acceleration by g, in which case the formulas mentioned become

$$v = gt = 32.16t \qquad (1)$$
$$v = \sqrt{2gh} = 8.02\sqrt{h} \qquad (2)$$
$$v = v_0 + gt = v_0 + 32.16t \qquad (3)$$
$$h = \tfrac{1}{2}vt \qquad (4)$$
$$h = \tfrac{1}{2}gt^2 = 16.08t^2 \qquad (5)$$
$$h = v_0t + \tfrac{1}{2}gt^2 = v_0t + 16.08t^2 \qquad (6)$$
$$h = \tfrac{1}{2}(v_0 + v)t \qquad (7)$$
$$h = \frac{v^2}{2g} = .015547v^2 \qquad (8)$$
$$t = \frac{v}{g} = .031095v \qquad (9)$$
$$t = \frac{2h}{v} \qquad (10)$$
$$t = \sqrt{\frac{2h}{g}} = .24938\sqrt{h} \qquad (11)$$
$$v = \sqrt{v_0^2 + 2gh} \qquad (12)$$

The last formula (which corresponds to formula 6 of Art. **133**), is derived as follows: from (3), $t = \frac{v - v_0}{g}$; from formula (7), $h = \frac{1}{2}(v_0 + v)t$; substituting the value of t, $h = \frac{1}{2}(v + v_0)\left(\frac{v - v_0}{g}\right) = \frac{v^2 - v_0^2}{2g} = h$; from which, $v^2 = 2gh + v_0^2$, and $v = \sqrt{v_0^2 + 2gh}$. This formula is useful when it is desired to find the velocity with which a body would strike the ground when thrown downward from a known height h with an initial velocity v_0. While all twelve formulas are used, it is probable that formulas (1), (2), (5), (8), and (11) are most frequently employed. In practice, .0155 is usually sufficiently accurate in

(8), .031 in (9), and .25 in (11), because of the limit of accuracy in measuring some factor.

143. If a body move upward instead of downward, it must have an initial velocity; this velocity will carry the body to a height indicated by formula (8), which is the velocity that would be attained by falling through the height h, the value obtained by applying formula (2), which is another way of writing formula (8). When the body has reached the height h, it stops, but not for any measurable length of time, since it immediately begins to fall, and on striking the earth, will have the same velocity v that it had on starting upwards; that is, the two velocities would be equal but for the resistance of the air. They are assumed to be equal in practice, when the velocity v_0 is not too great. In the case of a rifle or cannon ball, the velocity is so great that the resistance of the air must be considered, if the range is to be calculated with any degree of exactness. Applications of formulas (1)–(12) will now be shown by several examples. It may be remarked that it is useless to calculate results correct to more than four significant figures when $g = 32.16$.

EXAMPLE 1.—A stone is dropped from the top of the Woolworth building which is, say, 800 ft. above the ground; (*a*) how long will it take for the stone to reach the ground? (*b*) with what velocity will it strike?

SOLUTION.—(*a*) Here h and g are known; hence, use formula (11) to find t, and $t = \sqrt{\frac{2h}{g}} = .24938\sqrt{800} = 7.054-$ sec. *Ans.*

(*b*) The velocity may now be calculated by either formula (1) or by formula (2); using formula (2), $v = \sqrt{2gh} = 8.02\sqrt{h} = 8.02\sqrt{800} = 226.8$ ft. per sec. *Ans.*

EXAMPLE 2.—A baseball is thrown vertically upward with a velocity of 90 ft. per sec.; (*a*) how high will it rise after leaving the hand? (*b*) how long before it returns to the point from which it started?

SOLUTION.—(*a*) Here v and g are given and h is to be found; hence, using formula (8), $h = \frac{v^2}{2g} = .015547v^2 = .015547 \times 90^2 = 125.9$ ft. *Ans.*

(*b*) The time will be twice the time required for the ball to fall through the height h; hence, either formula (9), (10), or (11) may be used. Using formula (9), $t = \frac{v}{g} = .031095v = .031095 \times 90 = 2.7986$, say 2.8 sec.; and the total time is $2.8 \times 2 = 5.6$ sec. *Ans.*

EXAMPLE 3.—In order to find the depth of a deep well, a stone is dropped within it, the time that elapses between the instant of starting and the sound of its striking is accurately measured, and is found to be 3.5 sec. Taking the velocity of sound in air as 1090 ft. per sec., what is the depth of the well? Calculate only to the nearest foot.

SOLUTION.—The total time is 3.5 sec. The time required for the sound to reach the car is equal to the depth of the well h divided by the velocity of sound, or $\frac{h}{1090}$; hence the time of falling is $3.5 - \frac{h}{1090} = \frac{3815 - h}{1090} = t$. Using formula (5), $h = \frac{1}{2}gt^2 = 16.08\left(\frac{3815 - h}{1090}\right)^2 = h$. Squaring, transposing, and combining terms,

$$h^2 - 81517h = -14554225, \text{ from which } h = 179 \text{ ft.} \quad Ans.$$

To prove that the result is correct, the time required for the sound to travel 179 ft. is $179 \div 1090 = .164$ sec.; the time required for the stone to fall 179 ft. is $t = \sqrt{\frac{2h}{g}} = .24938\sqrt{179} = 3.336$ sec., and the total time is $3.336 + .164 = 3.500$ sec., as it should.

EXAMPLE 4.—If a body be thrown downwards with a velocity of 60 ft. per sec., what will be its velocity on striking the ground 600 ft. below?

SOLUTION.—Evidently formula (12) must be used in this case; consequently, $v = \sqrt{v_0^2 + 2gh} = \sqrt{60^2 + 2 \times 32.16 \times 600} = 205.4$ ft. per sec. *Ans.*

144. Projectiles.—Any freely moving body is a **projectile**; examples are a rifle bullet, a baseball, a jet of water, all moving freely through the air. For a body to become a projectile, it must have an initial velocity and must be acted on only by gravity and the resistance of the medium through which it passes, as air or water. A heavy (dense) body moving freely through still air at a comparatively low velocity may be considered as acted on only by gravity, and the resistance of the air may be neglected; the path of every such body is a parabola, except when moving in a vertical line. If the direction of the initial velocity be horizontal, as in Fig. 82, and the body falls downwards under the influence of gravity, the body will describe the path *Oab. . .h*, which is one-half of a parabola, the other half being on the other side of *O8′* and symmetrical to the half shown. The distance *8′h*, which is the horizontal distance between the starting and stopping points, is called the **range**. The range is evidently equal to the time required for the body to fall through the height *O8′* multiplied by the initial velocity. For instance, if the time required to pass through the height *O8′* is 8 sec. and the initial velocity is 100 ft. per sec., the range, when the direction of the initial velocity is horizontal, is $8 \times 100 = 800$ ft.

145. Let *hM* be a tangent to the parabola at *h*, and suppose that the body start from the point *h* with an initial velocity v_0 in the direction of the tangent *hM*. Resolve into two components, one horizontal and the other vertical, the initial velocity v_0, as

indicated in Fig. 83, the angle A being equal to the angle MhA in Fig. 82. If the value of v_0 be such that the horizontal component v'_0 equals the initial horizontal velocity along OC in Fig. 82, and the vertical component v''_0 equals the velocity acquired in falling through the height $h = OS'$, then the projectile on leaving the point h will follow the path $hgf. \ . \ .O$, Fig. 82, which is identically the same, but in the reverse direction, as $Oabc. \ . \ . h$. *On reaching the point O, all movement in a vertical direction ceases; the horizontal movement continues, however, and as the projectile falls, it describes the other half of the parabola.* The time in falling is the same as the time in rising; hence, the range is twice as great as is the case in Fig. 82.

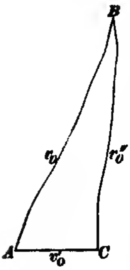


Fig. 83.

In Fig. 84, suppose the initial velocity has the direction AM, the projectile starting from A. The path described will be the parabola ABC; the range is AC; and the greatest height is OB.

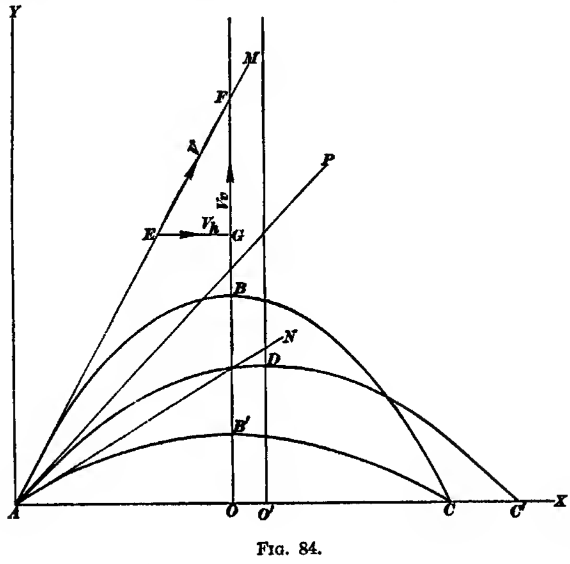


Fig. 84.

To find the values of the height OB and the range AC, let EF represent, to scale, the initial velocity V, EF being the direction in which the projectile starts and coinciding with or parallel

to AM, the tangent. Resolve V into two components, $EG = V_h$, the horizontal velocity, and $GF = V_v$, the vertical velocity; measure V_h and V_v; then, by formula (8), Art. **142**, $h = \frac{V_v^2}{2g} = .015547\, V_v^2 = OB$. Knowing the value of $h = OB$, the time required for the projectile to ascend this height may be found by (9) or (10); using formula (9), $t = \frac{V_v}{g} = .031095\, V_v$. The time that the projectile is in the air will be twice this, or $T = 2t$, and this multiplied by the horizontal velocity gives

$$\text{range} = AC = r = 2t\, V_h.$$

The angle which the tangent AM (the direction of the projectile at starting) makes with the horizontal (angle MAC) is called the **angle of elevation.**

146. In general, there are two angles of elevation that will give the same range for the same initial velocity. Since ACB, Fig. 83, is a right triangle, $B = 90° - A$; consequently, if A = the angle of elevation, an angle of elevation $= 90° - A$ will give the same range for the same initial velocity $v_0 = V$. Thus, referring to Fig. 84, if the range is AC when the angle of elevation is MAC and the initial velocity is V, the range will still be AC when the angle of elevation is $NAC = 90° - MAC = MAY$. The path of the projectile in the first case will be the parabola ABC; in the second case, the path will be $AB'C$. This fact can be readily proved by means of a garden hose; if the water leaves the nozzle in the direction AN, the range will be exactly the same as when it leaves the nozzle in the direction AM.

When the angle is 45°, the range will be a maximum, because $90° - 45° = 45°$, and there is only one angle that will give the same range. In Fig. 84, $PAC' = 45°$, the range is AC' and the greatest (maximum) height for this range is $O'D$.

In practice, the foregoing facts are not true for high velocities, such as those attained by rifle bullets and cannon shots, whose velocities vary between, say, 2000 and 3000 ft. per sec. The path is then no longer a parabola, the maximum range is attained when the angle of elevation is considerably less than 45°, and for a high angle of elevation the maximum height is so great that the air is less dense and offer less resistance than when the smaller angle is used.

EXAMPLE.—Suppose a baseball player can throw a ball with an initial velocity of 120 ft. per sec.; what is the greatest distance the ball can travel,

the range being measured on a horizontal line passing through the point at which the ball left his hand? What would have been the range if the angle of elevation had been 60°?

SOLUTION.—Referring to Fig. 84, if angle FEG is 45°, $FG = EG = V \times \frac{1}{2}\sqrt{2}$, $t = .031095V \times \frac{1}{2}\sqrt{2}$, and $r = EG \times 2t = V \times \frac{1}{2}\sqrt{2} \times .031095\,V \times \frac{1}{2}\sqrt{2} \times 2 = .031095V^2 = .031095 \times 120^2 = 447.8$ ft. *Ans.*

If angle of elevation $MEG = 60°$, $FG = V_v = V \times \frac{1}{2}\sqrt{3}$, $EG = V_h = \frac{1}{2}V$, and $r = .031095 \times V \times \frac{1}{2}\sqrt{3} \times 2 \times \frac{1}{2}\,V = .031095\,V^2 \times \frac{1}{2}\sqrt{3} = 387.8$ ft. *Ans.*

If the angle of elevation is $90° - 60° = 30°$, the range will also be 387.8 ft.

The maximum height that the ball rises in each case is, by formula (8), Art. **142**, for the first case, $h = .015547\,(\frac{1}{2}\sqrt{2} \times V)^2 = .015547 \times \frac{1}{2} \times 120^2 = 111.9$ ft.; for the second case, $h = .015547(\frac{1}{2}\sqrt{3} \times V)^2 = .015547 \times \frac{3}{4} \times 120^2 = 167.9$ ft.; for the third case, $h = .015547\,(\frac{1}{2} \times V)^2 = .015547 \times \frac{1}{4} \times 120^2 = 55.97$ ft.

EXAMPLE 2.—A jet of water issues from an orifice in a horizontal direction with a velocity of 113 ft. per sec.; if the center of the orifice is 10 ft. 3 in. above the ground, what is the range?

SOLUTION.—By formula (11), Art. **142**, $t = \sqrt{\frac{2h}{g}} = .24938\sqrt{h} = .24938\sqrt{10.25} = .80191$ sec., the time it will take the water to fall from the center of the orifice to the ground. Therefore, the range is $r = v_0t = 113 \times .80191 = 90.62$ ft. *Ans.*

147. The path of a projectile is frequently called its **trajectory**; and the higher the initial velocity the *flatter* will be the trajectory between two points. If a gun be pointed directly at the point it is desired to hit, the ball will necessarily strike below that point; for this reason, the sights are "raised," thus causing the gun to aim at a point above the point it is desired to strike. This adjustment of the sights is made according to the range and the velocity of the bullet.

ANGULAR VELOCITY AND ACCELERATION

148. Angular Velocity.—If a body in motion be acted upon by a force, its *velocity* may change or its *direction* may change or both may change. In the case of a projectile, both the direction and velocity change; this fact is indicated by the curved path and the further fact that the distances passed over in equal times are different. According to the first law of motion, the body must move in a right line unless acted upon by an *external* force; in the case of a projectile, the external force is the force of gravity.

Suppose a ball M to rest on a horizontal frictionless plane, with its center connected to a fixed pivot O, Fig. 85 (a), by a string CO.

If, now, the ball be struck a sharp blow in a direction parallel to the plane and perpendicular to *CO*, it will move in a circle having *O* for its center and *CO* for its radius; the velocity will be proportional to the effect of the blow, and if there is no friction or other resistances, the ball will move forever in the circle, and with undiminished velocity. The reason for this is that the direction of motion at any point is a tangent to the circle at that point; thus, for the point *C*, the direction of motion (velocity) is the same as that of the tangent *MA*, and if it were not for the string, the ball would move in the direction *CA*, when moving

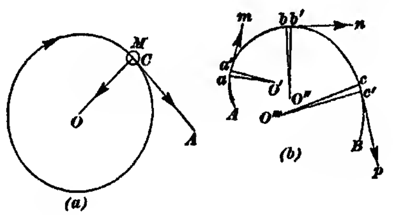


FIG. 85.

clockwise around the circle; in other words, if the string were cut at the instant the ball reached the point *C*, it would then continue in the direction *CA*, and with the same velocity that it had when it reached the point *C*. The ball is caused to move in a circle as the result of a pull exerted by the string; and since this pull is radial, its direction is that of the radius *CO*. But the radius drawn from the center to any point of a circle is perpendicular (normal) to the circle and to the tangent at that point. This is likewise true of any other point of the circle; hence, the pull exerted by the string is always perpendicular to the tangent, that is, to the direction of motion, and therefore has no influence in changing the velocity along the circle. Consequently, if the body meets with no resistances, such as friction, the atmosphere, etc., it will move forever with undiminished peripheral velocity.

If v = the linear (= peripheral) velocity in feet per second, r = the radius in feet, and n = number of revolutions per second, $v = 2\pi rn$. Let ω (Greek letter omega) = the number of radians turned through in one second; then, since one revolution = 2π

radians, $\omega = 2\pi n$ radians per second, which is called the **angular velocity** of the ball. Comparing the two velocities,

$$\omega : v = 2\pi : 2\pi r$$

from which,

$$\omega = \frac{v}{r}$$

That is, *the angular velocity is equal to the linear velocity divided by the radius of the arc in which the body moves;* and the linear velocity is equal to the angular velocity multiplied by the radius, $=$ or $v = r\omega$. For example, suppose a flywheel makes 150 r.p.m.; the angular velocity is $\omega = 2\pi n = 2 \times 3.1416 \times \frac{150}{60} = 15.708$ radians per second. If the radius to a point on the outside of the rim is 90 in., $v = 2\pi \times \frac{90}{12} \times \frac{150}{60} = 117.81$ ft. per sec. Then $\omega = \frac{v}{r} = \dfrac{117.81}{\frac{90}{12}} = 15.708$ rad. per sec., as before. Also, since $v = \omega r$, $v = 15.708 \times \frac{90}{12} = 117.81$ ft. per sec.

149. If the body move in a curve that is not a circle, as AB, Fig. 85 (b), the angular velocity is determined in the following manner: suppose the body to be at the point a and its linear velocity to be v'; suppose the body to move a very short distance (preferably, an infinitely small distance) to a'; then that part aa' of the path may be considered as an arc of a circle, the radius being $O'a$ and the center O'. Then O' is called the **instantaneous center** for the minute arc aa', and $O'a = r'$ is called the **radius of curvature**; a tangent to the radius at a will be perpendicular to the radius of curvature $O'a = r'$. The angular velocity at a is then $\omega' = \frac{v'}{r'}$. In a similar manner, let O'' be the instantaneous center for the point b, v'' and r'' the linear velocity and radius of curvature corresponding to b; then, $\omega'' = \frac{v''}{r''}$; etc. The tangents am, bn, and cp indicate the direction of the linear velocity at a, b, and c, respectively. In the case of a body moving in a circle, the instantaneous center is always the center of the circle, the radius of curvature is always the radius of the circle, and $\omega = \frac{v}{r}$, v being the linear velocity at any point. In what follows, only uniform velocity in a circle will be considered.

150. Radial Acceleration.—Referring again to Fig. 85 (a), the ball is acted upon by a force due to the pull of the string; so long as the angular velocity remains constant, this force also remains constant; and since a constant force always produces a constant acceleration, there is a constant acceleration directed toward the center. The ball, however, gets no nearer the center, because the acceleration is just sufficient to keep the ball moving in its circular path, and this is always the case, regardless of the form of the path. The manner of deriving the expression for the value of the acceleration toward the center, which may be termed the **radial acceleration**, is somewhat too technical to be given here; but, letting v = the linear velocity, ω = the angular velocity, and r = the radius,

$$\text{radial acceleration} = \frac{v^2}{r} = r\omega^2$$

FORCE AND MOTION

MOMENTUM

151. Force Required to Stop a Moving Body.—It was shown in *Physics* that if f = force in pounds, m = mass of body moved, w = weight of body moved, and a = acceleration of moving body, the force required to give the body an acceleration a is

$$f = ma \qquad (1)$$

If a body having a certain velocity v is brought to rest under the action of a constant force, it will have a constant negative acceleration $-a$, which will be the same in value as the acceleration required to give the body the velocity v when starting from 0; hence, the force required to bring the body to rest by making its velocity decrease uniformly from v to 0 with a constant acceleration a is also $f = ma$. By formula (1), Art. **131**, $v = at$, from which $a = \frac{v}{t}$. Substituting this value of a in formula (1), above, $f = m \times \frac{v}{t}$, or

$$ft = mv \qquad (2)$$

that is, the force required to bring a body having a velocity v to rest in a time t multiplied by the time is equal to the mass of the

body multiplied by the velocity. Since $m = \frac{w}{g}$, formula (2) may be written,

$$ft = \frac{wv}{g} \qquad (3)$$

If w is in pounds, v in feet per second, g in feet per second per second, and t in seconds, f will be in pounds; if w is in kilograms (or grams), v in centimeters per second, g in centimeters per second per second, and t in seconds, f will be in kilograms (or grams).

EXAMPLE 1.—A body weighing 128 pounds is started from rest under the action of a force and is given a velocity of 40 ft. per sec. in 3 sec.; neglecting friction and other resistances, what force was required?

SOLUTION.—Solving formula (3) for f,

$$f = \frac{wv}{gt} = \frac{128 \times 40}{32.16 \times 3} = 53.07- \text{ lb.} \quad Ans.$$

EXAMPLE 2.—What force is required to give a body weighing 280 lb. an acceleration of 12.4 ft. per sec.^{2}?

SOLUTION.—Since the acceleration is known, use formula (1), and

$$f = ma = \frac{280}{32.16} \times 12.4 = 107.96, \text{ say } 108 \text{ lb.} \quad Ans.$$

152. Formula (2) of the last article is the fundamental formula of dynamics; it is so important that the expressions on either side of the sign of equality have been given special names; that on the left-hand side, ft, is called the **time effect**; that on the right-hand side, mv, is called the **momentum** of the body. For any moving body, the mass multiplied by the velocity is called the momentum; the force required to stop the body in a given time is the time effect; if the time is one second, then the momentum is equal to the force, and momentum may be defined as the force that is required to stop a moving body in one second. This definition must be clearly understood; it is the steady (constant) force which, acting for one second will bring the body to rest, and it is equal to the mass of the body multiplied by its velocity in linear units per second. Thus, if a body weighing 448 kilograms have a velocity of 36 meters per second, its momentum (since 36 m. = 3600 cm.) will be $mv = \frac{448}{980.665} \times 3600 = \frac{448}{9.80665} \times 36 = 1644.6-$. If this body were brought to rest by a steady force acting for one second, the value of the force would be 1644.6 kilograms. Note that when g is taken as 980.665 cm. per sec.^{2}, the velocity must be in centimeters per second; but if the velocity

is in meters per second, g may be taken as 9.80665 meters per sec.^{2}

There is no name for the unit of momentum; if, however, the weight be taken as 1 lb., the velocity as 1 ft. per sec., and the unit of g as 1 ft. per sec.^{2}, then

$$mv = \frac{wv}{g} = \frac{1 \text{ lb.} \times \dfrac{1 \text{ ft.}}{1 \text{ sec.}}}{\dfrac{1 \text{ ft.}}{1 \text{ sec.}^2}} = 1 \text{ lb.} \times 1 \text{ sec.}$$

that is, the unit may be called the pound-second. For the time effect of the other side of the sign of equality, $ft = 1 \text{ lb.} \times 1 \text{ sec.}$, which agrees with the unit for momentum. In the metric system, the unit of momentum will be the gram-second or kilogram-second, according to whether the weight of the body is taken in grams or kilograms.

EXAMPLE.—A locomotive with its train weighs, say, 500 tons. Starting from rest, it gets up a speed of 30 miles per hour in just 4 minutes; if the acceleration be assumed to be constant, (*a*) what steady force must be exerted, neglecting all resistances? In other words, what force in addition to that required to overcome the resistances must be exerted to enable the locomotive and its train to get up to speed? (*b*) At 50 miles per hour what is its momentum?

SOLUTION.—(*a*) Reducing the tons to pounds, minutes to seconds, and miles per hour to feet per second and using formula (3) of Art. **151**, after solving for f,

$$f = \frac{wv}{gt} = \frac{500 \times 2000 \times 30 \times 5280}{32.16 \times 4 \times 60 \times 60 \times 60} = 5700 \text{ lb., very nearly.} \quad \textit{Ans.}$$

(*b*) At 50 miles per hour, the momentum is

$$mv = \frac{wv}{g} = \frac{500 \times 2000 \times 50 \times 5280}{32.16 \times 60 \times 60} = 2{,}280{,}265 \text{ lb.-sec.} \quad \textit{Ans.}$$

153. Generating Motion by Weights.—When motion in two or more bodies is due to the action of gravity on one of the bodies, the force causing the movement is, in general, easily found, and when divided by the mass of all the bodies moved, gives a quotient that is the acceleration of each of the moving bodies. Thus, from formula (1), Art. **151**, $a = \frac{f}{m}$; hence, if the force causing the movement is known and the mass moved by this force is also known, the acceleration can be found. The two problems that follow will make this clear.

Problem 1.—In Fig. 86, a weight is shown resting on the top of a table, which will be assumed to be a perfectly smooth horizontal plane surface; attached to the weight P is a cord, which

passes over a pulley and has weight W at the other end. If the part ab of the cord is horizontal and all resistances are neglected, find (*a*) an expression for the acceleration in terms of P and W; (*b*) find the tension in the cord in terms of P and W; if $P = 45$ lb. and $W = 72$ lb., what is (*c*) the acceleration? (*d*) the tension in the cord?

(*a*) W is caused to move downwards by the action of gravity, but gravity does not directly move P because the force that

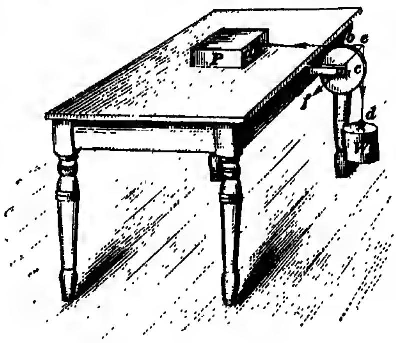


FIG. 86.

gravity exerts on P is counteracted by the reaction of the table; consequently, since P must move to the right when W moves downwards, the total mass moved by the action of gravity on W is $\frac{W+P}{g}$ + mass of cord + mass of pulley (in turning). The last two elements may be neglected in the present case, and the mass moved may be considered as $\frac{W+P}{g}$. The force producing this movement is the weight of W; hence, the acceleration is

$$a = \frac{f}{m} = \frac{fg}{w} = \frac{Wg}{W+P}. \qquad \textit{Ans.}$$

Here $f = w$ and $m = \frac{W+P}{g}$.

(*b*) If there were no friction (as is here assumed), and W were to move downwards with a uniform velocity, there would be no tension in the cord, because, since ab is horizontal and P moves in a horizontal direction, there is no component force acting upward. The conditions are exactly the same in respect to the

force acting on P as though the cord were cut and a force equal to that required to accelerate P were applied to the free end. The mass moved would then be $\frac{P}{g}$, and this multiplied by the acceleration, the value of which was found above, equals the force required to accelerate P, and is equal to the tension in ab $= T$; it also equals the tension in cd, since the tension of the string is the same throughout. Therefore,

$$T = ma = \frac{P}{g} \times \frac{Wg}{W + P} = \frac{PW}{W + P}. \quad Ans.$$

(c) Since $P = 45$ lb. and $W = 72$ lb.,

$$a = \frac{Wg}{W + P} = \frac{72 \times 32.16}{45 + 72} = 19.79 \text{ ft. per sec.}^2 \quad Ans.$$

If P and W are interchanged, the acceleration is $\frac{Pg}{W + P}$ $= \frac{45 \times 32.16}{45 + 72} = 12.37-$ ft. per sec.^{2}

(d) The tension of the cord is

$$T = \frac{PW}{W + P} = \frac{45 \times 72}{45 + 72} = 27.69+ \text{ lb.} \quad Ans.$$

If P and W be interchanged, the tension of the cord will be exactly the same as before, because, although the mass of the body on the table will then be greater, the acceleration will be less, and in the same proportion. If friction be considered, this will not be true, since the force of friction will be greater in the latter case than in the former.

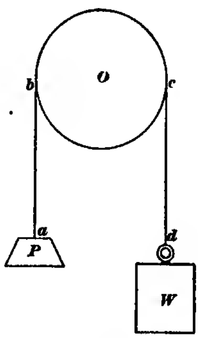


FIG. 87.

Problem 2.—Suppose that the weight P be suspended at one end of the cord as shown in Fig. 87; neglecting all resistances, find (a) the acceleration, (b) the tension in the cord, the weights of P and W being the same as in Problem 1.

(a) The moving force is evidently equal to $W - P$, since W and P act in opposite directions. If $W = P$, there will be no motion, since the system will then be in static equilibrium. As the result of the action of this force, P and W both move, and since the force is constant, both bodies receive a constant accel-

eration. The mass moved is $\frac{W+P}{g}$; consequently, the acceleration is

$$a = \frac{fg}{w} = \frac{(W-P)g}{W+P} = \frac{(72-45)32.16}{72+45} = 7.422- \text{ ft. per sec.}^2$$

Ans.

(*b*) The tension of the cord will be equal to the tension in *ab*, and this evidently equals the weight of P + the force required to accelerate P; that is, since the mass of P is $\frac{P}{g}$,

$$T = P + \frac{P}{g} \times a = P + \frac{P}{g} \times \frac{(W-P)g}{W+P} = P + \frac{P(W-P)}{W+P}$$

Clearing the right-hand member of fractions, combining and reducing,

$$T = \frac{2PW}{W+P} = \frac{2 \times 45 \times 72}{72+45} = 55.38+ \text{ lb.} \quad \textit{Ans.}$$

It will be observed that the tension of the cord is here twice as great as in the arrangement of Problem 1; note, also, that interchanging the two bodies P and W will produce no other effect than to reverse the direction of rotation of the pulley.

Considering the expression $a = \frac{(W-P)g}{W+P}$, the acceleration a may be made as small as desired by making the difference between the weights of W and P sufficiently small. It is by means of a device of this kind that an accurate measurement of a can be made experimentally; and when a is known, the value of g can be found at once. Thus, suppose the pulley is very light and that its journals turn on ball bearings, thus practically eliminating friction; suppose further that the cord is a fine silk thread, that $W = 2$ ounces, and $P = 1.9$ ounces. If, now, by accurate measuring and timing devices, it is found that W falls 3.96 ft. in 3.1 sec. then, by formula (3), Art. **131**, $a = \frac{2 \times s}{t^2} = \frac{2 \times 3.96}{3.1^2} = .8241$ ft. per sec.^{2} The expression found above for the acceleration is $a = \frac{(W-P)g}{W+P}$; from which $g = \frac{a(W+P)}{W-P}$. Substituting the value of a, as just found, and W and P,

$$g = \frac{.8241(2+1.9)}{2-1.9} = 32.14 \text{ ft. per sec.}^2$$

a very close approximation to the value of g at a place where

$g = 32.16$. The true value of g is one of the most important constants used in science.

154. Effect of Friction.—Friction always *opposes* the motion; for most cases that arise in practice, it may be regarded as a *constant* force acting in direct opposition to the force producing the motion. Thus, referring to Problem 1, of the last article, suppose the coefficient of friction is $\mu = .22$ between P and the table top; then the normal pressure is P, and a force equal to $.22P$ must be exerted before any movement can occur. Neglecting the friction of the pulley and cord, the force producing motion is $W - .22P$, and the acceleration is, the mass moved being the same as before, $a = \frac{(W - \mu P)g}{W + P} = \frac{(72 - .22 \times 45)32.16}{72 + 45}$ $= 17.07-$ ft. per sec.[2] If the weights P and W be interchanged, $a = \frac{(P - \mu W)g}{P + W} = \frac{(45 - .22 \times 72)32.16}{45 + 72} = 8.015+$ ft. per sec.[2]

To find the tension of the cord, the force required to move P and give it the acceleration due to the action of W is $\mu P + \frac{P}{g}a$; substituting the value of a just found,

$$T = \mu P + \frac{P}{g} \times \frac{(W - \mu P)g}{W + P} = \frac{(1 + \mu)PW}{W + P} = \frac{(1 + .22)45 \times 72}{72 + 45}$$
$$= 33.78 \text{ lb.}$$

The same value for T will be found if the weights are interchanged.

If it is desired to take into account the friction of the pulley also, the problem becomes indeterminate, because the normal pressure on the pulley bearing cannot be found until the tension in the cord is known. This may be approximated as closely as is desired by calculating T when the friction of the pulley is neglected; taking the value thus obtained as the tension, find the normal pressure on the bearing, and calculate the friction; this added to T will give a value T', which will be quite near the actual value of the tension. A repetition of the process will give a value T'' that is very near the correct value. Thus, the resultant of the tensions in *ba* and *cd*, Fig. 86, evidently has the direction *ef*; *bef* = *cef* = 45°; and *ef* = $T\sqrt{2}$. Taking the value of T just found, and assuming that the coefficient of friction for the bearing is .02, the force of friction for the pulley is $.02 \times 33.78\sqrt{2} = .96-$ lb. This does not increase the tension in *ab*, the part of the cord between the pulley and P, but it does increase

eration. The mass moved is $\frac{W+P}{g}$; consequently, the acceleration is

$$a = \frac{fg}{w} = \frac{(W-P)g}{W+P} = \frac{(72-45)32.16}{72+45} = 7.422- \text{ ft. per sec.}^2$$

Ans.

(*b*) The tension of the cord will be equal to the tension in *ab*, and this evidently equals the weight of P + the force required to accelerate P; that is, since the mass of P is $\frac{P}{g}$,

$$T = P + \frac{P}{g} \times a = P + \frac{P}{g} \times \frac{(W-P)g}{W+P} = P + \frac{P(W-P)}{W+P}$$

Clearing the right-hand member of fractions, combining and reducing,

$$T = \frac{2PW}{W+P} = \frac{2 \times 45 \times 72}{72+45} = 55.38+ \text{ lb.} \quad \textit{Ans.}$$

It will be observed that the tension of the cord is here twice as great as in the arrangement of Problem 1; note, also, that interchanging the two bodies P and W will produce no other effect than to reverse the direction of rotation of the pulley.

Considering the expression $a = \frac{(W-P)g}{W+P}$, the acceleration a may be made as small as desired by making the difference between the weights of W and P sufficiently small. It is by means of a device of this kind that an accurate measurement of a can be made experimentally; and when a is known, the value of g can be found at once. Thus, suppose the pulley is very light and that its journals turn on ball bearings, thus practically eliminating friction; suppose further that the cord is a fine silk thread, that $W = 2$ ounces, and $P = 1.9$ ounces. If, now, by accurate measuring and timing devices, it is found that W falls 3.96 ft. in 3.1 sec. then, by formula (3), Art. **131**, $a = \frac{2 \times s}{t^2} = \frac{2 \times 3.96}{3.1^2} = .8241$ ft. per sec.^{2} The expression found above for the acceleration is $a = \frac{(W-P)g}{W+P}$; from which $g = \frac{a(W+P)}{W-P}$. Substituting the value of a, as just found, and W and P,

$$g = \frac{.8241(2+1.9)}{2-1.9} = 32.14 \text{ ft. per sec.}^2$$

a very close approximation to the value of g at a place where

is very small, the result being that the force of friction is small as compared with that due to the weight sliding on the table top.

155. Motion on an Inclined Plane.—Let ABC, Fig. 88, be an inclined plane whose length is l and height is h, and on which rests a body M that is free to move down the plane. Suppose the weight of the body is P, represented by the vertical line oa, which resolves into two components ba and ob; then ba represents the force urging the body down the plane and ob represents the

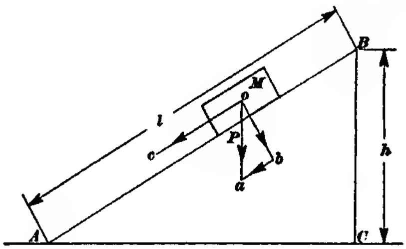


FIG. 88.

normal pressure against the plane. The triangles ACB and oba are similar; whence, the proportion $\frac{ab}{oa} = \frac{BC}{AB}$. But, $oa = P$, $AB = l$, and $BC = h$; letting $ab = f$, the force urging the body down the plane, and substituting in the proportion,

$$\frac{f}{P} = \frac{h}{l}, \text{ or } f = \frac{Ph}{l}$$

The mass of the body is $\frac{P}{g}$; hence, since $a = \frac{f}{m}$, the acceleration down the plane is $a = \frac{\frac{Ph}{l}}{\frac{P}{g}} = \frac{gh}{l}$, friction and other resistances being neglected. From formula (3), Art. **131**, $t = \sqrt{\frac{2s}{a}}$; if t is the time it takes the body to descend the entire length of the plane, then $s = l$, and $t = \sqrt{\frac{2l}{\frac{gh}{l}}} = \sqrt{\frac{2l^2}{gh}} = l\sqrt{\frac{2}{gh}}$. The velocity of the body when it reaches A, the bottom of the plane, is

$v = at = \frac{gh}{l} \times l\sqrt{\frac{2}{gh}} = \sqrt{2gh}$. If the body fell freely from B to C, its velocity on reaching C will be $v = \sqrt{2gh}$, which is exactly the same as that acquired in sliding down the plane. The time, however, will be different in the two cases. Thus, let t = the time required to fall through the height h, and let t' = the time required to slide down the plane; then, since $v = gt$ $= at'$, $t' = \frac{gt}{a} = \frac{gt}{\frac{gh}{l}} = \frac{tl}{h} = \frac{l}{h} \times t$. In other words, the time required for a body to slide down a smooth frictionless plane is equal to the time required for the body to fall through the height of the plane multiplied by the ratio of the length of the plane to the height of the plane.

The fact that the velocity is the same no matter how the body travels from a point of higher level to one of lower level is an extremely important principle in dynamics.

156. If the friction be considered, these results will, of course, be modified. Referring to Fig. 88, let μ = the coefficient of friction; then, the effective force f' acting in the direction oc will be $f' = f - \mu \times ob$. From the similar triangles oba and ACB, $\frac{ob}{oa} = \frac{AC}{AB}$, from which, $ob = \frac{P \times AC}{l}$. But $AC = \sqrt{l^2 - h^2}$; in the last article, the value of f was found to be $f = \frac{Ph}{l}$; hence, $f' = \frac{Ph}{l} - \frac{\mu P}{l}\sqrt{l^2 - h^2} = \frac{P}{l}(h - \mu\sqrt{l^2 - h^2})$, and the acceleration is $a = \frac{f'g}{P} = \frac{g}{l}(h - \mu\sqrt{l^2 - h^2})$.

Suppose, for example, that angle $A = 45°$; then $l = h\sqrt{2}$, and $l^2 = 2h^2$. Substituting these values in the last expression for a, and assuming that $\mu = .18$, $a = \frac{32.16}{h\sqrt{2}}(h - .18\sqrt{2h^2 - h^2})$ $= \frac{32.16}{\sqrt{2}}(1 - .18) = 18.65$ ft. per sec.²

Neglecting friction, the value of a is

$$a = \frac{fg}{P} = \frac{gh}{l} = \frac{32.16 \times h}{h\sqrt{2}} = 22.74 \text{ ft. per sec.}^2$$

If the angle A $= 30°$, $l = 2h$, and

$$a = \frac{32.16}{2h}(h - .18\sqrt{4h^2 - h^2}) = 11.07 \text{ ft. per sec.}^2$$

Neglecting friction, the value of a is

$$a = \frac{gh}{l} = \frac{32.16 \times h}{2h} = 16.08 \text{ ft. per sec.}^2$$

The ratio of the two accelerations in the first case is $\frac{18.65}{22.74}$ $= .826 -$, and in the second case, $\frac{11.07}{16.08} = .688+$. It is evident that the angle A may be decreased until there will be no motion at all; this angle is the same as the angle of friction, which was previously defined.

157. From what has been stated in Arts. **154–156**, the reader will obtain a good idea of the effects of friction in retarding motion. It is not advisable to pursue the subject further in an elementary work of this kind. It may be stated that in connection with the operation of machines, the friction of the separate parts as they move relatively to one another is seldom considered, the resistances being all grouped and considered as a whole in ascertaining the efficiency of the machine. The subject of efficiency was discussed in Arts. **94–97**.

CENTRIFUGAL FORCE

158. Central Forces.—Whenever a body moves in a curved path instead of a right line, it is caused to do so by the action of a force normal to the direction of motion, or else the acting force has a component acting in this direction. As was shown in connection with Fig. 85, a very short part of the path may be considered as a circular arc; then this normal force becomes a radial force, which acts toward the instantaneous center. Forces that act toward or away from a center are called **central forces,** and any force acting toward a center has received the special name of **centripetal force.** Referring to Fig. 85 (a), suppose the body to move in a circle, with a uniform angular velocity $= \omega$. The force exerted by the string, and which pulls on the ball, is the centripetal force. The reaction of the ball on the string is called the **centrifugal force.** The centrifugal force being a reaction only, it does not cause or tend to cause motion, but the centripetal force actually produces motion toward the center. If the ball revolve fast enough, the string will break, and the ball will no longer move in a curve, but in a right tangent to the circle that forms its path; whence the phrase "flying off at a tangent."

Although the centripetal force is the active force, still the centrifugal force is equal and opposite to it, and both originate and cease at the same instant; and any motion that the body may have after they have ceased to act is due to the linear velocity that it had while moving in the curved path.

159. Formulas for Centrifugal Force.—It was stated in Art. **150** that the radial acceleration when a body moves in a circle is $\frac{v^2}{r} = r\omega^2$, in which r is the distance from the center of the circle to the center of gravity of the revolving body, v is the linear and ω is the angular velocity of the center of gravity of the revolving body. Multiplying this by the mass of the body, the product will be the centripetal force = the centrifugal force = F. Therefore,

$$F = \frac{mv^2}{r} = \frac{wv^2}{gr} \qquad (1)$$

Also,

$$F = mr\omega^2 = \frac{wr\omega^2}{g} \qquad (2)$$

If n, the number of revolutions per minute is known, and r is taken in feet, then v (in feet per second) $= \frac{2\pi rn}{60} = .10472rn$. Substituting this value of v in formula (1),

$$F = \frac{w \times (.10472rn)^2}{32.16r} = .000341wrn^2 \qquad (3)$$

Formula (3) is the one most commonly used in practice for finding the value of the centrifugal force.

EXAMPLE.—If a ball weighing 12 lb. revolve in a horizontal plane at the rate of 180 r.p.m., and the distance of the center of gravity of the ball from the axis of revolution is 18 in., what is the centrifugal force, neglecting the weight of the arm connecting the ball to the shaft about which it revolves?

SOLUTION.—Using formula (3), and remembering that r must be expressed in feet, $F = .000341 \times 12 \times \frac{18}{12} \times 180^2 = 198.9 -$ lb. *Ans.*

160. Examples of centrifugal force are frequent in every-day life. When a street car goes around a curve, there is a tendency for the passenger to be thrown toward the outer rail. Where railway trains go around curves at high speed, the outer rail is raised, which counterbalances the centrifugal force; otherwise, the train would either leave the track or would turn over. A bicycle rider accomplishes the same result by bending his body sideways, so that he leans toward the center of the curve he is

traveling. When belts are run at high speed, the centrifugal force causes the two parts (driving side and driven side) to spread apart, thus increasing the belt strains and lessening the arc of contact, which reduces the hold of the belt on the pulley and decreases the driving power. When a locomotive scoops up water from a trough between the tracks while running, the effect is secured by centrifugal force, the water being guided in a curved pipe from the trough to the tank; the faster the speed of the locomotive, the greater the centrifugal force and the more water that will be supplied to the tank.

161. Flywheels and Disks.—When a flywheel or disk, an emery wheel, for example, revolves at a high rate of speed, the centrifugal force may become so great that the flywheel or disk will burst. In the case of flywheels having arms connecting the hub with the rim, it is not easy to calculate the exact effect of the centrifugal force, in fact it is practically impossible. In order to be on the safe side, it is usual to disregard the arms and consider the rim only, in which case the effect of the centrifugal force is to separate one half of the rim from the other half. The weight of one-half the rim is calculated and substituted in formula (3) of Art. **159**, and the result divided by $\pi = 3.1416$. The reason for dividing by π is that each particle of the rim is acted on in a *radial* direction, and the sum of the components normal to the plane of the section at which the break occurs, those on one side acting in the opposite direction to those on the other side, is equal to the centrifugal force of one-half the rim divided by π. It is also customary to take the radius r as the distance from the center of the shaft to the *inside* of the rim, instead of calculating the center of gravity of a cross section of the rim and using the radius to this point for r. For cast-iron flywheels, it is not considered advisable to have the peripheral velocity v exceed materially "a mile a minute;" it ought at any rate to be less than 6000 ft. per min.

For flywheels and disks, therefore, the centrifugal force F' may be expressed by the formula

$$F' = \frac{.000341wrn^2}{2 \times 3.1416} = .00005427wrn^2 \qquad (1)$$

in which w = the weight of the flywheel rim or disk. For flywheels, r is the radius to the inside of the rim; for disks, r is the radius to the center of gravity of one-half the disk = $.42441R$,

where R = radius of disk (see Art. **112**). Substituting the value of r in (1)

$$F'' = .00005427w \times .42441Rn^2 = .000023wRn^2 \qquad (2)$$

in which F'' = the centrifugal force of the disk.

NOTE.—It is to be understood that the value of the centrifugal force as calculated by formulas (1) and (2) of this article is the force that tends to separate one half of the flywheel rim or one half of a disk from the other half, due to the revolution of the flywheel or disk.

EXAMPLE.—A certain flywheel rim has the following dimensions: outside diameter = 14 ft., inside diameter = 12 ft. 6 in., width of face = 22 in. If the rim has a rectangular cross-section and is made of cast iron weighing .2604 lb. per cu. in., what is the centrifugal force of the flywheel when running at 120 r.p.m.?

SOLUTION.—First calculate the weight of the rim. The mean diameter is $\frac{14 + 12.5}{2} = 13.25$ ft. = 159 in. The thickness of the rim is $\frac{14 - 12.5}{2} = .75$ ft. = 9 in. Hence, the weight is $w = \pi \times 159 \times 22 \times 9 \times .2604 = 25{,}755$ lb., and the centrifugal force is, by formula (1),

$$F' = .00005427 \times 25{,}755 \times \frac{12.5}{2} \times 120^2 = 125{,}800 \text{ lb.} \quad \textit{Ans.}$$

EXAMPLES

(1) A ball weighing 5 lb. revolves in a horizontal plane; the distance from the center of gravity of the ball to the axis of revolution is $16\frac{1}{2}$ in. and the ball makes 220 r.p.m. What is (a) the centrifugal force of the ball? (b) the linear velocity of the ball? (c) the angular velocity?

Ans. (a) 113.5 —lb.
(b) 31.678 ft. per sec.
(c) 23.038 radians per sec.

(2) An emery wheel runs at 1600 r.p.m.; if its diameter is 10 in., what is the centrifugal force tending to burst it, its weight being 24 lb.?

Ans. 588.8 lb.

MECHANICS AND HYDRAULICS

(PART 3)

EXAMINATION QUESTIONS

(1) A ball rolls down a frictionless plane, the height of which is 18 ft. and length of base is 81 ft. If the ball has an initial velocity of 44 ft. per sec., (*a*) what will be the velocity at the lower end of the plane? (*b*) how long will it take the ball to travel the length of the plane?

Ans. (*a*) 55.62+ ft. per sec.
(*b*) 1.666− sec.

(2) Referring to Question 1, if the ball were caused to travel *up* the plane, the other conditions being the same as before, (*a*) what will be its velocity when it reaches the top? (*b*) how long will it take the ball to travel the length of the plane?

Ans. (*a*) 27.90− ft. per sec.
(*b*) 2.308+ sec.

(3) A heavy hammer is dropped from the top of a high building; a man stands 400 ft. from the point on the ground where it strikes. If 6.52 sec. elapse from the time the hammer starts to fall until the sound of the fall is heard, how high is the building? Take the velocity of sound in air as 1090 ft. per sec.

Ans. 609 ft.

(4) An automobile traveling at the rate of 50 mi. per hr. is stopped in 120 ft. Assuming that the force that brings the car to a stop is a steady one, (*a*) what is the acceleration? (*b*) how long will it take to stop the machine?

Ans. (*a*) 22.41 ft. per sec.^{2}
(*b*) $3\frac{3}{11}$ sec.

(5) A block of pulpwood, weighing 88 lb. leaves a horizontal overhead conveyor with a velocity of 45 ft. per min. and falls to the level ground below in 2.1 sec. How high is the conveyor?

Ans. 70.9 ft.

(6) Referring to example 3, Art. **133,** suppose the height of lift had been 625 ft. and that the total time of hoisting was 15 sec. If 3 sec. were required to accelerate the load from rest to the mean velocity and $2\frac{1}{4}$ sec. were required to stop it, then neglecting friction and other hurtful resistances, (*a*) what would be the tension in the rope at start? (*b*) in the middle of the hoist? (*c*) when stopping? Total load moved is 8500 lb.

Ans. (*a*) 12,950 lb. (*b*) 8,500 lb (*c*) 2,567 lb.

(7) A stream of water issues from a nozzle in a horizontal direction and strikes the ground 25 ft. (horizontally) from a point vertically under the nozzle. If the nozzle is 6 ft. 9 in. above the ground, what is the velocity of the jet?

Ans. 38.6 −ft. per sec.

(8) A ball is thrown vertically upward, and 7.8 sec. elapse before it returns to the hand; how high did it go? *Ans.* 245 ft.

(9) The outside diameter of the flywheel of an engine is 92 in., width of face is 14 in., and thickness of rim is $5\frac{1}{2}$ in. Taking the weight of a cubic inch of the metal as .28 lb., what is the centrifugal force tending to separate one half from the other when the flywheel is making 250 r.p.m.? *Ans.* 67,070 lb.

(10) The dryers on a paper machine are 6 feet in diameter, if the machine is running 900 ft. per. min how fast are the dryers turning in revolutions per minute? *Ans.* 47.75 r.p.m.

(11) A beater roll, 60 in. in diameter, makes 106 r.p.m. what is (*a*) the angular velocity? (*b*) the linear velocity of a point on the circumference in feet per minute.

Ans. 11.1 radians per sec. 1665 ft. per min.

(12) A shell weighing 880 lb. is fired from a gun with a velocity of 2150 ft. per sec.; what is its momentum? *Ans.* 58,831 lb.-sec.

(13) An automobile weighing 3300 lb. is traveling at the rate of 2 mi. per min.; (*a*) what steady force will bring it to rest in 10 sec.? (*b*) how far will it travel in being brought to rest?

Ans. (*a*) 1806 lb. (*b*) 880 ft.

(14) Referring to Fig. 86 and taking the coefficient of friction between the weight and the table as .25, what is (*a*) the tension in the cord when P = 112 lb. and W = 86 lb.? (*b*) what is the acceleration of P? Neglect all other hurtful resistances.

Ans. (*a*) 60.8 lb. (*b*) 9.421 ft. per sec.[2]

(15) A ball weighing 28 lb. revolves in a horizontal plane against a ring, making 150 r.p.m., as in a certain type of pulverizers; what is the centrifugal force, if the distance between the axis of revolution and the center of the ball is 45 in.?

Ans. 805.6 lb.

(6) Referring to example 3, Art. **133**, suppose the height of lift had been 625 ft. and that the total time of hoisting was 15 sec. If 3 sec. were required to accelerate the load from rest to the mean velocity and $2\frac{1}{4}$ sec. were required to stop it, then neglecting friction and other hurtful resistances, (*a*) what would be the tension in the rope at start? (*b*) in the middle of the hoist? (*c*) when stopping? Total load moved is 8500 lb.

Ans. (*a*) 12,950 lb. (*b*) 8,500 lb (*c*) 2,567 lb.

(7) A stream of water issues from a nozzle in a horizontal direction and strikes the ground 25 ft. (horizontally) from a point vertically under the nozzle. If the nozzle is 6 ft. 9 in. above the ground, what is the velocity of the jet?

Ans. 38.6 —ft. per sec.

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Ans. 11.1 radians per sec. 1665 ft. per min.

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Ans. (*a*) 1806 lb. (*b*) 880 ft.

(14) Referring to Fig. 86 and taking the coefficient of friction between the weight and the table as .25, what is (*a*) the tension in the cord when P = 112 lb. and W = 86 lb.? (*b*) what is the acceleration of P? Neglect all other hurtful resistances.

Ans. (*a*) 60.8 lb. (*b*) 9.421 ft. per sec.[2]

MECHANICS AND HYDRAULICS

(PART 4)

DYNAMICS

WORK, ENERGY, AND POWER

162. Relation between Work and Kinetic Energy.—In *Physics*, it was stated that energy is the ability to do work and that kinetic energy is energy of motion; it was also stated that energy and work are equivalent and both are measured in the same units.

The work done in raising a body whose weight is w through a vertical height h is wh; the same work will be done if the body falls through the height h, and if the body falls freely, the kinetic energy after falling through the height h will be exactly equal to the work that must be done to raise the body through the height h. Let E_k = the kinetic energy; then $E_k = wh$. By formula (8), Art. **142**, $h = \frac{v^2}{2g}$; hence, $E_k = w \times \frac{v^2}{2g} = \frac{wv^2}{2g} = \frac{1}{2}mv^2$, since $\frac{w}{g} = m$. Therefore, the kinetic energy of a moving body may be written

$$E_k = \frac{1}{2}mv^2 = \frac{wv^2}{2g}$$

If v is expressed in feet per second, g must be expressed in feet per second per second; that is, the velocity element (say feet per second) in g must be the same as that used to express v. The weight w may be in pounds, grams, kilograms, or tons; in the latter case, the unit of energy (and work) is the **foot-ton,** when the unit of length is taken as the foot. The above formula will give the kinetic energy of any moving body when its weight and velocity are known. Thus, if a shell fired from a gun weighs 960 lb. and has a velocity on leaving the muzzle of the gun of

2150 ft. per sec. (called the *muzzle velocity*), the energy of the shell is $E_k = \frac{960 \times 2150^2}{2 \times 32.16} = 69{,}000{,}000$ ft.-lb. $= \frac{69{,}000{,}000}{2000}$ $= 34{,}500$ ft.-tons; this is exactly equal to the amount of work that would be required to stop the shell. If this work were all to be turned into heat, it would be equivalent to $\frac{69{,}000{,}000}{778}$ $= 88{,}700$ B.t.u. If the specific heat of the material composing the shell be taken as .117, the number of B.t.u. required to heat the shell 1 degree is $960 \times .117 = 112.3$ B.t.u. Consequently, if all the energy were expended in heating the shell, its temperature would be raised $\frac{88{,}700}{112.3} = 790°$ F.

Observe that if a moving body have a velocity v and its direction be vertically upwards, it will rise to a height $h = \frac{v^2}{2g}$; in falling, it can do work to the amount wh. Therefore, the kinetic energy E_k is equal to the work wh that the body can do.

163. Units of Work.—When using the English system of measures, the unit of work is always taken as the *foot-pound;* it is the work done when a force of one pound acts continuously through a distance of one foot, and is equivalent to raising a weight of one pound through a height of one foot. When the C.G.S. system is used, the unit of work is called the **erg,** which is the work done when a force of one dyne acts continuously through a distance of one centimeter; in other words, 1 erg = 1 dyne-centimeter.

The dyne is defined as the force that will give a mass of 1 gram a velocity of 1 cm. per sec. in 1 second; that is, it will give a mass of 1 gram an acceleration of 1 cm. per sec^2. The relation between the dyne and the pound is easily found: thus, to calculate very accurately, 1 kilogram = 2.204622341 lb.; hence, 1 gram = .002204622341 lb. 1 meter = 3.28084275 ft.; hence, 1 cm. = .0328084275 ft. $g = 980.665$ cm. per sec^2. Then, since by formula (1), Art. **151,** $f = ma$, and as f in this case is 1 dyne, m is $\frac{1 \text{ gram}}{g}$, and $a = 1$ cm. per sec.^{2},

$$1 \text{ dyne} = \frac{.002204622341}{980.665 \times .0328084275} \times .0328084275$$
$$= \frac{.002204622341}{980.665} = .000002248089 \text{ lb.}$$

The erg = 1 dyne × 1 cm. = $\frac{.002204622341}{980.665}$ × .0328084275 = .00000007375627 ft.-lb. The erg is too small a unit for practical work; hence, what is called the **joule** is used in place of it, 1 joule being 10,000,000 = 10^7 ergs. Therefore,

$$1 \text{ joule} = .7375627 \text{ ft.-lb.}$$

since .00000007375627 × 10^7 = .7375627.

From the foregoing,

$$1 \text{ ft.-lb.} = \frac{1}{.7375627} = 1.355817 \text{ joules}$$

The dyne, erg, and joule belong to what is called the C.G.S. system of units, which is used universally by scientists and in electrical engineering. The expression C.G.S. is an abbreviation for centimeter-gram-second. In practical calculations, the joule may be taken as .73756 ft.-lb. and one foot-pound may be taken as 1.3558 joules.

164. In the metric system, the unit of work is the meter-kilogram, which corresponds to the foot-pound in the English system; it is the work required to raise 1 kilogram through a height of 1 meter, and it is equal to 1 kilogram × 1 meter = 2.204622341 × 3.28084275 = 7.2330192 ft.-lb. (approximately 7.233 ft.-lb.). The relation between the meter-kilogram (m.-Kg.) and the erg is easily found. From the definition of the dyne, 1 dyne = $\frac{1 \text{ gram}}{980.665}$; hence, 1 erg = $\frac{1 \text{ gram}}{980.665}$ × 1 cm. = $\frac{1 \text{ gram} \times 1 \text{ cm.}}{980.665}$, from which, 1 gram × 1 cm. = 980.665 ergs.

Multiplying both sides of this equation by 1000,

$$1000 \text{ grams} \times 1 \text{ cm.} = 1 \text{ kilogram} \times 1 \text{ cm.} = 980665 \text{ ergs.}$$

Multiplying both sides of the last equation by 100,

$$1 \text{ kilogram} \times 100 \text{ cm.} = 1 \text{ kilogram} \times 1 \text{ m.} = 98{,}066{,}500 \text{ ergs.}$$

But, 1 kilogram × 1 meter = 1 meter-kilogram; hence,

$$1 \text{ meter-kilogram} = 98{,}066{,}500 \text{ ergs} = 9.80665 \text{ joules.}$$

Now, since 1 joule = .7375627 ft.-lb., 1 m.-Kg. = 9.80665 × .7375627 = 7.2330192 ft.-lb., the same value that was previously found.

It is also evident from the above that

$$1 \text{ gram} = 980.665 \text{ dynes,}$$

$$1 \text{ kilogram} = 980{,}665 \text{ dynes,}$$

$$1 \text{ ft.-lb.} = \frac{1}{7.2330192} = .13825485 \text{ m.-Kg.}$$

165. Power.—Suppose a certain machine can raise a load of 500 lb. 6 ft. in 3 sec., and that it takes another machine 15 sec. to accomplish the same thing. The useful work done is 500 $\times$ 6 = 3000 ft.-lb. in both cases; both machines have performed the same work, but the first machine has done it in one-fifth the time that it took the second machine. The first machine is therefore more *powerful* than the second; it can do five times the work in the same time, and is said to have five times the *power* of the second machine.

Power is the rate of doing work; it is equal to the work done in a certain time divided by the time. Thus, let f = the force acting or the resistance overcome, s = the space through which the force acts or through which the resistance was overcome, and t = the time; then,

$$\text{Power} = \frac{fs}{t} = \frac{\text{work}}{\text{time}}$$

Referring to the last paragraph, the power of the first machine is $\frac{500 \times 6}{3} = 1000$ ft.-lb. per sec., and the power of the second machine is $\frac{500 \times 6}{15} = 200$ ft.-lb. per sec.

When using the English system of units, the unit of power is generally taken as 1 foot-pound per second or 1 foot-pound per minute, the work being measured in foot-pounds and the time being taken in seconds or minutes. For example, suppose a man can exert an average force of 30 lb. on the handle of a windlass for 55 sec.; if the radius of the handle is 16 in., and he makes 20 complete turns of the handle during the time stated, how many units of power does he expend? Here, the force acts through a distance of 2 $\times$ 16 $\times$ 3.1416 $\times$ 20 $\div$ 12 = 167.55 ft. Hence, the power expended is $\frac{30 \times 167.55}{55} = 91.39$ ft.-lb. per sec. = 91.39 $\times$ 60 = 5483 ft.-lb. per min. That is, he works at the rate of 91.39 ft.-lb. of work per sec. or at the rate of 5483 ft.-lb. of work per min.

Example.—A man lifts a barrel of flour weighing 204 lb. 15 in. in $\frac{1}{2}$ sec.; what power does he exert?

Solution.—Power $= \frac{204 \times 15}{12 \times .5} = 510$ ft.-lb. per sec. *Ans.*

166. Horsepower.—The unit of power as above defined is too small for measuring the power of large machines and mechanism; that is, the resulting numbers would be too large for convenient

use. For this reason, the practical unit for power measurement is the **horsepower**, which is defined as 33,000 ft.-lb. of work performed in 1 min. Letting H = the horsepower, f = the force in pounds, s = the distance (space) in feet through which the force acts, and t = the time in minutes,

$$H = \frac{fs}{33000t} \qquad (1)$$

If t be measured in seconds, let t_s be the time in seconds; then $t = \frac{t_s}{60}$; substituting this value of t in formula (1),

$$H = \frac{60fs}{33000t_s} = \frac{fs}{550t_s} \qquad (2)$$

Therefore, a horsepower may be defined as 550 ft.-lb. of work performed in 1 second. A horsepower may also be defined as $33000 \times 60 = 1{,}980{,}000$ ft.-lb. of work performed in one hour.

It is to be noted that power and horsepower are *rates* of doing work; they do not mean that the time spent in doing the work is 1 sec., 1 min., or 1 hr., but that the work is proportional to the work that would be done in those times. In the example of the last article, the time was .5 sec., which is equal to $\frac{.5}{60 \times 60} = \frac{1}{7200}$ hr. $= \frac{.5}{60} = \frac{1}{120}$ min. $= \frac{1}{2}$ sec.; therefore, the horsepower of the man is $H = \frac{204 \times 15}{33000 \times \frac{1}{120} \times 12} = \frac{204 \times 15}{550 \times .5 \times 12} = \frac{204 \times 15}{1980000 \times \frac{1}{7200} \times 12} = .927 + = .9\frac{3}{11}$ horsepower. No man could exert so great a power for any length of time, but he may be able to do so for a very short time. If the work were performed by a machine, the horsepower of the machine, neglecting all hurtful resistances, would be $.9\frac{3}{11} = \frac{51}{55}$ horsepower; and if the machine were to operate continuously for 1 hour at this rate, it would perform $19800000 \times .9\frac{3}{11} = 1{,}836{,}000$ ft.-lb. of work.

167. In the C.G.S. system, the unit of power is 1 joule per second, which is called a **watt.** Letting p = power, in watts, w = work or energy, in joules, and t = time, in seconds,

$$p = \frac{w}{t} \qquad (1)$$

and

$$w = pt \qquad (2)$$

The relation between the watt and the horsepower is easily found. By definition, 1 watt = 1 joule per second; 1 horsepower = 550 ft.-lb. per sec.; hence, since 1 ft.-lb. was found to equal 1.355817 joules, 550 ft.-lb. = 550 × 1.355817 = 745.69935 joules, and 1 horsepower = 745.7 joules per second = 745.7 watts. In practical engineering calculations, it is customary to consider 1 horsepower as equal to 746 watts. Since the watt is rather small for a practical unit, it is customary to express the power of large machines in terms of the **kilowatt**, which is 1000 watts. If the horsepower be taken as 746 watts, 1 kilowatt $= \frac{1000}{746}$ = 1.340483 horsepower, and is commonly taken as 1.34 horsepower or, roughly, as $1\frac{1}{3}$ horsepower, which is close enough for most practical purposes. The true value, however, is 1 kilowatt $= \frac{1000}{745.69935}$ = 1.341023 horsepower. Since it is a difficult matter to measure the output of a machine with any great degree of exactness, 1 kilowatt may usually be taken as equal to $1\frac{1}{3}$ horsepower and 1 horsepower as $\frac{3}{4}$ kilowatt. If greater exactness be required,

$$1 \text{ kilowatt} = 1.341 \text{ horsepower, and}$$
$$1 \text{ horsepower} = .7457 \text{ kilowatt}$$

168. In the metric system, 1 horsepower is considered to be 75 meterkilograms per second. Since 1 m.-kg. = 7.2330192 ft.-lb.,

1 metric horsepower = 75 × 7.2330192 = 542.47644 ft.-lb. per sec.

Therefore, the metric horsepower is only $\frac{542.47644}{550} = .9863208$, say 98.632% of the English horsepower. Also, 1 English horsepower $= \frac{550}{542.47644} = 1.013869$ metric horsepower, 1 metric horsepower = .74569935 × .9863208 = .7354988, say .7355 kilowatt, and 1 kilowatt $= \frac{1}{.7354988} = 1.359622$, say 1.3596 metric horsepower.

EXAMPLE.—If the output of a certain dynamo is 436 kilowatts, (*a*) how many horsepower is this equivalent to? (*b*) how many metric horsepower?

SOLUTION.—(*a*) The number of horsepower would usually be estimated as 436 × $\frac{4}{3}$ = 581 horsepower. More accurately, it would be 436 × 1.34 = 584 H.P. Very accurately, it would be 436 × 1.341 = 584.68 H.P. *Ans.*

(*b*) The metric horsepower is 436 × 1.3596 = 592.79 m. H.P. *Ans.*

use. For this reason, the practical unit for power measurement is the **horsepower**, which is defined as 33,000 ft.-lb. of work performed in 1 min. Letting H = the horsepower, f = the force in pounds, s = the distance (space) in feet through which the force acts, and t = the time in minutes,

$$H = \frac{fs}{33000t} \qquad (1)$$

If t be measured in seconds, let t_s be the time in seconds; then $t = \frac{t_s}{60}$; substituting this value of t in formula (1),

$$H = \frac{60fs}{33000t_s} = \frac{fs}{550t_s} \qquad (2)$$

Therefore, a horsepower may be defined as 550 ft.-lb. of work performed in 1 second. A horsepower may also be defined as $33000 \times 60 = 1{,}980{,}000$ ft.-lb. of work performed in one hour.

It is to be noted that power and horsepower are *rates* of doing work; they do not mean that the time spent in doing the work is 1 sec., 1 min., or 1 hr., but that the work is proportional to the work that would be done in those times. In the example of the last article, the time was .5 sec., which is equal to $\frac{.5}{60 \times 60} = \frac{1}{7200}$ hr. $= \frac{.5}{60} = \frac{1}{120}$ min. $= \frac{1}{2}$ sec.; therefore, the horsepower of the man is $H = \frac{204 \times 15}{33000 \times \frac{1}{120} \times 12} = \frac{204 \times 15}{550 \times .5 \times 12} = \frac{204 \times 15}{1980000 \times \frac{1}{7200} \times 12} = .927+ = .9\frac{3}{11}$ horsepower. No man could exert so great a power for any length of time, but he may be able to do so for a very short time. If the work were performed by a machine, the horsepower of the machine, neglecting all hurtful resistances, would be $.9\frac{3}{11} = \frac{51}{55}$ horsepower; and if the machine were to operate continuously for 1 hour at this rate, it would perform $19800000 \times .9\frac{3}{11} = 1{,}836{,}000$ ft.-lb. of work.

167. In the C.G.S. system, the unit of power is 1 joule per second, which is called a **watt.** Letting p = power, in watts, w = work or energy, in joules, and t = time, in seconds,

$$p = \frac{w}{t} \qquad (1)$$

and

$$w = pt \qquad (2)$$

other. Let OC, measured on OX, represent the distance through which the force acts; at various points a, b, c, etc. between O and C (the more the better), erect ordinates aa', bb', cc', etc. and make their lengths equal the forces acting at a, b, c, etc. Through these points, draw the irregular line $ADEFB$; then, the area $OADEFBC$ represents the work done while the variable force acts through the distance OC. For, finding the area by Simpson's or the trapezoidal rule, and dividing it by the length OC (actual area and actual length), the quotient will be the mean ordinate $OA' = CB'$; drawing $A'B'$ parallel to OX, $OA'B'C$ is a rectangle, and its area is necessarily equal to the area of the figure $OADEFBC = OA' \times OC = f_m \times s$, in which $f_m =$ the mean ordinate OA', which, in turn, is the *average force* exerted throughout the distance OC; in other words, the work done is equivalent to the work done by a *constant* force OA' acting through the distance OC. It is to be noted that the area of $DEF =$ area $A'DA + B'FB$; that is, the line $A'B'$ cuts off as much area *above* the line and under the curve $ADEFB$ as is included between the line and the curve *below* it.

171. A case of steady pressure, practically speaking, is the discharge of a steam pump; here a column of water whose length is equal to the total height of lift is raised a distance equal to the stroke at every stroke of the piston or plunger. A case of varying pressure is the cylinder of a steam engine; the steam follows the piston at full pressure for a part of the stroke; it is then cut off and expands, the pressure rapidly falling, until the exhaust port opens; the pressure then drops very nearly to the pressure of the atmosphere, and remains at that pressure on the return stroke until the exhaust port closes, when the steam is compressed, the pressure rapidly rising, until the end of the return stroke is reached. By means of an instrument called an **indicator,** a line is drawn on a sheet of paper that indicates the pressure on the piston at every part of the stroke; the outline so drawn is called an **indicator diagram.**

An indicator diagram is shown in Fig. 90. Before any steam is allowed to enter the indicator from the cylinder, the line AB is drawn; this is called the **atmospheric line,** and it indicates the pressure of the atmosphere. The diagram begins at C, where the full steam pressure is on the piston urging it ahead; this continues until the point D is reached, when the steam is cut off. The steam expands from D to E, where the exhaust port opens.

F is the end of the stroke. On the return stroke, the pencil traces the line *FGHC*, the exhaust port closing at *H* and the steam is compressed during that part of the stroke included between *H* and *AC*. While not strictly correct, the pressures indicated by the line *FGHC* may be considered as being on the opposite side of the piston during the forward stroke of the piston, and as opposing the pressures indicated by the line *CDEF*. Hence, the *working pressures* will be difference between these, and they will be represented by lines drawn perpendicular to *AB* and in-

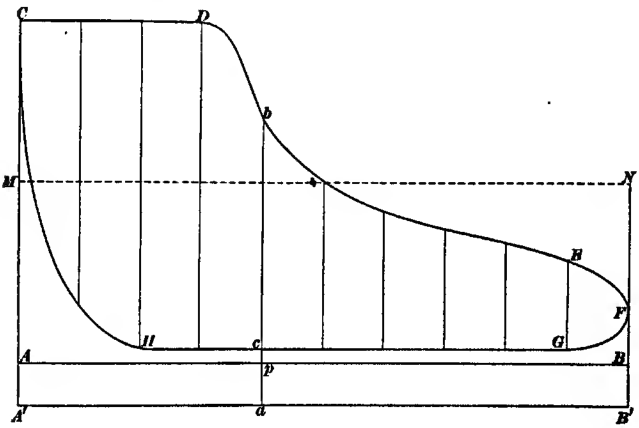


Fig. 90.

cluded between *CDEF* and *FGHC*. The area *CDEFGHC* divided by the length *AB* gives what is called the **mean effective pressure**; it is the average pressure (specific pressure) transmitted to the crosshead. Suppose the indicator spring is such that 1 inch measured on an ordinate of the diagram represents 60 lb per sq. in., then, if *AC* is 1.97 in. long, it represents $1.97 \times 60 = 118.2$ lb. per sq. in. Divide the diagram into 10 equal parts, as shown, and find the area by Simpson's or the trapezoidal rule (the latter will usually be accurate enough for this purpose); suppose the area so found is 4.42 sq. in. The line *AB* (length of diagram) can generally be made to have any convenient length; suppose that in this case, the length is 4 in. Then the mean effective pressure is $4.42 \div 4 \times 60 = 66.3$ lb. per sq. in. If *AM* be laid off equal to $4.42 \div 4 = 1.105$ in. and *MN* be

drawn parallel to AB, the rectangle $AMNB$ will have the same area as the diagram.

The pressures above AB are *gauge* pressures. To represent *absolute* pressures, take a reading of the barometer to find what the atmospheric pressure is; otherwise, call it 14.7, say 15, pounds per sq. in., and lay off $AA' = \frac{15}{60} = \frac{1}{4}$ in. Draw $A'B'$ parallel to AB; then any ordinate measured from $A'B'$ to the curve will give the absolute pressure at that point. For instance, if the point p on AB is 1.6 in. from A and the stroke of the piston is 28 in., p represents the position of the piston when it has completed $28 \times \frac{1.6}{4} = 11.2$ in. of its stroke. Draw the ordinate ab, and if ab measures 1.64 in. and ac measures .33 in., the pressure urging the piston ahead of this point is $1.64 \times 60 = 98.4$ lb. per sq. in., absolute, the pressure on the other side of the piston opposing this motion is $.33 \times 60 = 19.8$ lb. per sq. in., absolute, and the difference $= 98.4 - 19.8 = 78.6$ lb. per sq. in. is the effective pressure urging the piston ahead and acting on the crosshead. *The length* of cb is $1.64 - .33 = 1.31$ in., and $1.31 \times 60 = 78.6$ lb. per sq. in., as before.

Suppose the diameter of the piston discussed above is 22 in. and that the engine runs at the rate of 160 r.p.m. If the engine is double-acting, as is usually the case, it makes 2 strokes for every revolution, or $160 \times 2 = 320$ strokes per minute. The piston therefore travels $\frac{28 \times 320}{12} = 746\frac{2}{3}$ ft. per min. The total average pressure on the piston is $.7854 \times 22^2 \times 66.3 = 25{,}203$ lb. = the force, which in one minute acts through a distance of $746\frac{2}{3}$ ft. Consequently, the work done in 1 minute is $25203 \times 746\frac{2}{3} = 18{,}818{,}240$ ft.-lb. The horsepower of the engine is, therefore, $\frac{18818240}{33000} = 570.25$ H.P.

Let H = the horsepower, P = the mean effective pressure in pounds per square inch (determined from the indicator diagram), L = length of stroke in feet, A = area of piston in square inches, and N = number of strokes per minute; then

$$H = \frac{PLAN}{33000}$$

That this formula is correct is readily seen from the preceding calculation. Thus, $P \times A$ is the force, $L \times N$ is the distance, and $P \times A \times L \times N = P \times L \times A \times N = PLAN$ = work done

in one minute, and this divided by 33000 is the horsepower. This formula is very easy to remember, as the letters form the word *plan*.

172. The above formula for horsepower may be given a form that will adapt it to any machine that is operated by a fluid or which discharges a fluid. Let P = pressure in pounds per square foot, p = pressure in pounds per square inch, A = area in square feet, and a = area in square inches; then the above formula would be written

$$H = \frac{pLaN}{33000}$$

But, $A = \frac{a}{144}$, from which, $a = 144A$; substituting this value of a in the formula,

$$H = \frac{144pLAN}{33000}$$

Now $L \times A$ = the volume of the cylinder in cubic feet = volume displaced by the piston in one stroke, and $L \times A \times N$ = volume in cubic feet displaced by the piston in N strokes = volume displaced in 1 minute; representing this volume by V and substituting in the last equation,

$$H = \frac{144pV}{33000} = \frac{PV}{33000},$$

since $\frac{p}{144} = P$, the pressure in pounds per square foot.

This last formula may be applied to any machine operated by a fluid (liquid or gas) or which discharges a fluid, p being the average pressure of the fluid in pounds per square inch, P the pressure in pounds per square foot, and V the volume displaced or discharged in cubic feet per minute.

EXAMPLE 1.—A mine ventilating fan delivers 22,000 cu. ft. of air per minute under a pressure of 4.25 lb. per sq. ft. If the efficiency of the fan is 78%, what horsepower is required to operate it?

SOLUTION.—Since the pressure is in pounds per square foot,

$$H = \frac{PV}{33000} = \frac{4.25 \times 22000}{33000} = 2.833 \text{ H. P.} = \text{horsepower required}$$

to move the air. Since the efficiency of the fan is 78%, the power required to operate it = 2.833 ÷ .78 = 3.63 H. P. *Ans.*

In any problem involving horsepower or power measurements of any kind, it is not advisable to use more than three or four significant figures in the final result, on account of difficulties in making accurate measurements.

EXAMPLE 2.—An electrically driven pump raises water to a total height of 57 ft. If the efficiency of the pump is 84% and it delivers 90,000 gallons of water per hour, what power in kilowatts is required to operate the pump?

SOLUTION.—Taking the weight of a cubic foot of water as 62.4 lb., 1 United States gal. weighs $\frac{231 \times 62.4}{1728} = 8.34\frac{1}{6}$ lb. For practical purposes, this is best taken as $8\frac{1}{3} = \frac{25}{3} = \frac{100}{12}$ lb. = weight of 1 gal. of water (the Imperial-British-gal. = 10 lb.); hence, 90,000 gal. weigh $90000 \times \frac{25}{3}$ = 750,000 lb., which is raised 57 ft. in 1 hour. Using formula (1), Art. **166**,

$$H = \frac{750000 \times 57}{33000 \times 60} = 21.6 \text{ horsepower.}$$

Since the efficiency is .84, the horsepower required to operate the pump is $21.6 \div .84 = 25.7$ H.P. Taking 1 horsepower as $\frac{3}{4}$ kilowatt, the power in kilowatts is $25.7 \times \frac{3}{4} = 19.3$ k.w. *Ans.*

The calculation might have been performed as follows: 90000 gal. $= \frac{90000 \times 231}{1728} = 12{,}030$ cu. ft. = the volume of water discharged. A column of water 1 ft. square and 1 ft. high contains 1 cu. ft. and exerts a pressure of 62.4 lb.; hence, a column of water 1 ft. high exerts a pressure of 62.4 lb. per sq. ft. Since the water is raised 57 ft., the total pressure exerted is 62.4×57 lb. per sq. ft. Therefore, using the formula above given, $H = \frac{PV}{33000} = \frac{62.4 \times 57 \times 12030}{33000 \times 60} = 21.6$ H.P. This result is the same as was previously found. Division by 60 is required in order to reduce the volume per hour to volume per minute. Values have been calculated only to three significant figures because the weight and volume of the water as given and calculated is not correct to more than that number of figures. In cases of this kind, greater accuracy in calculation is not only unnecessary but it is also misleading.

173. Buying and Selling Power.—Power, or rather work, can be bought and sold as though it were a commodity; in fact it may be considered as a commodity, an article of commerce. Note, however, that power is rate of doing work; it is a unit of comparison, not of quantity, and can consequently neither be bought nor sold, though the term "buying power" frequently occurs in engineering transactions. What is really bought and sold is work, as will be readily apparent from the following considerations.

Suppose a man has a 10-horsepower engine, and instead of furnishing his own "power" to operate it, he buys it in the form of steam delivered from a heating plant. Obviously, he cannot buy 10 H.P., because the length of time he runs his engine will determine the amount of steam used, and the engine will be rated at 10 H.P. whether it runs for 1 second or 1 year. He may arrange to pay a certain price for every hour that his engine runs, in which case, the price will be based on what is called the **horse-**

power-hour. Let H = the horsepower of the engine, w = the work done in t hours; then, $H = \frac{w}{33000 \times 60 \times t} = \frac{w}{1980000t}$. If $t = 1$ hr., then $H = \frac{w}{1980000 \times 1 \text{ hr.}}$, and 1 horsepower-hour $= \frac{w}{1980000 \times 1 \text{ hr.}} \times 1 \text{ hr.} = \frac{w}{1980000}$. Hence, in other words, 1 horsepower-hour = 1,980,000 ft.-lb. of work. Therefore, what he really buys is work, not power. If his engine runs 8 hours each working day, he will require $10 \times 1980000 \times 8 = 158{,}400{,}000$ ft.-lb. of work to operate it. Since this is an extremely inconvenient number to use, it is customary to say that he uses $10 \times 8 = 80$ horsepower-hours each day, and he will buy on that basis. It should always be kept in mind that although one may speak of buying "power," what is actually bought is work.

174. In the case of a motor operated by electric current, the power will be bought on the basis of the watt-hour or kilowatt-hour (abbreviated to k.-w.h.). Since a watt is 1 joule per second, a watt-second is 1 joule, a watt-hour is $1 \times 60 \times 60 = 3600$ joules, and a kilowatt-hour is $1000 \times 3600 = 3{,}600{,}000$ joules. But, 1 joule = .7375627 ft.-lb.; hence, 1 k.-w.h. $= 3600000 \times .7375627 = 2{,}655{,}226$ ft.-lb. Or, since 1 kilowatt = 1.341023 h.p., 1 k.-w.h. $= 1.341023 \times 1980000 = 2{,}655{,}226$ ft.-lb.

EXAMPLES

(1) How many horsepower is equivalent to 256 kilowatts?

Ans. 343.3 H.P.

(2) A certain machine does work at the rate of 315 meter-kilograms per sec. (*a*) what is its rating in metric horsepower? (*b*) in kilowatts?

Ans. (*a*) 4.2 m.H.P. (*b*) 3.089 k.w.

(3) The diameter of a steam-engine cylinder is 26 in., the stroke is 32 in., and the mean effective pressure is 72.6 lb. per sq. in., and the fly-wheel makes 128 r.p.m.; what is (*a*) the horsepower of the engine? (*b*) what is the power in kilowatts?

Ans. (*a*) 797.4 H.P. (*b*) 594.6 k.w.

(4) In a certain paper mill, a pump raises water 39 ft. and discharges 7500 gal. per hour, If the efficiency of the pump is 75%, what is (*a*) the actual power of the pump in kilowatts? (*b*) If the pump is operated by electricity, which is furnished at 3 cents per kilowatt-hour (k.w.-h.), and runs an average of 40 hr. per week, how much does the electricity cost per week?

Ans. (*a*) 1.224 k.w. (*b*) \$1.47.

(5) If the pump in Example 4 were in a Canadian mill, the Imperial gallon might be the unit of volume. Calculate the problem on that basis.

Ans. { (a) 1.47 k.w.; (a) $1.77 }

(6) How many British thermal units (B.t.u.) are equivalent to one horsepower? (b) to one horsepower-hour?

Ans. { (a) 2545 – B.t.u.; (b) 3413 – B.t.u. }

(7) If the weight of an elevator and its load of pulp is $2\frac{3}{4}$ tons, and it hoists the load 80 ft. in 12 sec., what horsepower must be used, (a) neglecting all hurtful resistances? (b) What is the horsepower if the total efficiency is 73%?

Ans. { (a) $66\frac{2}{3}$ H.P.; 91.3 H.P. }

HYDRAULICS

MEASURING FLOW OF WATER

175. Definition.—**Hydraulics,** which is also called **hydrokinetics** and **hydrodynamics,** is that branch of hydromechanics that deals with the *flow* of fluids. Although, properly speaking, any liquid or gas is a fluid, it is customary to restrict the meaning of the word hydraulics, applying the term only to the flow of water; and this is the sense in which it is here used. The principles of hydraulics apply also to the paper pulp in water at low concentrations, to solutions, and to some mixtures.

176. Due to various causes, some of which will soon be mentioned, it is practically impossible to calculate the flow of water (or any fluid) with any high degree of accuracy. As a consequence, it is not advisable to express calculated values of velocities, discharges, etc. to more than three significant figures, and all numbers used in such calculations may and ought to be restricted to not more than four significant figures.

177. Mean Velocity.—If a cross-section be taken of a flowing stream and the velocity be measured at different points of the section, considerable variation in velocities will be found; in other words, at hardly any two points of the section will the velocity be the same. The surface touched by the water when flowing—the inside of a pipe, the sides and bottom of an open channel, etc.—is called the **rubbing surface;** that part of the flowing water that touches the rubbing surface is retarded by friction, and this hinders the movement of the layer or layers

next to it; the top surface moves at a different velocity from that of the bottom surface, etc. For these reasons and others, it is generally the practice to use the **mean velocity** in connection with calculations pertaining to the flow. Let v_m = the mean velocity in feet per second, Q = the total quantity that flows past the section in cubic feet, and let A = area of section in square feet;

then $$v_m = \frac{Q}{A} \qquad (1)$$

and $$Q = Av_m \qquad (2)$$

If for any reason it is desired to express the area in square inches, let a = the area in square inches; then $a = 144A$,

and $$v_m = \frac{144Q}{a} \qquad (3)$$

and $$Q = \frac{av_m}{144} \qquad (4)$$

EXAMPLE.—If a 6-inch pipe discharges 54.2 cu. ft. per min. what is the mean velocity of the water?

SOLUTION.—Applying formula (3), reducing the discharge to cubic feet per second,

$$v_m = \frac{144 \times 54.2}{.7854 \times 6^2 \times 60} = 4.6 \text{ ft. per sec.} \quad Ans.$$

In what follows, unless otherwise specially stated, all velocities will be understood to be mean velocities.

178. While the discharge is usually *calculated* in cubic feet per second or per minute, it is generally expressed in gallons per second, per minute, per hour, or per day, particularly in commercial transactions. Since 1 cu. ft. $= \frac{1728}{231} = 7.48052 -$ gal., the discharge can be converted into gallons by multiplying the number of cubic feet by 7.48052. For most cases arising in practice. results sufficiently exact are obtained by multiplying the cubic feet by 7.48, and this value will be used hereafter. If it be desired to convert gallons into cubic feet, divide the gallons by 7.48 or multiply gallons by $\frac{231}{1728} = .13368$; it will be sufficiently exact to multiply by .1337, and this value will be used in what follows. It is also to be understood that the U. S. gallon is referred to. The British, or Imperial, gallon is equal to 1.20114 U. S. gal.

EXAMPLE.—(*a*) How many gallons are equivalent to 3275 cu. ft.? (*b*) How many cubic feet are equivalent to 63,800 gal.?

SOLUTION.—(a) Since 1 cu. ft. = 7.48 gal., 3275 cu. ft. = 3275 × 7.48 = 24,497 gal., say 24,500 gal. *Ans.*

(b) Since 1 gal. = .1337 cu. ft., 63,800 gal. = 63800 × .1337 = 8,530 cu. ft. *Ans.*

EFFLUX THROUGH STANDARD ORIFICES

179. The Velocity of Efflux.—The word **efflux** means the process of flowing; hence, efflux of water means the flow of water; it does not mean the discharge in the sense of quantity, but simply the flowing or discharging without regard to quantity. An **orifice** is an opening in a vessel through which the water or other fluid issues or flows.

Suppose the water to issue from a small orifice A, Fig. 91, in the bottom of a vessel, the velocity being v ft. per sec. (It is understood that v = mean velocity.) After a time t sec., a quantity $Q = Av$ cu. ft. will have discharged, A being the area of the orifice in square feet. The weight of the water will be $w = 62.4Q$ lb. Suppose that as the water flows out an equal quantity flows into the vessel, thus keeping the height of the upper surface ab of the water in the vessel above the level cd constant; represent this height by h_1. For convenience, let the time be 1 second; then in 1 sec., w lb. of water have flowed into the vessel and w lb. will have flowed out of it. Evidently, any water that flows into the vessel will flow out of it, if the action continue long enough; that is w lb. will fall from level ab to level cd, and the work it could do is wh_1. The kinetic energy of the water as it issues from the orifice at level cd is $w \times \frac{v^2}{2g}$, and this must equal the work;

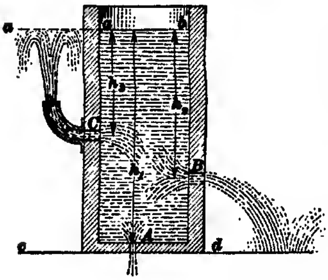


FIG. 91.

hence, $wh_1 = \frac{wv^2}{2g}$, from which

$$v = \sqrt{2gh_1} = 8.02\sqrt{h_1}$$

In other words, the velocity is the same as though the water had fallen freely through the height h_1, which is equal to the difference of levels between the upper surface of the water and the

point of discharge. This velocity is called the **theoretical velocity of efflux**; the height h_1 is called the **hydrostatic head** or, simply, the **head**; and the velocity v is said to be the **velocity due to the head.**

If the orifice be in the side of a vessel, as at B, the head h_2 is measured to the *center* of the orifice, and the velocity v_2 is

$$v_2 = \sqrt{2gh_2}$$

If the water flows directly into the atmosphere in a horizontal direction, as indicated, the path will be a parabola, and the range may be calculated as in example 2, Art. **146.** If a pipe be connected to the vessel, so the water can flow in an upward (vertical) direction, as shown at C, it will rise to the same level as the upper surface of the water in the vessel, since the velocity of efflux is due to the head h_3, and this velocity will carry the water to a height h_3.

180. From the foregoing, it is plain that if the velocity of efflux be known, the head that produces it, called the *head due to the velocity*, can be found, since

$$h = \frac{v^2}{2g}$$

Likewise, if the head be known, the velocity of efflux can be found.

EXAMPLE.—What must be the head in order to produce a velocity of efflux of 42 ft. per sec.?

SOLUTION.—The head required to produce a velocity of 42 ft. per sec. is

$$h = \frac{v^2}{2g} = \frac{42^2}{2 \times 32.16} = 27.4 \text{ ft.} \quad \textit{Ans.}$$

181. If the top surface of the water be subjected to an additional pressure of, say, p lb. per sq. in., as by fitting the vessel with a piston and placing a weight on the piston, the resulting velocity of efflux will be exactly the same as though the head had been increased until the specific pressure on the section ab, Fig. 91, is equal to p. A column of water 1 in. square and 1 ft. high weighs $\frac{62.4}{144} = .4\frac{1}{3}$, say .433 lb., and this equals the specific pressure, in pounds per square inch, exerted by water for each foot of depth. Consequently, to produce a specific pressure of p lb., the depth of the water in feet must be $p \div .433 = 2.308p$, say $2.31p$. Let h' = the head in feet equivalent to this additional pressure; then

$$h' = 2.31p \qquad (1)$$

The total head, called the **equivalent head,** is $h + h'$, and

$$v = \sqrt{2g(h + h')} = 8.02\sqrt{h + h'} \qquad (2)$$

The pressure, in pounds per square foot multiplied by velocity in feet per minute gives available power in foot-pounds per minute. To convert this in to horsepower, divide by 33000.

EXAMPLE.—Suppose water to stand in a cylinder 15 in. in diameter to a vertical height of 36 ft. If the cylinder is fitted with a piston weighing 180 lb. which rests on top of the water, and on which is laid a weight of 450 lb., what will be the velocity of efflux through a small orifice in the bottom of the cylinder?

SOLUTION.—The total pressure on top of the water is $180 + 450 = 630$ lb.; the area of the piston is $.7854 \times 15^2 = 176.7$ sq. in.; and the specific pressure is $\frac{630}{176.7}$ lb. $= p$. Hence, $h' = 2.31 \times \frac{630}{176.7} = 8.24$ ft., and $v = 8.02\sqrt{36 + 8.24} = 54.5$ ft. per sec. *Ans.*

182. Size of Orifice.—In connection with the preceding formulas, it has been assumed that the orifice was small compared with a section of the water taken at right angles to the direction of efflux. Referring to Fig. 91, if the area a of the orifice A is less than $\frac{A}{20}$, in which A is the area of the bottom of the vessel, then the formula of Art. **179** may be used; but, if a is equal to or greater than $\frac{A}{20}$, the following formula must be used:

$$v = \sqrt{\frac{2gh}{1 - \left(\frac{a}{A}\right)^2}}$$

EXAMPLE.—A 22-inch round pipe is filled to a height of 84 ft.; what will be the velocity of efflux through a round hole in the bottom, 9 in. in diameter?

SOLUTION.—Since areas of circles are proportional to the squares of the diameters, the above formula may be written $v = 8.02\sqrt{\frac{h}{1 - \left(\frac{d^2}{D^2}\right)^2}}$

$= 8.02\sqrt{\frac{h}{1 - \left(\frac{d}{D}\right)^4}} = 8.02\sqrt{\frac{84}{1 - \left(\frac{9}{22}\right)^4}} = 74.6$ ft. per sec. *Ans.*

183. If the orifice is in the side of the vessel, as the opening in the head (flow) box admitting stock to the paper-machine wire, the head on the center of the orifice (distance of center of orifice below the upper surface of the water) must be greater than four times the depth of the orifice. For example, suppose the orifice is rectangular, and measures $2\frac{1}{4}$ in. by 40 in., with the short side

vertical; then if the head on the center is equal to or greater than 2.25 × 4 = 9 in., the velocity of efflux may be calculated by the formula of Art. **179.** Otherwise, let h_1 be the head on the bottom of the orifice, h_2 = the head on the top of the orifice, and b = the breadth; then,

$$Q = \tfrac{2}{3}b\sqrt{2g}(\sqrt{h_1^3} - \sqrt{h_2^3}) = \tfrac{2}{3}b\sqrt{2g}(h_1^{\frac{3}{2}} - h_2^{\frac{3}{2}}) = 5.347b(\sqrt{h_1^3} - \sqrt{h_2^3}).$$

In this formula, Q is the discharge in cubic feet per second when b is measured in feet. It is to be noted that $\sqrt{a^3} = a^{\frac{3}{2}}$, which accounts for the two ways of writing the above formula.

Example.—The slot in the flow box on a news print paper machine is 2 in. deep and 160 in. wide; the depth of stock in the flow box is 27 in. above the center of the opening. The stock contains .5% by weight of dry paper fiber. What is (*a*) the velocity of stock leaving the box? (*b*) the weight of paper fed to the machine per hour? Neglect the effect of friction and consistency of stock. The weight of a cubic foot of the stock may be taken as 62.4 lb.

Solution.—(*a*) The formula in Art. **179** is the one to use, and $V = 8.02\sqrt{2.25}$ = 12.03 ft. per sec. = 721.8 ft. per min. *Ans.*

(*b*) The area of the slot is $\frac{2 \times 160}{144} = \frac{20}{9}$ sq. ft.; hence, the discharge is $\frac{20}{9} \times 12.03$ = 26.73 cu. ft. per sec.; and the weight of the paper fed to the machine per hour is 26.73 × 62.4 × 60 × 60 × .005 = 30,023 lb., say 30,000 lb. *Ans.*

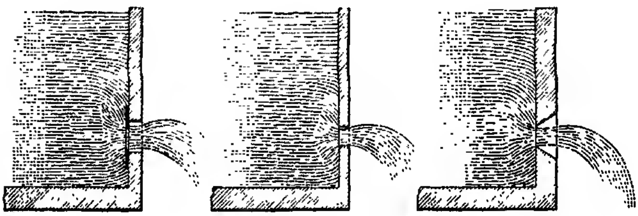
Note.—As will be seen later, these are theoretical figures and will be reduced by several factors in practice.

184. Standard Orifice.—All the foregoing formulas give what are termed *theoretical results*, called theoretical because they assume conditions that never occur in practice, although the results may be approximately correct. The discharge is greatly influenced by the character of the edges of the orifice—whether the edges are rounded or square, whether the side of the vessel is thin or thick, etc. Consequently, in order to obtain accurate results, it is necessary to have what is termed the standard orifice.

An orifice is a **standard orifice** when the water flowing through it touches only the inside edge of the opening. Three standard orifices are shown in Fig. 92. At (*a*), the orifice is in a thin plate through which the water flows without touching the wall of the vessel; at (*b*) the wall of the vessel is thin, being less in thickness than the diameter of the orifice, and the edges are square, as indicated by the shape of the cross-section; at (*c*), the edges have been beveled, so that the area of the opening at the outside of

the wall is greater than at the inside, thus preventing the water from touching the surface of the opening.

185. The Vena Contracta.—When water issues from a standard orifice, it first contracts and then expands, as indicated in Fig. 92; and it does this no matter what the shape of the orifice or what its size. If a section of the stream be taken at the point of greatest contraction, this section is called the **vena contracta** or *contracted vein*, a name given to it by Sir Isaac Newton. For a circular orifice or for a square orifice, the distance of the vena contracta from the edge of the standard orifice is about one-half the depth of the orifice, and the vertical depth of the vena



(a) (b) (c)

FIG. 92.

contracta is quite closely .8 that of the orifice. Consequently, the area of the vena is about $.8^2 = .64$ that of the orifice. This area varies slightly for different heads and different sizes of the orifice, a fair average value being .627, which is called the **coefficient of contraction.** Therefore, if the coefficient of contraction be denoted by c_c, then letting A = the area of the orifice and v_a = the actual mean velocity,

$$Q = c_c v_a A$$

Since the average value of c_c is .627,

$$Q = .627 v_a A$$

186. The actual velocity at the vena contracta is not quite equal to the velocity due to the head at that point. This loss in velocity is due to the friction of the water at the edges of the orifice and to its viscosity, water not being a perfect fluid. The ratio of the actual velocity to the theoretical velocity due to the head is called the **coefficient of velocity.** Let v_a = the actual

velocity, $v = \sqrt{2gh}$ = velocity due to the head, and c_v = the coefficient of velocity; then

$$c_v = \frac{v_a}{v}$$

from which, $v_a = c_v v$

Substituting this value of v_a in the equation of the last article,

$$Q = c_c c_v v A = c_e v A$$

in which $c_e = c_c \times c_v$, and is called the **coefficient of efflux.** From this last equation,

$$c_e = \frac{Q}{Av}$$

and the coefficient of efflux (also called the **coefficient of discharge**) can be found experimentally by carefully measuring the discharge for a certain time, calculating the theoretical discharge for the same time by the formula $Q = Av = A\sqrt{2gh}$, and dividing the first result by the second. Experiments show that c_e varies somewhat for different heads and sizes of orifice, principally because of variations in c_v, the coefficient of velocity, which varies from about .97 to .99. A fair average value is .98, thus making the average value of c_e, the coefficient of efflux,

$$c_e = c_c c_v = .627 \times .98 = .61446$$

Taking the average value of c_e as .615 and letting Q_a = actual discharge and letting $v = \sqrt{2gh}$ = velocity due to the hydrostatic head,

$$Q_a = .615Av$$

EXAMPLE.—Water issues from a round standard orifice, the diameter of which is 4½ in. If the head on the center of the orifice is 23 ft. 9 in., what is (*a*) the velocity of efflux? (*b*) the discharge in gallons per minute?

SOLUTION.—(*a*) Taking the coefficient of velocity as .98, and letting v_a represent the actual velocity of efflux,

$$v_a = c_v\sqrt{2gh} = .98 \times 8.02\sqrt{23.75} = 38.3 \text{ ft. per sec.} \quad \textit{Ans.}$$

(*b*) Let Q_a = the actual discharge; then

$$Q_a = .615Av = .615 \times \frac{.7854 \times 4.5^2}{144} \times 8.02\sqrt{23.75} \times 60 = 159.3 \text{ cu. ft.}$$

and $159.3 \times 7.48 = 1191$ gal. per min. *Ans.*

187. Discharge through a Short Tube.—If instead of discharging through a standard orifice, the discharge is through a short tube, as shown in Fig. 93, the quantity discharged in a unit of time is increased. The tube is straight and of the same diameter as the orifice; the edges are square, as indicated at *a* and *b*; and

the length of the tube must be at least 2½ times the diameter of the orifice. As the water enters the tube, it contracts, in the same manner as for a standard orifice; it then expands and fills the tube before it emerges into the atmosphere. The result is that the average value of the coefficient of discharge is about .815, instead of .615, the average value for a standard orifice. For a short tube, therefore,

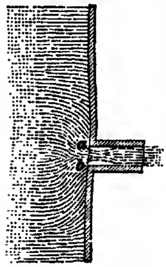


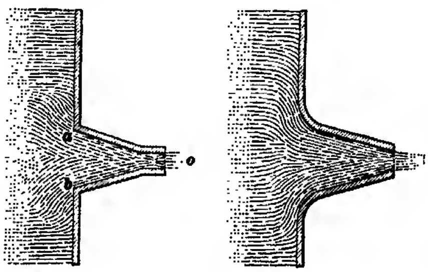
Fig. 93.

$$Q_a = .815Av = .815A\sqrt{2gh}$$

where h = the head on the axis of the tube.

The value of the coefficient .815 varies somewhat with the head, which should not exceed about 40 or 50 feet; it is smaller for high heads than for low heads, its value being as low as .80 for the former and as high as .83 for the latter. For heads higher than 50 feet, use .8 for the coefficient.

188. Discharge through Conical Tubes.—If instead of a straight tube, a conical tube be used, the discharge is further increased.



(a) (b)

Fig. 94.

If the edges are sharp, as indicated at a and b, Fig. 94 (a), the coefficient of efflux varies with the angle aob of the cone, its greatest value being about .95 when the angle aob = 13½°. If the edges are well rounded, as shown in Fig. 94 (b), the coefficient of efflux is still further increased, and may usually be taken as 1; that is, the discharge is $Q_a = Q = Av = A\sqrt{2gh}$, in which A = the area of the end of the tube from which the water issues, and h = the head on the axis of the tube.

189. Discharge through Nozzles.—A **nozzle** is a cone-shaped piece attached to the end of a pipe or hose, the *tip* being usually cylindrical, as indicated in Fig. 95. The diameter D is the same as that of the pipe or hose, and the diameter d of the tip is much smaller. The object of a nozzle is to increase the coefficient of the velocity of efflux, which increases the range of the water, a very desirable result in connection with fire hose. The theoretical velocity of efflux may be calculated by the formula of Art. **182,** in which a = area of cross-section of tip, A = area of cross-section

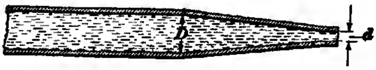


FIG. 95.

of pipe, and v = the velocity of efflux. Since these cross-sections are usually circles, the formula may be more conveniently written as follows: let d = diameter of tip, D = diameter of pipe or hose; then, $a = \frac{1}{4}\pi d^2 = .25\pi d^2$, $A = .25\pi D^2$, and $\frac{a}{A} = \frac{.25\pi d^2}{.25\pi D^2} = (\frac{d}{D})^2 = n^2$ when $n = \frac{d}{D}$. Substituting in the formula of Art. **182,** $\left(\frac{a}{A}\right)^2 = n^4$, and

$$v = \sqrt{\frac{2gh}{1 - n^4}}$$

The coefficient of velocity varies from $c_v = .97$ to $c_v = .99$, a fair average being $c_v = .98$; hence,

$$v_a = .98\sqrt{\frac{2gh}{1 - n^4}} = 7.86\sqrt{\frac{h}{1 - n^4}} \qquad (1)$$

The coefficient of efflux may also be taken as $.98 = c_e$; hence, $Q_a = .98av = .98 \times \frac{\pi \times d^2}{4 \times 144} \times 8.02\sqrt{\frac{h}{1 - n^4}}$, from which ($d$ being taken in inches),

$$Q_a = .0429d^2\sqrt{\frac{h}{1 - n^4}} \qquad (2)$$

If, instead of the head h, the specific pressure p is given, then (by Art. **181** $h = 2.31p$; substituting this value of h in formulas (1) and (2) and reducing,

$$v_a = 11.95\sqrt{\frac{p}{1 - n^4}} \qquad (3)$$

$$Q_a = .0652d^2\sqrt{\frac{p}{1 - n^4}} \qquad (4)$$

EXAMPLE.—The diameter of a fire hose is $2\frac{1}{4}$ in., the diameter of the tip of the nozzle is $\frac{3}{4}$ in., and the pressure at the nozzle is 61 lb. per sq. in.; what is (a) the velocity of discharge? (b) the number of gallons discharged per minute? (c) neglecting the resistance of the air, to what height will the water rise if the nozzle is pointed vertically upward?

SOLUTION.—(a) Applying formula (3), $n = \frac{.75}{2.25} = \frac{1}{3}$, and

$$v_a = 11.95\sqrt{\frac{61}{1 - (\frac{1}{3})^4}} = 93.9 \text{ ft. per sec. } \quad Ans.$$

(b) Applying formula (4),

$$Q_a = .0652(\tfrac{3}{4})^2\sqrt{\frac{61}{1 - (\frac{1}{3})^4}} = .288 \text{ cu. ft. per sec.}$$

The number of gallons per minute is, therefore,

$$.288 \times 7.48 \times 60 = 129 \text{ gal. per min. } \quad Ans.$$

(c) The height to which the water will rise is

$$h = \frac{v_a^2}{2g} = \frac{93.9^2}{2 \times 32.16} = 137 \text{ ft. } \quad Ans.$$

190.—The Venturi Meter.—This is a device for measuring the flow of water at some point in a pipe line; it is very simple, but accurate, and consists essentially of two conical surfaces (frustums) joined at their small ends, as indicated in Fig. 96, where

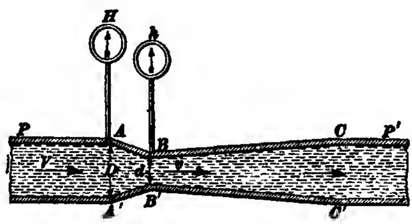


FIG. 96.

AB and BC are the two conical frustums. The angle which AB makes with the axis may be from 12° to 15°, while the angle that BC makes with the axis should be only about one-fourth as great, say from 3° to $3\frac{3}{4}$°; this makes the angles included between AB and $A'B'$ and between BC and $B'C'$ 24° to 30° and 6° to $7\frac{1}{2}$°, respectively. The throat BB' is rounded to reduce friction. The meter is inserted in the pipe line PP', the large ends having the same inside diameter as the pipe, denoted by D. The diameter at the small end, called the *throat*, is denoted by d. Small pipes are inserted at A and B, which have gauges H and

189. Discharge through Nozzles.—A **nozzle** is a cone-shaped piece attached to the end of a pipe or hose, the *tip* being usually cylindrical, as indicated in Fig. 95. The diameter D is the same as that of the pipe or hose, and the diameter d of the tip is much smaller. The object of a nozzle is to increase the coefficient of the velocity of efflux, which increases the range of the water, a very desirable result in connection with fire hose. The theoretical velocity of efflux may be calculated by the formula of Art. **182**, in which a = area of cross-section of tip, A = area of cross-section

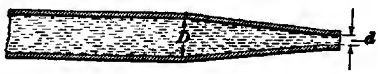


FIG. 95.

of pipe, and v = the velocity of efflux. Since these cross-sections are usually circles, the formula may be more conveniently written as follows: let d = diameter of tip, D = diameter of pipe or hose; then, $a = \frac{1}{4}\pi d^2 = .25\pi d^2$, $A = .25\pi D^2$, and $\frac{a}{A} = \frac{.25\pi d^2}{.25\pi D^2} = \left(\frac{d}{D}\right)^2 = n^2$ when $n = \frac{d}{D}$. Substituting in the formula of Art. **182**, $\left(\frac{a}{A}\right)^2 = n^4$, and

$$v = \sqrt{\frac{2gh}{1 - n^4}}$$

The coefficient of velocity varies from $c_v = .97$ to $c_v = .99$, a fair average being $c_v = .98$; hence,

$$v_a = .98\sqrt{\frac{2gh}{1 - n^4}} = 7.86\sqrt{\frac{h}{1 - n^4}} \qquad (1)$$

The coefficient of efflux may also be taken as $.98 = c_e$; hence, $Q_a = .98av = .98 \times \frac{\pi \times d^2}{4 \times 144} \times 8.02\sqrt{\frac{h}{1 - n^4}}$, from which ($d$ being taken in inches),

$$Q_a = .0429d^2\sqrt{\frac{h}{1 - n^4}} \qquad (2)$$

If, instead of the head h, the specific pressure p is given, then (by Art. **181** $h = 2.31p$; substituting this value of h in formulas (1) and (2) and reducing,

$$v_a = 11.95\sqrt{\frac{p}{1 - n^4}} \qquad (3)$$

$$Q_a = .0652d^2\sqrt{\frac{p}{1 - n^4}} \qquad (4)$$

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have been calculated as follows: $Q = AV = \frac{\pi}{4} \times \frac{6^2}{144} \times 10$ $= 1.96$ cu. ft. per sec.

In the foregoing formulas, it has been assumed, as is generally the case, that the diameter of the meter at A is equal to the diameter of the pipe at that point. In any case, D in the formula is the diameter of the pipe and V is the velocity of the water as it enters the meter.

FLOW OF WATER IN PIPES

191. Loss of Head.—When water flows through a pipe under the influence of gravity only, the head that induces the flow is the hydrostatic head measured by the difference of level between the upper surface of the water at the entrance to the pipe and the horizontal plane passed through the middle point of the section where the water is discharged. Thus, suppose that a pipe is connected to the bottom of a reservoir, and the depth of the water at the entrance to the pipe is 18 ft.; if the vertical distance between the point where the water enters the pipe and the point at which the water is discharged is 45 ft., the hydrostatic head is $45 + 18 = 63$ ft., and this is the head that induces the flow, provided the water is discharged freely into the atmosphere. Suppose, however, that the water discharges into another tank or reservoir, and that the depth of the water in the second reservoir above the point of discharge is 12 ft.; this acts as a head that tends to prevent the water from entering the second reservoir—it tends to make the water move in the opposite direction in the pipe. Consequently, the *effective hydrostatic head* is $63 - 12 = 51$ ft., and this is the head inducing the flow from the first reservoir to the second. It is easy to see that the effective hydrostatic head is the difference of level between the upper surface of the water at entrance and the upper surface of the water at discharge, and the length or shape (straight or curved) of the pipe has nothing to do with the effective hydrostatic head, which will here be called, simply, the hydrostatic head. This consideration is of special importance where water wheels are so situated that high water in the tailrace or river may seriously affect the effective head on the wheels.

The velocity with which water discharges from a pipe is not the velocity due to the hydrostatic head. As the water flows through the pipe, it meets with certain resistances, principally friction, the

effect of which is exactly the same as that of a head opposing the flow. Denoting the hydrostatic head by h_s and the head that is equivalent to the resistances by h_r, the effective head h that causes the flow is

$$h = h_s - h_r$$

If h_r is equivalent to *all* the resistances, then the velocity of efflux will be

$$v = \sqrt{2gh}$$

The head h_r is called the **loss of head,** and it is made up of a number of elements, some of which are:

(1) There is a loss of head when the water enters the pipe, unless the end of the pipe is flush with the side of the reservoir and is well rounded, which is not usually the case.

(2) As the water flows through the pipe, it rubs against the sides of the pipe and there is a loss of head due to friction. Except in the case of very short pipes, this is the principal loss of head.

(3) If the pipe is suddenly enlarged or suddenly contracted, as when water flows from a small pipe into a larger one or from a large pipe into a smaller one, there is a loss of head due to this.

(4) Bends, particularly sharp bends and those having a short radius, also produce a loss of head.

(5) Any obstruction of any nature whatever, such as rivets, flanges, valve openings, or foreign substances lodged inside the pipe, reduce the cross-sectional area and also the flow, and act as a loss of head.

If pipes are smooth (inside), have no projecting edges, all enlargements or contractions are made gradual, bends are well rounded and to a large radius, then the loss of head due to friction is so great in comparison with the others that they may be neglected in calculating the discharge.

192. Actual Velocity of Discharge.—There are so many factors entering into and affecting the flow of water that it is practically impossible to calculate the discharge of a pipe with any great degree of accuracy. As the result of a large number of experiments, carefully conducted by many different observers and under various conditions, it has been found that the friction loss varies directly as the length of the pipe, inversely as the diameter, and nearly as the square of the velocity, and that it is independent of the pressure. Many different formulas have

been suggested for the velocity, the following being as satisfactory as any for accuracy:

$$v = 2.315\sqrt{\frac{hd}{fl + .1d}} = \sqrt{\frac{5.36hd}{fl + .1d}} \qquad (1)$$

Here v = mean velocity of efflux in feet per second;
h = hydrostatic head in feet;
l = total length of pipe in feet from the point of entrance to the point of discharge;
d = diameter of pipe in inches;
f = coefficient of friction. For paper stock or pulp, this factor varies with many conditions, principally with the consistency of the stock.

It is assumed that the pipe is a straight cylindrical pipe of uniform cross-section (diameter) throughout its length; or, if the diameter varies, the change from one size to another is gradual, as in the case of the venturi meter. It is also assumed that the pipe is smooth, is either new or has been in use for but a short time, and is made of cast iron, steel, or wrought iron. According to Weisbach, the coefficient of friction f may be expressed by the formula

$$f = .01439 + \frac{.017155}{\sqrt{v}} \qquad (2)$$

193. Formula (1) of the last article may be used when the length of the pipe exceeds about 60 times its diameter. The term $.1d$ allows for the loss of head due to entrance; and if the entrance is well rounded and the end of the pipe does not project into the water, this term may be neglected; it may also be neglected if the pipe is longer than 1000 times its diameter. In either case, the formula becomes

$$v = 2.315\sqrt{\frac{hd}{fl}} = \sqrt{\frac{5.36hd}{fl}} \qquad (1)$$

A pipe whose length is less than 3 times its diameter is called a **short tube,** and its discharge may be calculated by the formula of Art. **187.** If the length is greater than 3 times but less than 60 times its diameter, the pipe is called a **long tube** or a **very short pipe;** the velocity of the efflux is then given by the formula

$$v = (.832 - .004\frac{l}{d})\sqrt{2gh} = (6.67 - .032\frac{l}{d})\sqrt{h} \qquad (2)$$

If the pipe is longer than 60 times its diameter but less than 1000

times its diameter, it is called a **short pipe**, and the velocity of efflux should be calculated by formula (1) of Art. **192**. A pipe longer than 1000 times its diameter is called a **long pipe**, and the velocity of efflux may be calculated by formula (1) above.

EXAMPLE.—What is (*a*) the velocity of efflux from an 8-inch pipe, 24 ft. long, under a head of 5 ft.? (*b*) the discharge in gallons per minute?

SOLUTION.—(*a*) Since 8 in. = $\frac{2}{3}$ ft., $\frac{l}{d} = 24 \div \frac{2}{3} = 36$; hence, the length of the pipe is 36 times its diameter and formula (2) must be used.

$$v = (6.67 - .032 \times 36)\sqrt{5} = 12.34 \text{ ft. per sec.} \quad Ans.$$

(*b*) The discharge in cubic feet per second is $Q = av = \frac{.7854d^2v}{144}$ $= .005454d^2v$; in gallons per second, it is $\frac{.7854d^2v}{144} \times 7.48 = .0408d^2v$; in gallons per minute, it is $.0408d^2v \times 60 = 2.448d^2v = 2.448 \times 8^2 \times 12.34$ $= 1933$ gal. per minute. *Ans.*

When the diameter of the pipe is taken in inches and v in feet per second,

$$Q = .005454d^2v = \frac{6d^2v}{1100} \text{ cu. ft. per sec.} \quad (3)$$

$$Q = .0408d^2v \text{ gal. per sec.} \quad (4)$$

$$Q = 2.448d^2v \text{ gal. per min.} \quad (5)$$

Knowing the velocity, it can be substituted in one of these three formulas to find the discharge.

194. To calculate accurately the velocity of efflux from a long or a short pipe by either of the two formulas just given requires that the coefficient of friction f be known; but before f can be determined, it is necessary to know the velocity (the very quantity it is desired to find) to substitute in formula (2) of Art. **192.** This difficulty is overcome by assuming a value for f, say .024, and calculate v; then calculate f or take it from the table below. If this value of f is greater or less than the assumed value, recalculate v, using the new value of f.

COEFFICIENTS OF FRICTION FOR v IN FEET PER SECOND

$v =$ 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$f =$ .0686	.0527	.0457	.0415	.0387	.0365	.0349	.0336	.0325
$v =$ 1.0	1.2	1.4	1.6	1.8	2	3	4	5
$f =$ .0315	.0300	.0289	.0280	.0272	.0265	.0243	.0230	.0221
$v =$ 6	7	8	9	10	11	12	14	16
$f =$ .0214	.0209	.0205	.0201	.0198	.0196	.0193	.0190	.0187

EXAMPLE.—What is the velocity of efflux from a 4-inch pipe that is 740 ft. long, under a head of 58 ft.?

SOLUTION.—Here 4 in. = $\frac{4}{12}$ ft., and $740 \div \frac{4}{12} = \frac{740 \times 12}{4}$ is evidently greater than 1000; hence, use the velocity formula for long pipes. Taking $f = .024$,

$$v = 2.315\sqrt{\frac{58 \times 4}{.024 \times 740}} = 8.37 \text{ ft. per sec.}$$

Referring to the table, the value of f for $v = 8$ is .0205 and for $v = 9$, .0201. Since v will be greater the smaller f is, and since there is considerable difference between .0240 and .0205 or .0201, try .02 for f. Then,

$$v = 2.315\sqrt{\frac{58 \times 4}{.02 \times 740}} = 9.17 \text{ ft. per second. } \textit{Ans.}$$

For this value of v, f is equal almost exactly to .0200.

Had formula (1) of Art. **192** been used, the value for v would have been

$$v = 2.315\sqrt{\frac{58 \times 4}{.02 \times 740 + .1 \times 4}} = 8.73 \text{ ft. per sec.}$$

To find the value of f for this value of v by using the table, proceed as follows: let v' = the next smaller value in the table and v'' = the next larger value; let f' = the value corresponding to v', and f'' = the value corresponding to v''; then, letting v = the given (or calculated) value and f = the required value,

$$f = f' + (f'' - f')\frac{v - v'}{v'' - v'}$$

In the present case, $v = 8.73$, $v' = 8$, $v'' = 9$, $f' = .0205$, and $f'' = .0201$; hence, $f = .0205 + (.0201 - .0205)\frac{8.73 - 8}{9 - 8} = .0205 - .0004 \times .73 = .0202$. Substituting this value of f,

$$v = 2.315\sqrt{\frac{58 \times 4}{.0202 \times 740 + .1 \times 4}} = 8.67 \text{ ft. per sec.}$$

This last value of v is as close as can be obtained by the formula; and it will be noted that it differs quite a little from the value first obtained, which was 9.17 ft. per sec., being $\frac{9.17 - 8.67}{9.17} = .055 - = 5\frac{1}{2}\%$ smaller. This may seem considerable, but the values obtained by the different formulas recommended by the various authorities will vary as much or more. It is safer to use the smaller value, however, and it is therefore suggested and recommended that formula (1) of Art. **192** be used for all pipes the length of which is less than 5000 times the diameter. In any case, regardless of what formula is used or by what

authority recommended, it is useless to express final results to more than 2 significant figures, except, perhaps, when the first figure is 1, in which case, 3 significant figures may be used, though the last figure will probably not be accurate.

195. Interpolation.—The method given above for finding a value intermediate between those given in a table is called the **method of interpolation** or, simply, **interpolation**, and the formula may be used in connection with almost any table arranged for practical use. To illustrate the method more fully, consider the portion of a table in the margin, which gives the total heat of steam corresponding to a given number of inches of vacuum. The first (left-hand) column gives the vacuum in inches of mercury, and is called the column of **arguments**; the second column gives the total heats in B.t.u., corresponding to the given vacuums and is called the column of **functions**. If the table in the last article were arranged in this same order, the velocities would be the arguments and the values of f would be the functions.

Vacuum (Inches)	Total Heat (B.t.u.)
20	1130.1
18	1133.4
16	1136.1
14	1138.6
12	1140.7
10	1142.3

Suppose, now, that it were desired to find the total heat corresponding to a vacuum of 15.2 in. Here the given argument 15.2 lies between 14 and 16 in the table. In all cases of this kind, let x' = the argument next *above* and x'' = the argument next *below* the given argument, the given argument being supposed to be written in its proper place in the table; let u' = the function corresponding to the argument x' and u'' = the function corresponding to the argument x''; also, let x = the given argument and u = the required function, which corresponds to the argument x. Then

$$u - u' : u'' - u' = x - x' : x'' - x'$$

From this proportion, u can be found if x is given or x can be found if u is given; thus, solving for u,

$$u = u' + (u'' - u')\frac{x - x'}{x'' - x'} \qquad (1)$$

and solving for x,

$$x = x' + (x'' - x')\frac{u - u'}{u'' - u'} \qquad (2)$$

To find the value of u (the total heat) for a vacuum of 15.2 in.

by means of the above table, $x = 15.2$, $x' = 16$, $x'' = 14$, $u' = 1136.1$, $u'' = 1138.6$, and u is found to be, by formula (1),

$$u = 1136.1 + (1138.6 - 1136.1)\frac{15.2 - 16}{14 - 16} = 1137.1 \text{ B.t.u.}$$

If it were desired to find the vacuum corresponding to a total heat of 1135 B.t.u. $= u$, u falls between $1133.4 = u'$ and $1136.1 = u''$; the corresponding values of the arguments are $x' = 18$ and $x'' = 16$. Substituting in formula (2),

$$x = 18 + (16 - 18)\frac{1135 - 1133.4}{1136.1 - 1133.4} = 16.8 \text{ in. of vacuum.}$$

When finding values of u (the function), it is useless to express them to a greater number of significant figures than are given to the functions in the table.

196. Quantity Discharged.—Having calculated or measured the velocity of efflux, the discharge is found by the formula $Q = Av$. In the case of pipes, the discharge is generally expressed in gallons per minute, and may be calculated by formula (5) of Art. **193** when v is known. Substituting in this formula the value of v in formula (1) of Art. **192**,

$$Q = 5.667d^2\sqrt{\frac{hd}{fl + .1d}} \text{ gal. per min.} \qquad (1)$$

By this formula, the discharge in gallons per minute can be calculated directly when the length and diameter of the pipe and the head are known.

To find the mean velocity of efflux when the discharge in gallons per minute and the diameter of the pipe in inches are known, solve formula (5) of Art. **193** for v, obtaining

$$v = \frac{.4085Q}{d^2} \qquad (2)$$

EXAMPLE 1.—How many gallons of water will a 6-inch pipe deliver in 24 hours under a head of 170 ft., if the length of the pipe is 5780 ft.?

SOLUTION.—Since formula (1) requires that f be known, this must be determined first, and this requires the calculation of the velocity v. Also, since the length of the pipe divided by the diameter $= \frac{5780 \times 12}{6}$, is greater than 5000, formula (1) of Art. **193** may be used. Assuming that $f = .024$, $v = \sqrt{\frac{5.36 \times 170 \times 6}{.024 \times 5780}} = 6.27$ ft. per sec. For $v = 6$, $f = .0214$, which being smaller than the assumed value .0240, shows that the value of v will be greater than 6.27 if .0214 were substituted for f. Taking v as, say, 6.6,

$f = .0211$. Now v may be recalculated or formula (1) above may be applied to find the discharge, in which case,

$$Q = 5.667 \times 6^2 \sqrt{\frac{170 \times 6}{.0211 \times 5780 + .1 \times 6}} = 590 \text{ gal. per min.}$$

$$= 590 \times 60 \times 24 = 849{,}600, \text{ say } 850{,}000 \text{ gal. in 24 hours.} \quad \textit{Ans.}$$

If it is desired to calculate f, find that $f = .0213$ for $v = 6.27$. Substituting in formula (1) of Art. **192**, $v = \sqrt{\dfrac{5.36 \times 170 \times 6}{.0213 \times .5780 + .1 \times 6}} = 6.65$. For $v = 6.65$, $f = .0211$, and Q will be found to have the same value as before, since f and all the other quantities are the same. If .0211 be substituted for f in the above formula, the value of v will be found to be 6.68, for which $f = .0211$ to four decimal places, which are all that can be relied on. Note that a change of 2 units in the number expressed by the significant figures of f, makes a change of 3 units in the significant figures of v; thus for $f = .0213$, $v = 6.65$ and for $f = .0211$, $v = 6.68$. The difference between $f = .0240$ and the calculated value of f multiplied by $\frac{3}{2}$ will therefore be approximately equal to the difference between the velocities. For instance $(.0240 - .0213) \times \frac{3}{2} = .00405$, or, say, 41 units; since v increases when f decreases, $627 + 41 = 668$, or 6.68, which is the exact value of f to 3 significant figures. This method of approximating f from the value .0240 and calculated values of v and f is usually exact enough for all practical purposes.

EXAMPLE 2.—An 8-inch pipe discharges 1,480,000 gal. of water per day of 24 hours; what is the average velocity of efflux?

SOLUTION.—The discharge in gallons per minute is $\dfrac{1{,}480{,}000}{60 \times 24} = 1028 -$ gal. per min. Substituting in formula (2) above,

$$v = \frac{.4085 \times 1028}{8^2} = 6.58, \text{ say } 6.6 \text{ ft. per sec.} \quad \textit{Ans.}$$

197. Head Required to Produce a Given Discharge.—Suppose a certain discharge is required from a pipe of a given size; the head necessary to produce this discharge may be found by solving formula (1) of the last article for h, obtaining

$$h = \left(\frac{Q}{5.667 d^2}\right)^2 \left(f\frac{l}{d} + .1\right) \qquad (1)$$

If it is desired to find the head that will produce a certain velocity, solve formula (1) of Art. **192** for h, obtaining.

$$h = \frac{v^2}{5.36d}(fl + .1d) = .187v^2\left(f\frac{l}{d} + .1\right) \qquad (2)$$

EXAMPLE.—It is desired to have an 8-inch pipe discharge 1,200,000 gal. per day of 24 hours; if the length of the pipe is 7500 ft., what must be the head?

SOLUTION.—The discharge per minute is $\dfrac{1200000}{60 \times 24} = 833$ gal. The velocity of efflux is, by formula (2) of the last article,

$$v = \frac{.4085 \times 833}{8^2} = 5.32, \text{ say } 5.3 \text{ ft. per sec.}$$

From the table of Art. **194**, $f = .0219$, by interpolation, for $v = 5.3$. Substituting in formula (1) above.

$$h = \left(\frac{833}{5.667 \times 8^2}\right)^2 \left(\frac{.0219 \times 7500}{8} + .1\right) = 109\text{-ft.} \quad \textit{Ans.}$$

198. To Find the Diameter of the Pipe for a Given Discharge. If the velocity of efflux is known, the diameter is readily found by solving formula (2) of Art. **196** for d, obtaining

$$d = \sqrt{\frac{.4085Q}{v}} \qquad (1)$$

If, however, v is not known, neglect the term $.1d$ in formula (1) of Art. **196** and solve for d, obtaining

$$d = \sqrt[5]{\frac{flQ^2}{32.11h}} \qquad (2)$$

In order to apply this formula, it is first necessary to assume a value for f, calculate d, then apply formula (2) of Art. **196** to find v, and, finally, find the value of f corresponding to this v, substitute in formula (2) again, and calculate d. For example, suppose it is desired to find the diameter of a pipe that will discharge 900,000 gal, of water per day of 24 hours under a head of 80 ft. the length of the pipe being 9600 ft. The discharge per minute is $\frac{900000}{24 \times 60} = 625$ gal. Assuming that $f = .024$, as recommended in Art. **194**, formula (2) above may be written

$$d = \sqrt[5]{\frac{.024Q^2l}{32.11h}} = \sqrt[5]{\frac{Q^2l}{1340h}} \qquad (3)$$

Substituting the values given in this last formula,

$$d = \sqrt[5]{\frac{625^2 \times 9600}{1340 \times 80}} = 8.1 \text{ in.}$$

Substituting this value of d in formula (2) of Art. **196**,

$$v = \frac{.4085 \times 625}{8.1^2} = 3.89 \text{ ft. per sec.}$$

From the table of Art. **194**, $f = .0231$ for $v = 3.89$; hence, substituting this value of f in formula (2),

$$d = \sqrt[5]{\frac{.0231 \times 9600 \times 625^2}{32.11 \times 80}} = 8.0+$$

Consequently, if the pipe is new, an 8-inch pipe will probably answer.

199. It will be noted that in the preceding calculation, it was necessary to extract the fifth root of a number. It is never

necessary to obtain more than two significant figures for the diameter of a pipe, because fractions of an inch rarely occur in the diameter of a pipe as ordinarily manufactured and the largest commercial size is less than 100 in. in diameter. From 1 in. to 72 in., the ordinary sizes for cast-iron or wrought-iron pipe are as follows: 1, 2, 3, 4, 6, 8, 10, 12, 14, 16, 18, 20, 24, 30, 36, 42, 48, 54, 60, 72 inches in diameter. Therefore, in calculating the diameter of a pipe as illustrated in the last article, if a fraction occur in the calculated diameter, always take the commercial size next larger than the integral part of the calculated diameter. The diameter of the pipe as calculated will be for new pipe; but as the pipe continues in use, the discharge for the same head will become considerably reduced. For this reason, it would be better to use a 10-inch pipe than an 8-inch one for the case mentioned in the last article, if the supply is to be kept up indefinitely at 900,000 gal. per day. If the pipe is foul from long use, or is quite rough (due to the corroding action of impure water or to other causes), the coefficient of friction f should be doubled in calculating the discharge, that is, the value $2f$ should be used instead of f; this will make the discharge from a foul pipe about 70% of that from a clean pipe having the same diameter and head. Since the fifth root is desired to only two significant figures, it may be obtained directly from the table of powers given in *Elementary Applied Mathematics*.

EXAMPLE.—If the head is 64 ft. and the length of the pipe is 370 ft., what should be the diameter to discharge 200 gal. of water per minute?

SOLUTION.—Applying formula (3),

$$d = \sqrt[5]{\frac{200^2 \times 370}{1340 \times 64}} = 2.8, \text{ say } 3 \text{ in.}$$

Since the selected value, 3 in., is relatively considerably larger than the calculated value, 2.8 in., and since a 2-inch pipe will evidently be too small, it is not necessary to calculate the diameter any closer. It will be well, however, to calculate the discharge for a 3-inch pipe. Doing so, the approximate velocity may be found roughly by formula (2) of Art. **196**, in order to determine an approximate value for f. Thus,

$$v = \frac{.4085 \times 200}{3^2} = 9 + \text{ ft. per sec.}$$

From the table of Art. **194**, $f = .0201$ when $v = 9$. Substituting in formula (1) of Art. **196**,

$$Q = 5.667 \times 3^2\sqrt{\frac{64 \times 3}{.0201 \times 370 + .1 \times 3}} = 254 \text{ gal. per min.}$$

Therefore, the 3-inch pipe will be sufficiently large. *Ans.*

200. A little consideration will show that insofar as commercial sizes of pipe are concerned, the diameter may be calculated by formula (3) of Art. 198. It is only when exact dimensions are required that the more refined method of Art. 196 is necessary.

Paper pulp suspended in water has a higher coefficient of friction than water, and the coefficient varies with the kind of stock and its consistency. This necessitates larger pipe, pumps, etc. than would be required for water, as is explained in the chapter on *pumps* under *General Mill Equipment.*

EXAMPLES

(1) What is (*a*) the velocity of efflux from a standard square orifice, 3 in. square, the head on the center being 18 ft. 4 in.? (*b*) What is the discharge in gallons per minute?

Ans. { (*a*) 33.6 ft. per sec.
(*b*) 592 gal. per min.

(2) The pressure at the entrance to a venturi meter is 32.7 lb. per sq. in. the pressure at the throat is 21.5 lb. per sq. in., diameter at entrance is 4 in., diameter at throat is 1¾ in. What is (*a*) the velocity at entrance? (*b*) the discharge?

Ans. { (*a*) 7.8 ft. per sec.
(*b*) 306 gal. per min.

(3) What is the discharge through a short tube having sharp edges, if the diameter of the tube is 2½ in. and the head on the center is 18 ft. 6 in.?

Ans. 430 gal. per min.

(4) A nozzle has a diameter of 3 in. at the large end and 1⅛ in. at the tip. When the pressure is 75 lb. per sq. in., (*a*) what will be the velocity of efflux? (*b*) To what height can the water be thrown if the nozzle is pointed vertically upward? (*c*) What is the discharge?

Ans. { (*a*) 105 ft. per sec.
(*b*) 170 ft.
(*c*) 324 gal. per min.

(5) What is (*a*) the velocity of efflux from a pipe 6 ft. long, 1¾ in. in diameter, under a head of 18 ft. 5 in.? (*b*) What is the discharge?

Ans. { (*a*) 23.0 ft. per sec.
(*b*) 172 gal. per min.

(6) A 4-inch pipe, 1875 ft. long, discharges water under a head of 156 ft.; what is (*a*) the velocity of efflux? (*b*) the discharge?

Ans. { (*a*) 9.39 ft. per sec.
(*b*) 368 gal. per min.

(7) What should be (*a*) the diameter of a pipe, commercial size, to discharge 2,400,000 gal. per day of 24 hours, if the length of the pipe is 14,400 ft. and the head is 240 ft? (*b*) How many gallons per day will a pipe of this diameter discharge under the same conditions when new?

Ans. { (*a*) Diameter = 12 in.
(*b*) 3,500,000 gal.

(8) What must be the head in order that a 2-inch pipe that is 1025 feet long may discharge 3000 gal. of water per hour? Ans. 55 ft.

MECHANICS AND HYDRAULICS

(PART 4)

EXAMINATION QUESTIONS

(1) A steam pump is rated at 28 horsepower; what (*a*) would be its rating in kilowatts? (*b*) in metric horsepower?

Ans. (*a*) 20.9 k.w. (*b*) 28.4 metric h.p.

(2) The m.e.p. (mean effective pressure) as measured from an indicator diagram of a steam engine is 65.15 lb. per sq. in.; if the diameter of the cylinder is 28 in., length of stroke 36 in., and revolutions per minute is 125, what is the horsepower?

Ans. 912 h.p.

(3) The power of an electric current in watts is equal to the strength of the current in amperes multiplied by the pressure in volts. If a dynamo deliver 65 amperes of current at 225 volts, (*a*) what is the power of the current in kilowatts? (*b*) If the efficiency of the dynamo is 88.6%, what horsepower is required to operate it? (*c*) If the dynamo be driven by a steam turbine having an efficiency of 91%, what power must it generate to operate the dynamo?

Ans. (*a*) 14.625 k.w. (*b*) 22.14 h.p (*c*) 24.33 h.p.

(4) A paper mill has a contract with an electric power plant to furnish current for lighting and power at the rate of $1\frac{3}{4}$ cents per k.w.-h. During one week of 7 days, it burned 28 25-watt lamps, 56 40-watt lamps, and 24 125-watt lamps an average of $11\frac{1}{2}$ hours per day; (*a*) what was the daily cost for lighting? (*b*) what was the equivalent of the work paid for in horsepower-hours?

Ans. (*a*) \$1.195, say \$1.20. (*b*) 91.6 h.p.-hr.

(5) To what height will a 50-horsepower pump deliver 540 gal. of water per minute, if the efficiency of the pump (allowing for friction, leakage, etc.) is 78%? *Ans.* 286 ft.

(6) Water flows through a rectangular orifice 3½ in. by 5¼ in. at the rate of 1240 gal. per min.; what is its mean velocity?

Ans. 21.6 ft. per sec.

(7) If the orifice in Question 6 is a standard orifice, what is the head on the center to obtain the same discharge? *Ans.* 19.2 ft.

(8) The nozzle of a fire hose is 3 in. in diameter at entrance and ¾ in. in diameter at the tip; (*a*) if the head is 180 ft., what is the velocity of discharge? (*b*) to what vertical height can the water be thrown? (*c*) what is the discharge?

Ans. (*a*) 105.7 ft. per. sec. (*b*) 173.7 ft. (*c*) 145.5 gal. per min.

(9) What is (*a*) the velocity and (*b*) the discharge through a pipe 1½ in. in diameter and 70 in. long under a head of 21 ft.?

Ans. (*a*) 23.7 ft. per sec. (*b*) 131– gal. per min.

(10) What is (*a*) the velocity at entrance and (*b*) the discharge through a venturi meter if the head at entrance is 59 ft., head at throat is 42 ft., diameter at entrance is 3½ in., and diameter at throat is 1½ in.?

Ans. (*a*) 6.06 ft. per sec. (*b*) 182 gal. per min.

(11) What is (*a*) the discharge and (*b*) the velocity of discharge from a 5-inch pipe, 1280 ft. long, under a head of 210 ft.?

Ans. (*a*) 924 gal. per min. (*b*) 15.1 ft. per sec.

(12) What commercial size of pipe should be laid to deliver 3,000,000 gal. of water to a paper mill per day under a head of 356 ft., if the length of the pipe is 12,700 ft.? *Ans.* 10 in.

(13) What head is required for a 3-inch pipe, 756 ft. long, to deliver 5000 gal. of water per hour? *Ans.* 16 ft.

(14) Assuming that the water in the last example were used to drive a small turbine having an efficiency of 77.8%, what would be the horsepower of the turbine? *Ans.* 0.262, say ¼ h.p.

(15) A 450-horsepower steam engine is operated an average of 7 hr. 24 min. for 6 days each week. Counting 52 weeks per year, (*a*) what will be the cost per kilowatt-hour if the power is bought for $36 per horsepower-year? (*b*) what is the power cost per hour of operation?

Ans. (*a*) $0.0209. (*b*) $7.017.

SECTION 2

ELEMENTS OF ELECTRICITY

By J. J. CLARK, M.E.

(PART I)

NATURE AND KINDS OF ELECTRICITY

INTRODUCTION

1. Purpose of Study.—The subject of electricity has been included in this course in order that the student may understand the application of its principles in connection with the operation of the different electrical machines and apparatus used in pulp and paper mills. How electricity is generated and controlled will be explained, and such of its principles will be discussed as will enable the student to understand the construction and operation of electric motors and other apparatus used about the plant; that is, he will understand their object, how to select them for different purposes, and what makes them "go." It is beyond the scope of this work to teach the design of electrical machinery, but the student who has made a thorough study of the principles here explained ought to be able to select intelligently the proper machine or apparatus for any specific purpose, to determine whether the apparatus already in use is that best adapted to the fulfillment of the desired purpose, and should be able to understand what is the matter with it if it get out of order.

The connection between electricity and magnetism is extremely close, and the student is urged to pay particular attention to the explanations here given; a thorough understanding of the principles of magnetism will make the subsequent part of the text comparatively easy. More complete information relating to the entire subject will be found in textbooks that can usually be obtained from the local library or in the library of the engineering department of the plant.

STATIC ELECTRICITY

2. Nature of Electricity.—According to the Standard Dictionary, electricity is *"an imponderable and invisible agent producing various manifestations of energy, and generally rendered active by some molecular disturbance." Imponderable* means without weight. The exact nature of electricity is not known; many theories have been advanced, but no one of them has been accepted as satisfactory. Just what it is does not matter to the practical man; his only concern is to know how to generate it, control it, and make it do useful work. He is equally ignorant concerning the forces called gravitation, chemical affinity, and many other of nature's phenomena, but that does not deter him from utilizing them to the best advantage. It is very doubtful if he could derive any greater benefit from them if he knew what is their ultimate cause, since he could not, in any case, create them.

3. Generating Electricity.—There is really no such thing as generating or producing electricity, in the sense that something is obtained where nothing was before. Electricity is probably all-pervasive and is in equilibrium, and it manifests itself only when its equilibrium is disturbed by the action of mechanical or chemical forces; when these forces are directed in a specified and definite manner, the state of equilibrium is altered in a specified and definite manner also, and before the state of equilibrium can be restored, work must be done. If properly directed, this work may be utilized in a definite manner. The process is exactly analogous to the action of a pile driver. When the weight rests on the pile, everything is in equilibrium. In raising the weight to a particular height, a certain force must be exerted; and since this force acts through a distance equal to the height, a certain amount of work is done in raising the weight. Now for the system to return to its previous state of equilibrium, the weight must be restored to its previous level—the height of the pile—and in doing this, work may be accomplished. If the weight is allowed to fall on the pile—by removing its support and guiding it—the work done will be measured by the force of the blow and the distance that the pile is driven.

Thus, when anything is done to disturb the equilibrium of the electrical state of matter, work must be done to produce this disturbance; and work, useful or otherwise, may be obtained as

the result of the action. The action produces what may be termed an *electric stress*, and this is what is called **producing** or **generating electricity.** Always bear in mind the unalterable fact that electricity is not something that can be obtained from nothing; it is merely an agent, and gives back only what it receives. In other words, in order to obtain a certain amount of work or energy by means of electricity, it is necessary to do at least as much work in producing the electric stress as is given out when the stress is removed. For example, suppose one pulley to drive another pulley by means of a belt. As much work must be imparted to the driving pulley as is received by the driven pulley; in fact, more work must be imparted to the driving pulley, because of the hurtful resistances—friction, bending the belt, heating the bearings, etc. So it is when electricity is the agent; more work must be done than is obtained through the agency of electricity.

4. Electricity is an *agent* (see definition, Art. 2), an extremely useful one, and has two uses: (1) to transform energy from one state or kind to another; (2) to transfer energy from one point to another. By means of the dynamo, for instance, mechanical energy may be changed into electrical energy, which, in turn, may be used to drive a motor, heat an electric iron, light the house, cause the release of chlorine from salt, and in innumerable other ways. By means of wires, called *conductors*, electrical energy may be conveyed long distances—many miles, in fact—before it is again transformed and utilized.

5. Kinds of Electricity.—There is really only one kind of electricity; but since it manifests itself in two widely different forms, in so far as their effects are concerned, it is customary to divide it into two classes: (1) *static, or frictional, electricity;* (2) *dynamic, or current, electricity.*

Static, or frictional, electricity is generated by friction, by rubbing together certain unlike substances. For example, if a stick of sealing wax be rubbed with a piece of flannel or a glass rod be rubbed with a piece of silk, it will be found that the stick and the rod will both attract light substances, such as pith balls, pieces of paper, etc., the attraction being caused by the static electricity generated by the friction. Under certain conditions, when an electrified body is brought into contact with another body, a spark of light will pass between the points of contact just

before they touch. This may be well illustrated in the following manner: let a person stand on a piece of rubber or a glass plate, or take four glass tumblers, set them on the floor about a foot apart in the form of a square, lay a board on top of the tumblers, and let the person stand on the board; let a second person rub the back of the first person (who keeps on his coat, which is presumably made of wool) with a piece of fur, say a muff; then, if either person (or a third person) bring his finger to the skin of the other, a spark will pass, and the skin will feel a sensation like the prick of a pin. By continuing the rubbing, as many sparks may be obtained as are desired. The best results are obtained when the room is cool and the air is dry. Whenever, in combing the hair, the hair follows the comb, this effect is produced by static electricity. Sometimes, when walking under and near a rapidly moving belt, the hair on one's head will be drawn toward the belt with a slight pull; this is also caused by static electricity.

6. Positive and Negative Electricity.—By experiment, it will be found that there are two kinds of electrification, to which have been given the names of **positive electricity** and **negative electricity.** Both kinds are always generated at the same time When the glass rod is rubbed with silk, one kind is excited on the rod and the other kind on the silk; that excited on the rod is called *positive electricity*, and that excited on the silk is called *negative electricity*. The order is reversed when the stick of sealing wax is rubbed with flannel; here the electricity excited on the stick is negative and that excited on the flannel is positive. These facts can easily be proved by the circumstance that if both bodies are electrified positively or both negatively, they repel each other; while if one is electrified positively and the other negatively, they attract each other. This fact may be stated in the form of the following very important law:

Law.—*Bodies that are similarly electrified repel one another, while two bodies dissimilarly electrified attract each other.*

It is to be noted that either kind of electrification might have been termed positive, in which case, the other would have been negative; but, having decided to call the kind on the glass rod when rubbed with silk positive, or $+$, the kind produced on the silk is negative, or $-$.

7. It is to be noted that the electricity produced by friction stays where it is generated until it is **discharged,** as it is termed,

by bringing the electrified body into contact with or near another body. For instance, when the back of the person standing on the tumblers has been rubbed, he will retain the electricity indefinitely; but as soon as he steps on the floor or comes into contact with another person or object in contact with the floor, the electricity is discharged. For this reason, frictional electricity is called static electricity, the word *static* meaning a state of rest or equilibrium.

8. In the case of static electricity, the resulting electric stress is very high, but what may be called the electric quantity is very low. As a consequence, static electricity is practically useless as an agent for doing useful work; indeed, it is something to be avoided in paper mills or any other place where machinery is employed. When a moving belt passes close to a permanent object, tiny blue sparks may frequently be seen in the dark; these sparks are caused by the friction between the belt and the pulley, which results in an electric stress, and causes an accumulation of *static electric charges* (as they are termed), which leave the belt by the way of the permanent object and escape to the earth, thus restoring the equilibrium. In cases where this action takes place to a large extent, a metal wire or bar is fixed close to the belt, to remove these charges at a place where it is most convenient instead of allowing them to escape at random. Similar effects are observed on the winding rolls of a paper machine (because of the friction between the calenders and the paper); here a flexible copper wire or a brass chain is used to remove the electric charge, which passes along the wire or chain to the frame and thence to the ground. If the charge be not removed, more work must be expended in winding and unwinding the paper, on account of the attractive forces between one layer of paper and the next on the roll. Much of the trouble experienced by the printer in feeding sheets of paper to the press is due to these static charges.

Static charges of this kind are not dangerous to human life; but if they are not removed by a wire or chain or other conductor, they may escape to the body, head, or hand. The sudden shock is not dangerous in itself, but it may so startle a man as to cause him to make a quick, unusual movement, which may bring him into contact with moving parts of a machine. These sparks are identical in character with lightning, the only difference being that in the case of lightning, the electrical stress is enormously

higher. Still, sparks from a high-speed belt have been known to ignite a wiping rag that was wet with gasoline.

9. Conductors.—*An electric charge may be conveyed from one point to another by means of what is called a* **conductor.** *While all substances will conduct electricity,* some of them conduct so badly that for all practical purposes, they do not conduct at all; such substances are called **insulators** or **non-conductors.** Glass, rubber, and gutta percha are the best examples of insulators; practically, they do not conduct electricity at all. Pure water and air are also very poor conductors. All the metals are good conductors, though some are far better than others; silver is the best conductor known, and copper ranks next. As copper is very nearly as good a conductor as silver, and is much cheaper, it is the one most used in practice.

In the experiment in Art. **5**, glass tumblers were used to keep the electricity from passing from the body to the earth. The glass being a non-conductor, the electricity remained stored in the body until a path was made for it to escape to the earth, thus restoring the equilibrium. The path was the body of the second man, and the reason that it was necessary to approach so close before the spark could take effect was because air is such a poor conductor that the length of the path through the air had to be exceedingly short. Had the first person stood on the floor instead of the insulators, the electricity would have been generated as before; but it would have been conducted to the earth, the body acting as a conductor, as fast as formed, and no spark or other effect could be obtained. If several persons stand on insulators, the charge may be passed from one to another, a spark taking place each time, until the last person touches something not insulated from the earth. If the persons are insulated from the floor by wearing rubber boots or having rubber soles and heels on their shoes, the one having the charge may walk around the room and wait indefinitely before giving the charge to anyone else, provided no part of his body comes into contact with another object. It should also be noted that the spark may come from any part of the body of the person having the charge. Thus, if the second person brings any part of his body, say a finger, to any exposed part of the person charged, the back of the neck, hand, chin, etc., the spark will pass; this shows that the charge covers the entire body, and when discharged, is concentrated and passes along the conductor, entering it at the point of contact. Moreover,

the charge concentrates and passes to the ground almost instantaneously.

CURRENT ELECTRICITY

10. Definition.—If by some means, electricity can be supplied as fast as it flows away, the result will be a **continuous current** of electricity. This effect may be secured very simply by taking a glass jar, Fig. 1, and partly filling it with a solution of sulphuric

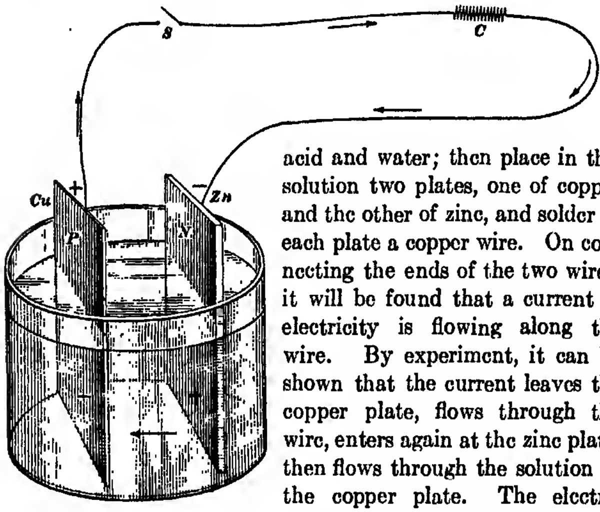

FIG. 1.

acid and water; then place in this solution two plates, one of copper and the other of zinc, and solder to each plate a copper wire. On connecting the ends of the two wires, it will be found that a current of electricity is flowing along the wire. By experiment, it can be shown that the current leaves the copper plate, flows through the wire, enters again at the zinc plate, then flows through the solution to the copper plate. The electric current is caused by chemical action, the acid in the solution dissolving the zinc gradually. Electricity produced in this manner is called **current** or **dynamic electricity,** because the electricity is in motion, moving (flowing) as fast as it is generated.

11. The apparatus described in the last article is called a **primary element** or **cell;** and when two or more cells are properly connected, the arrangement is called a **battery.** The copper plate is designated in Fig. 1 by Cu, the chemical symbol for copper, and the zinc plate is designated by Zn, the chemical symbol for zinc. The point or place where the wire is attached to the plate, and all that part of the plate above and not touched by the solution, is called the **electrode** or **pole,** terms often incorrectly

applied to the whole plate. In Fig. 1, P is the positive electrode (pole) and N is the negative electrode (pole). It will be observed that the current (as indicated by the arrows) leaves the cell at the positive electrode (positive pole), flows through the wire and coil C, enters the cell at the negative electrode (negative pole), then flows through the solution, which is called the **electrolyte,** to the negative plate, continuing in this manner until the electrolyte will dissolve no more zinc (has become *saturated*) or until the zinc in contact with the electrolyte has been dissolved. The positive and negative plates and the positive and negative poles are indicated by + and −, respectively. The path of the current, which is made up of the two plates, the electrolyte, the wire, and the coil, is called the **electric circuit,** or, simply, **the circuit.** That part of the circuit not in contact with the electrolyte is called the **external circuit**; the parts of the plates touched by the electrolyte and the part of the electrolyte included between the plates is called the **internal circuit.**

12. If the circuit be interrupted at *any* point, as by separating the wire into two parts (by opening the switch S), by disconnecting the wire at one of the electrodes, or by lifting one or both plates out of the electrolyte, it is called an **open circuit**; but when the circuit forms a continuous path from one plate to the other and through the electrolyte, it is called a **closed circuit.** A current will flow only when the circuit is *closed;* if there is a break anywhere in it, no matter how far the break may be from the point where the current is generated, it is an *open* circuit, and no current will flow.

Without regard to how the current is generated, the circuit must be complete, that is, must be closed, before the current can flow. In the experiment described in Art. **5**, the circuit was completed when the person standing on the insulators touched another person or object, the current flowing through the body of the person touched or through the object touched to the earth, and through the earth to the place where it was generated.

13. Another way of obtaining dynamic (current) electricity is by means of a **dynamo,** which is a machine for transforming mechanical energy into electrical energy. The mechanical energy is supplied usually by a steam engine, steam turbine, water motor, or gas engine, and the dynamo converts it into electrical energy.

14. In accordance with its manner of flowing, current electricity is divided into two classes: *direct*, or *continuous*, *currents* and *alternating currents*. When the current flows through a closed circuit continuously and always in the same general direction, it is called a **direct**, or **continuous**, **current**; all currents from primary cells are direct currents. The currents generated by a dynamo, as will be explained hereafter, frequently flow first in one direction and then in the opposite direction, then in the first direction again, and reversing once more; such a current is called an **alternating current**, because the direction of its flow *alternates*. These alternations are very rapid, as a rule. In an ordinary alternating current system, these changes take place at the rate of 120 times per second. The change in direction and the change back again is called a cycle; evidently, it takes two changes to make one cycle, and 120 changes per second are equal to $120 \div 2 = 60$ cycles per second. In technical language, the ordinary alternating current system "has a frequency of 60 cycles per second." If an incandescent lamp is lighted by this alternating current, the current through the filament reverses its direction 120 times every second, and every time it reverses, the current falls to zero. If the rate of alternation were very low, the lamp would, of course, flicker; but the rate is so high that the eye cannot perceive any flickering. In some cases, the frequency is exceedingly high; thus, in wireless telegraphy, currents may alternate with a frequency of 1,000,000 cycles per second.

THEORY OF CURRENT ELECTRICITY

FLOW OF CURRENT

15. Meaning of Flow.—From what has been stated previously, it is clear that there is no actual transference of a substance from one place to another when electricity flows, in the sense that when water, for example, flows through a pipe, a certain amount of water is transferred from one point to another. Electric energy, however, is transferred when the current flows. This may seem at first to be somewhat mysterious, but a little consideration will make the matter clear. Suppose that one end of a wire be inserted in the flame of a gas jet and held there; if the wire is not too long, and if it be covered with some non-heat-conducting

applied to the whole plate. In Fig. 1, P is the positive electrode (pole) and N is the negative electrode (pole). It will be observed that the current (as indicated by the arrows) leaves the cell at the positive electrode (positive pole), flows through the wire and coil C, enters the cell at the negative electrode (negative pole), then flows through the solution, which is called the **electrolyte,** to the negative plate, continuing in this manner until the electrolyte will dissolve no more zinc (has become *saturated*) or until the zinc in contact with the electrolyte has been dissolved. The positive and negative plates and the positive and negative poles are indicated by + and −, respectively. The path of the current, which is made up of the two plates, the electrolyte, the wire, and the coil, is called the **electric circuit,** or, simply, **the circuit.** That part of the circuit not in contact with the electrolyte is called the **external circuit;** the parts of the plates touched by the electrolyte and the part of the electrolyte included between the plates is called the **internal circuit.**

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Without regard to how the current is generated, the circuit must be complete, that is, must be closed, before the current can flow. In the experiment described in Art. **5**, the circuit was completed when the person standing on the insulators touched another person or object, the current flowing through the body of the person touched or through the object touched to the earth, and through the earth to the place where it was generated.

13. Another way of obtaining dynamic (current) electricity is by means of a **dynamo,** which is a machine for transforming mechanical energy into electrical energy. The mechanical energy is supplied usually by a steam engine, steam turbine, water motor, or gas engine, and the dynamo converts it into electrical energy.

Coulomb. The **coulomb** is the amount of (quantity of) electricity that is required to deposit 0.001118 gram of silver out of a solution of nitrate of silver and water, the solution consisting of 15 parts of silver nitrate and 85 parts of water, both by weight. Consequently, the number of coulombs required to deposit 2.5 grams of silver from such a solution is $\frac{2.5}{.001118} = 2236.1+$ coulombs.

Comparing the flow of electricity with the flow of water through a pipe, the coulomb corresponds to the quantity of water discharged, measured in gallons, cubic feet, or pounds.

17. The Ampere.—In connection with problems concerning flow, quantity alone is of very little value; what is wanted is the *rate of flow*, as the number of gallons or cubic feet or pounds of water discharged per second, per minute, or per hour. The unit of rate of flow of a current of electricity is *one coulomb per second*, which is called one **ampere** (named after Andre Marie Ampere). There is no single word for the term, one foot per second, but the single word, ampere, includes both quantity and time. An analogous term is the word *knot*, to indicate the speed of a ship; it means one nautical mile per hour.

18. Whenever the word *per* occurs in a unit that is made up of two or more quantities, it always indicates *division;* hence, an ampere, which is one coulomb per second, means $\frac{\text{one coulomb}}{\text{one second}}$, one pound per square foot means $\frac{\text{one pound}}{\text{one square foot}}$, etc. Therefore, if 2.5 grams of silver are deposited out of a standard silver nitrate solution in 320 seconds, the rate of flow of the current is $\frac{2.5}{.001118} \div 320 = \frac{2.5}{.001118 \times 320} = 6.988-$, say 7, amperes, because $\frac{2.5}{.001118} = 2236.1$ coulombs, and $2236.1 \div 320 = 7$ coulombs per second = 7 amperes. The rate of flow of the current is called the **strength** of the current; consequently, the strength of the current required to deposit 2.5 grams of silver from a standard silver nitrate solution in 320 seconds is 7 amperes.

Whenever a unit is made up of two (or more) quantities united by a hyphen, their multiplication is understood; thus, one foot-pound means one foot $\times$ one pound. Hence, one ampere-second

means one ampere × one second $= \frac{\text{one coulomb}}{\text{one second}} \times$ one second = one coulomb; that is, if the strength of the current in amperes is known, and the current flows with unvarying strength for a certain number of seconds, the product of the amperes and the seconds is the number of coulombs. For instance, if a pipe discharges 1260 gallons of water per minute, it discharges $\frac{1260}{60}$ = 21 gal. per sec., and the number of *gallons* discharged in 15 seconds is 21 × 15 = 315 gal. This result may be arrived at by using the actual units when multiplying; thus, $21 \left(\frac{\text{one gal.}}{\text{one sec.}}\right) \times 15 \text{ (sec.)} = 21 \times 15 \left(\frac{\text{one gal.}}{\text{one sec.}} \times 1 \text{ sec.}\right) = 315$ gal., the seconds canceling.

19. The current flow is always measured in amperes, and it is for this reason that the expression, ampere-seconds, is frequently used instead of its equivalent value, coulombs, when it is desired to specify the quantity of electricity and not the quantity per unit of time. An ordinary 16-candlepower, carbon-filament lamp takes about ½ ampere, that is, a current of about ½ ampere is flowing through it; a tungsten lamp of the same candlepower takes about ⅕ ampere. An ordinary electric iron takes about 5 amperes, and a 10-horsepower motor about 75 amperes. The foregoing figures imply that the voltage is about 110.

20. Potential and Electromotive Force.—When the weight that drives the pile (in the operation of a pile driver) is raised, it possesses what is termed *potential energy*, or *energy due to position*. As previously pointed out, when an electrical stress (see Art. 3) is produced, work must be done, and if the current does not flow, this work is stored up as potential energy, in the same manner as when the weight of a pile driver is raised and held, not being allowed to fall. In works on electricity, the single word *potential* is used instead of the term *potential energy;* hence, before an electric current can flow, there must be a difference of potential between one end of the circuit and the other or between one point of the circuit and another. In the case of the pile driver, when the weight rests on the earth, it may be said to have zero potential; and when it rests on top of the pile, it has a potential E' equal to the work necessary to raise the weight from the earth to the top of the pile; and when it is raised to

the height from which it is to be allowed to fall, it has a potential E'' equal to the work required to raise it from the earth to that point. The difference of potentials $E' - E''$ is the potential available for doing work on the pile.

In the case of electric currents, the potential of the earth is taken as zero, and the difference of potential between any two points of a circuit is the potential available for overcoming resistances and doing work. Since every electric current possesses capacity for doing work, every such current possesses potential.

The difference of potential between any two points of a circuit is what causes a current to flow between those points; and, since the current always flows from the positive towards the negative or from a higher potential to a lower, it is assumed that the positive end of a circuit is at a *higher* potential than the negative end.

21. Energy and work are mutually equivalent, work being the actual manifestation of energy. But work is equal to a force multiplied by the distance through which it acts; hence, when energy is turned into work, a force must act, and the value of this force depends upon the difference of potential available and the distance through which it acts. Every electric current, therefore, may be regarded as being caused by the action of a force, the value of the force being determined by the difference of potential. In the case of the pile driver, the force of gravity is the acting force and equals the weight of the falling weight. Another illustration is the flow of water through a pipe, when the water flows from a certain level to a lower level. The force causing the flow is the *pressure* created by the difference of level between where the water enters the pipe and where it discharges, called the *head*. As the head increases the discharge increases, and vice versa. So it is with the electric current, the acting force or pressure, called the **electromotive force**, causes the current to flow, and the greater the electromotive force the greater the strength of the current in amperes, and vice versa.

22. Some writers use the word potential in the sense of a force, giving to the words potential and electromotive force practically the same meaning; this is an error, as potential is energy, and not a force. The electromotive force, however, is caused by the difference of potential, and if there were no difference of potential there would be no electromotive force. It is to be

noted that an electromotive force may exist whether the current move or not. For instance, if the discharge end of a pipe be closed by turning a cock or closing a valve, the water in the pipe will still be subjected to a pressure; in fact, the pressure at the closed end will be greater than when the current is flowing, because no work is being done and no resistances are being overcome. This is exactly what happens in connection with an electric current; if the circuit is open, so the current cannot flow, the electromotive force will be greater at some particular point in the circuit than when the current is flowing.

23. The practical unit of electromotive force is the **volt** (named after Alessandro Volta), and is usually defined as *the electromotive force that is required to cause a current of one ampere to flow against a resistance of one ohm. Instead of the words electromotive force, the abbreviation E.M.F. or e.m.f. is* freely used, both in writing and speaking; also, the word **voltage** is frequently used for the same purpose. The voltage, or e.m.f., between the electrodes of a standard Carhart-Clarke cell is 1.434 volts; that is, *it requires an e.m.f. of* 1.434 volts to force *a current of 1 ampere through the electrolyte of a standard Carhart-Clarke cell. Consequently, a volt may be defined as* $\frac{1000}{1434}$*ths of the e.m.f. between the electrodes of a Carhart-Clarke* standard cell.

24. Usual Voltages.—The voltages in regular commercial use cover a wide range of values; they often go as high as 110,000 volts. Lighting systems usually carry current at 110 volts; transmission systems for long distances carry current under a pressure of 6600 to 220,000 volts. Generators (dynamos) are made that produce voltages as high as 6600, and sometimes 13,200 volts. In order to obtain higher voltages, the voltage is increased by means of what are called *transformers*, which step up the alternating current voltage from that of the generator to that of the line, and then step it down at the other end to that of the apparatus using the current. In mills and factories, the voltage for motor circuits is usually 125 or 250 volts, where direct current is used, and 220, 440, or 550 volts where alternating current is used. In many cases, alternating currents operate motors at 2200 volts, and sometimes at 6600 volts.

25. Direction of Flow.—The current always flows from a point of *high* potential to one of *lower* potential. From what was

stated in Art. **11**, the point of high potential may be indicated by the sign + and any point of lower potential by −; thus, if one end of a wire through which a current is flowing or will flow when the circuit is closed be marked + and the other end be marked −, the current will flow from the positive, or +, end

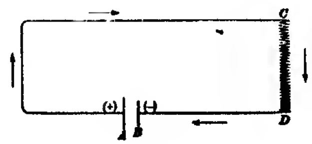

Fig. 2.

to the negative, or −, end. In Fig. 2, the long light line (A) of the primary cell represents the positive (+) terminal and the short heavy line (B) represents the negative (−) terminal; the current thus flows from A to C along the upper wire, through the resistance CD, and back to B.

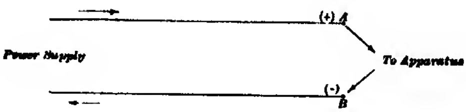

Fig. 3.

In Fig. 3, consider the terminals A and B, across which an e.m.f. is maintained. Since A is marked +, the current will flow from A into electrical apparatus between A and B, and then away at B, and back to its source.

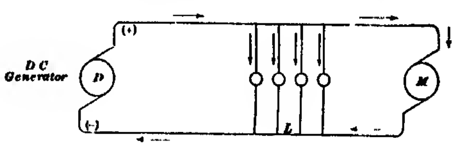

Fig. 4.

Fig. 4 represents a d.-c. (direct current) dynamo, D, lighting a bank of four incandescent lamps L, and driving a motor M. The current is flowing from the + terminal of the dynamo, along the top wire of the circuit, to the lamps and the motor, and back to the negative (−) terminal of the dynamo.

26. Resistance.—All substances offer a **resistance** to the passage of an electric current when they are used as conductors. If an incandescent 16-candlepower lamp having a carbon filament be placed in a circuit having an e.m.f. of 110 volts, a current of .5 ampere will flow through the lamp; but, if a tungsten lamp of the same candlepower be placed on the same circuit, only .2 ampere will be forced through the lamp. That is, 110 volts forces .5 ampere through the carbon filament and only .2 ampere through the tungsten filament; hence, the resistance of the carbon filament is *less* than that of the tungsten filament. Resistance may thus be defined as that property of a substance which resists or limits the flow of an electric current through it.

The amount of resistance offered by a conductor depends upon the material of which it is composed, its length, its shape and its cross-sectional area. The resistance also depends upon the temperature of the conductor, increasing as the temperature increases, in the case of metals, and decreasing as the temperature increases, in the case of carbon, the various insulators, and electrolytic solutions.

27. The unit of resistance used in practice is the **ohm** (named after George Simon Ohm). Several values have been used for the ohm at various times, but the value now universally used is that of the International Ohm, which may be defined as the resistance offered by a column of mercury 1 square millimeter in cross section and 106.3 centimeters in height, at a temperature of 0° C. (32° F.). The resistance of 1000 feet of No. 10, B. and S. gauge, copper wire is very nearly one ohm. For wire of the same material and at the same temperature, the resistance varies directly as the length and inversely as the cross section; that is, as the length of the wire increases the resistance increases in the same proportion, and as the cross-sectional area increases the resistance decreases in the same proportion. This is what would naturally be expected, since if the length be doubled, the wire will be equivalent to two wires of the same length, and the resistance of two equal wires will be twice that of one wire. However, if the cross-sectional area be doubled, the wire will be equivalent to two wires each having the same length, area, and resistance as the original wire, and the same e.m.f. will transmit twice the current. Hence, the resistance of the larger wire will be half that of the smaller wire.

OHM'S LAW

28. The relation between the voltage, resistance, and strength of current was first stated by Dr. G. S. Ohm, and this relation is known as Ohm's law. This law is extremely important, and it should be carefully committed to memory.

Ohm's Law.—*The strength of a continuous current in a circuit varies directly as the electromotive force and inversely as the resistance; the strength (in amperes) is equal to the electromotive force (in volts) applied to that circuit divided by the total resistance of the circuit (in ohms).*

Let I = strength of current in amperes;
E = electromotive force applied to the circuit; in volts;
R = total resistance of circuit in ohms;

then,

$$I = \frac{E}{R} \qquad (1)$$

From formula (1),

$$R = \frac{E}{I} \qquad (2)$$

and

$$E = IR \qquad (3)$$

For example, if a lamp having a carbon filament uses .5 ampere of current and the voltage across the lamp is 110 volts, the resistance is, by formula (2), $R = \frac{110}{.5} = 220$ ohms. In the case of a tungsten lamp having the same voltage and using only .2 ampere of current, the resistance is $R = \frac{110}{.2} = 550$ ohms.

EXAMPLE 1.—If the e.m.f. of a Grove cell is 1.8 volts, the internal resistance is .32 ohm, and the external resistance is 8 ohms, what is the strength of the current in the circuit?

SOLUTION.—The total resistance is the sum of the internal and external resistances, or $8 + .32 = 8.32$ ohms. Applying formula (1),

$$I = \frac{1.8}{8.32} = .216 + \text{ampere. } \textit{Ans.}$$

EXAMPLE 2.—If the internal resistance of a Daniel cell is 2.2 ohms, the resistance of the external circuit is 33 ohms, and the strength of the current is .03125 ampere, what is the e.m.f. of the cell?

SOLUTION.—The total resistance is $2.2 + 33 = 35.2$ ohms. Applying formula (3),

$$E = .03125 \times 35.2 = 1.1 \text{ volts. } \textit{Ans.}$$

EXAMPLE 3.—The wire in the external circuit of a certain cell has a re-

sistance of 20 ohms, the cell has an e.m.f. of 1.44 volts, and the strength of the current is .066 ampere. What is the internal resistance of the cell?

SOLUTION.—Representing the internal resistance by r, the total resistance is $r + 20$ ohms. By formula (2), $r + 20 = \frac{1.44}{.066} = 21.818+$ ohms, and $r = 21.818 - 20 = 1.818$ ohms. *Ans.*

ELECTRIC CIRCUITS

29. It was stated in Art. **12** that before a current can flow the circuit must be closed; it was also stated that the current must return to the point or place where it was generated; and these statements are true, no matter how the current is produced. It will be convenient in what follows under the above heading to consider the current as produced by a battery of cells, but all statements will apply with equal truth when the current is generated by a dynamo.

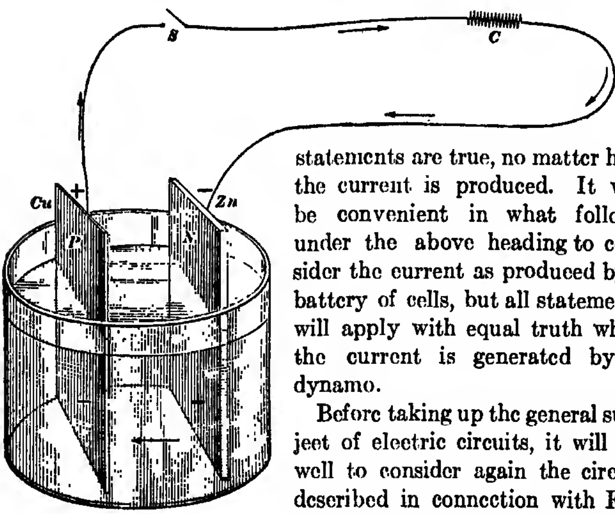

FIG. 5.

Before taking up the general subject of electric circuits, it will be well to consider again the circuit described in connection with Fig. 5, which is Fig. 1 repeated. Where the current leaves the cell, or where it leaves its place of generation, is always *positive* and is marked +; the potential at this point is always higher than at any other point of the circuit. The point where the current enters (returns to) the cell is *negative* and is marked minus (−); the potential at this point is always lower than at any other point of the external circuit. The direction of the current is indicated by the arrows. The current passes from the zinc plate, through the electrolyte to the copper plate; hence, that part of the zinc plate touched by the electrolyte is positive, and that part of it

above the electrolyte is negative. This corresponds to a bar magnet, in which one end is positive and the other negative. For the same reason, that part of the copper plate touched by the electrolyte is negative (since the current flows to it, and the current always flows towards a negative), and the part above it is positive. The plate that is *consumed* by the electrolyte is called *positive;* in this case, the zinc plate is the positive plate. The other plate, the copper plate in this case, is the negative plate. Instead of positive and negative plates, the words anode and cathode are used, the **anode** being the electrode (pole) by which the current *enters* the cell, and the **cathode** being the electrode (pole) by which the current *leaves* the cell. In Fig. 5, the current flows from the anode (the zinc plate) through the electrolyte to the cathode (the copper plate), leaves the cell at the cathode, flows through the wire to the resistance coil C, through the coil and the other wire to the anode, where it enters the cell. S is a switch, the opening or closing of which opens or closes the circuit. Thus, if the switch is open, no current flows; but if it is closed, the current flows through the entire circuit. Electricity produced by a cell or a battery of cells is usually called *voltaic*, or *galvanic, electricity*, named after Alessandro Volta and Alvisio Galvani, respectively.

30. Series Circuit.—When the current leaves its source and returns to it by a single path, the path traveled is called a **series circuit;** the circuit in Fig. 6 is a series circuit. The circuit may

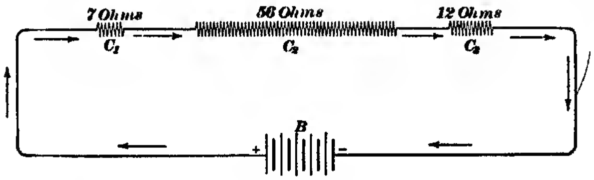

FIG. 6.

be composed of wires of different sizes and made of different materials (and, hence, of different resistances) and may have any number of resistances, such as lamps, motors, rheostats, etc.; but if the current pass through each in succession and back to the source, the whole making a *single* path, the whole is a series circuit, and the various resistances are said to be connected in series. Referring to Fig. 6, B is a battery of cells generating

electricity. The current leaves the battery at the point marked +, flows to and through the resistance coil C_1, to and through the resistance coil C_2, to and through the resistance coil C_3, and then back to the battery. The path is therefore a single one, and the circuit is a series circuit. Referring now to the battery B, the heavy lines represent anodes and the (longer) light lines cathodes, each anode and its adjacent cathode representing one cell. The cells are so connected that the cathode of the first (right-hand) cell is joined to the anode of the second cell; the cathode of the second cell is joined to the anode of the third cell, etc. The current, consequently, passes through each cell in succession, and the cells are said to be *connected in series*. The result of this arrangement is that the entire circuit—wires, coils, and cells—is a series circuit. Considered as poles, the cathode is the positive pole and is marked +, the anode is the negative pole, and is marked −.

31. Resistance of a Series Circuit.—When all the resistances are in series, as in Fig. 6, the total resistance of the circuit is equal to the sum of the separate resistances that make up the circuit, or the resistance of the wire + resistances of coils + internal resistances of the cells composing the battery. The diagram shows that there are 5 cells, and if the internal resistance of each cell is 1.7 ohms, the resistance of the battery is $1.7 \times 5 = 8.5$ ohms. If the total resistance of the conducting wire is 4.5 ohms, the total resistance of the circuit is $8.5 + 4.5 + 7 + 56 + 12 = 88$ ohms.

32. The e.m.f. of Cells Connected in Series.—The e.m.f. of a battery of cells connected in series is equal to the sum of the e.m.f.'s of the cells. This is evident, since it is equivalent to piling several weights on top of one another, thus making a single weight whose value is equal to the sum of the separate weights. If each cell of the battery in Fig. 6, has a voltage of 1.32 volts, the e.m.f. of the battery is $1.32 \times 5 = 6.6$, and the strength of the current in the circuit is $\frac{6.6}{88} = .075$ ampere.

EXAMPLE.—Suppose it had been desired to pass a current of about .36 ampere through the above circuit; how many cells of the same voltage (1.32 volts) would have been required?

SOLUTION.—By formula (3), Art. **28**, $E = IR = .36R$. The resistance of the external circuit is $4.5 + 7 + 56 + 12 = 79.5$ ohms. Let n = the number of cells, then the resistance of the internal circuit is $1.7 \times n = 1.7n$, and the voltage of all the cells is $1.32n$. The total resistance is $R = 1.7n$

$+ 79.5$, and $E = 1.32n = .36(1.7n + 79.5)$. Dividing both sides of this equation by .36,

$$3\tfrac{2}{3}n = 1.7n + 79.5$$

Multiplying by 3, $11n = 5.1n + 238.5$

Whence, $5.9n = 238.5$

and $n = 40.4 +$, say 40 cells. *Ans.*

To ascertain whether this result is correct, calculate the strength of the circuit when 40 cells are used. The internal resistance is $1.7 \times 40 = 68$ ohms; the total resistance is $68 + 79.5 = 147.5$ ohms. The total e.m.f. is $1.32 \times 40 = 52.8$ volts. Then, by formula (1), Art. **28**, $I = \frac{52.8}{147.5} = .358-$ ampere. Had 41 cells been used, the total resistance would have been $1.7 \times 41 + 79.5 = 149.2$ ohms; the total voltage, $1.32 \times 41 = 54.12$ volts, and the strength of current, $\frac{54.12}{149.2} = .363-$ ampere.

When cells are connected in series, the current generated in the first cell flows into the second, the current generated in the first two cells flows into the third, etc., and the total current, which is generated in all the cells, flows through the entire outer, or external circuit, when all the external resistances are in series. Note that: *the current flowing at any point of the external circuit of a series circuit is exactly the same as at any other point of the external circuit.*

33. Parallel or Multiple Circuits.—When a circuit is divided into two or more branches, uniting later at some point in the circuit, as in Fig. 7, where the circuit is divided at A and united

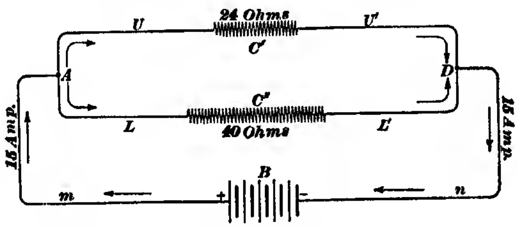

Fig. 7.

at D, the circuit is called a **parallel** or **multiple circuit.** The reason for using the names is evident; in the diagram, the two wires U and L are *parallel*, and since there is more than one wire, this fact is indicated by the word *multiple.* The current flowing through the wire m is divided at A, a part going through the wire

U, the coil C', and wire U', and the remainder through the wire L, the coil C'', and the wire L'; both parts of the current unite at D, and the entire current flows back to the battery through the wire n. The two circuits between A and D are called **shunts**; L is said to shunt U and U is said to shunt L, meaning thereby that a part of the current which would otherwise go through U (or L), if there were but one wire; is deflected or *shunted* through L (or U). The case is similar to that of a water main having branches; the water flowing through the main will divide, a part going through one branch, a part through another branch, etc. If the branches were all to unite again, forming another main, all the current flowing through the first main would flow through the second, and also through the branches. This is what happens in a multiple electric circuit; all the current flows through the main conductors, and this equals the sum of the currents flowing through the shunts. If the resistances of the shunts are all equal, the amount of current flowing through each will be equal to the current in the main conductors divided by the number of shunts; if they are not equal the current divides *inversely* as their resistances. Thus, in Fig. 7, if the current in the mains is 15 amperes and the resistance of the upper circuit is equal to the resistance of the lower circuit, the current in the two shunts will be $15 \div 2 = 7.5$ amperes in each. If, however, the resistance in the upper shunt is 24 ohms and in the lower shunt 40 ohms, it is evident that more current will flow through U than through L, since the resistance of U is less than the resistance of L. Let $x =$ the current flowing through U; then $15 - x =$ the amount of current flowing through L. By Ohm's law, the strength of the current varies inversely as the resistance; the e.m.f. causing the flow is determined by the difference of potential between the terminals of the circuit, and for any particular value, the variation in current flow will be due to a variation in the resistances. If the current varied directly as the resistance, the current in the two shunts could be determined by the proportion

$$x : 15 - x = 24 : 40$$

But, since the proportion is an inverse one, it must be written either as

$$x : 15 - x = \tfrac{1}{24} : \tfrac{1}{40} \qquad (a)$$

or as

$$x : 15 - x = 40 : 24 \qquad (b)$$

Solving either proportion, (a) or (b), $x = 9.375 = 9\frac{3}{8}$ amperes, and $15 - x = 15 - 9\frac{3}{8} = 5\frac{5}{8}$ amperes. Therefore, the upper wire U carries $9\frac{3}{8}$ amperes and the lower wire L carries $5\frac{5}{8}$ amperes. The sum of the currents carried by the two shunts is $9\frac{3}{8} + 5\frac{5}{8} = 15$ amperes, the total current.

34. These results might have been obtained in another manner, using a method similar to that of partnership in arithmetic. Thus, the current going through the upper shunt is proportional to $\frac{1}{24} = .041\frac{2}{3}$; the current going through the lower shunt is proportional to $\frac{1}{40} = .025$; the sum is $.066\frac{2}{3}$; hence, the upper shunt carries $\frac{.041\frac{2}{3}}{.066\frac{2}{3}} \times 15 = 9.375$ amperes, and the lower shunt carries $\frac{.025}{.066\frac{2}{3}} \times 15 = 5.625$ amperes. Or, instead of reducing the fractions to decimals (that is, to a common denominator of 100), reduce the fractions to fractions having a common demoninator, preferably the least common denominator, and then operate with the numerators in the same manner as with the decimals. Thus, the L.C.M. of 24 and 40 is 120; $\frac{1}{24} = \frac{5}{120}$ and $\frac{1}{40} = \frac{3}{120}$; $5 + 3 = 8$; then, the current in the upper shunt is $\frac{5}{8} \times 15 = 9\frac{3}{8}$ amperes, and the current in the lower shunt is $\frac{3}{8} \times 15 = 5\frac{5}{8}$ amperes.

35. This last method may be applied to any number of shunts. Thus, suppose there were four shunts, A, B, C, and D, in a multiple circuit, and that 24 amperes are flowing through the main. If the resistances are $A = 6$ ohms, $B = 10$ ohms, $C = 3$ ohms, and $D = 11$ ohms, what is the current in each shunt? The fractions are $\frac{1}{6}$, $\frac{1}{10}$, $\frac{1}{3}$ and $\frac{1}{11}$; the L.C.D. is 330, and the fractions reduced to this denominator are $\frac{55}{330}$, $\frac{33}{330}$, $\frac{110}{330}$, and $\frac{30}{330}$. The sum of the numerators is 228; and the current in shunt A is $\frac{55}{228} \times 24 = 5.789$ amp., the current in shunt B is $\frac{33}{228} \times 24 = 3.474$ amp., the current in shunt C is $\frac{110}{228} \times 24 = 11.579$ amp., and the current in shunt D is $\frac{30}{228} \times 24 = 3.158$ amp. The sum of the currents in the four shunts is $5.789 + 3.474 + 11.579 + 3.158 = 24.000$ amp., the same as the current in the main.

It may be remarked that if the resistances contain decimals of more than two figures, it will usually be easier to reduce the fractions to decimals than to reduce them to a common denominator.

Another example of a multiple circuit is shown in Fig. 4. Here the current is divided so that it passes through each of

the four lamps and also through the motor M. In this case, there are evidently 5 shunts.

36. Conductance.—The **conductance** of a circuit is the reciprocal of its resistance. Thus, if the resistance of a certain circuit is 34.72 ohms, its conductance is $1 \div 34.72 = .02880+$; if the resistance of a circuit is .00637, its conductance is $\frac{1}{.00637} = 157-$. Consequently, (see Art. **35**) in a multiple circuit, the current divides in direct proportion to the conductances of the shunts. The unit of conductance is the **mho** (pronounced *mo*), this word being *ohm* written backwards. In the first case above, the conductance is .0288 mho; in the second case, it is 157 mhos.

If the conductance in mhos is known, the reciprocal of it will be the resistance in ohms; for instance, if the conductance of a certain circuit is .0625 mho, the resistance of this circuit is $1 \div .0625 = 16$ ohms.

37. Connecting Cells in Multiple or Parallel.—Two or more cells may be connected in multiple or parallel by connecting each anode to a main and each cathode to a main, as shown in Fig. 8, where all the cathodes are connected to the main conductor M and all the anodes are connected to the main conductor N. The current flows from the cathodes through wire M, the coil C, the wire N, and back to the anodes. The e.m.f. of the circuit is the same as the e.m.f. of one cell, assuming that the cells are alike and that they all have the same e.m.f.

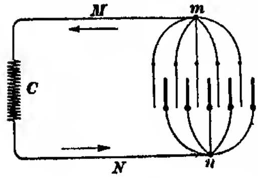

FIG. 8.

To understand this statement, assume that there are 5 blocks of wood, each block being one foot square and of such thickness as to weigh 8 pounds; then the pressure per square foot due to the weight of one block will be 8 pounds. If the blocks are piled, one on top of the other, they will be arranged in series, and the pressure per square foot due to their weight will be $5 \times 8 = 40$ pounds. If, however they are arranged alongside one another, they will be arranged in multiple, and the pressure per square foot will be the same as that of one block, or 8 pounds. This is exactly what happens in the case of the voltage of a multiple circuit—the voltage of any one shunt is the same as the voltage of the main.

As another illustration, a cell will generate the same e.m.f. regardless of its size; but if the distance between the plates remain constant, the current strength will be in proportion to the area of the plates in contact with the electrolyte, assuming that this area (which may be called the *wetted area*) is the same for both plates; but if not, the smaller wetted area should be used in forming the proportion. Suppose the wetted area is 1 square millimeter; then the internal resistance may be considered as equal to that of a wire of the same substance as the electrolyte, having a cross-section of 1 sq. mm. and a length equal to the distance between the plates. If the area be doubled, the resistance will be halved; but the e.m.f. being the same as before, the current will be doubled, all in accordance with the statements in Art. **27**. If a number of cells, say 5, are connected in series, the current must pass through all 5 cells; this is equivalent to lengthening the path through the electrolyte, making it, in this case, 5 times as long, and increasing the resistance 5 times. But, if the resistance be increased 5 times, the e.m.f. must also be increased 5 times to get the same strength of current. Therefore, when cells are connected in series, the electromotive force of the combination is equal to the e.m.f. of 1 cell multiplied by the number of cells.

If the cells are connected in parallel (multiple), the effect will evidently be the same as though one large cell were used whose plates had the same wetted area as the sum of the wetted areas of the small cells, assuming that the distance between the plates were the same in both cases. The resistance of the electrolyte in the large cell will, however, be only $\frac{1}{n}$th as great, the number of cells being n, since the wetted area of the large cell is n times that of one of the small cells. The e.m.f. (voltage) generated by the chemical action of the electrolyte will evidently be the same as for one of the small cells. Thus, suppose there are 5 cells; then, if the 5 anode plates and 5 cathode plates are placed in the same large cell, and are situated the same distance apart as in the small cells, and the corresponding anodes and cathodes are connected by wires, the conditions are exactly the same as though there were 5 separate cells. But, if all the anodes are connected to a single wire and all the cathodes are connected to the same wire, the result is the same as though there were a single anode and a single cathode, each having a wetted area 5 times that

of a single cell. This, however, has not in any way changed the e.m.f.

Assuming that there is no external resistance, connecting the cells in multiple increases the current strength as many times as there are cells; because, assuming that there are n cells, the resistance of the electrolyte is only $\frac{1}{n}$th as great. Since $I = \frac{E}{R}$, and E remains the same, $I = E \div \frac{R}{n} = E \times \frac{n}{R} = n \times \frac{E}{R}$.

When the cells are connected in series, the *current strength* remains the same. The current passes through all the cells in succession; the e.m.f. is n times as great, but the resistance is n times as great also; and $I = \frac{n \times E}{n \times R} = \frac{E}{R}$, as before. It is here assumed, as in the preceding case, that there is no external resistance.

What is true of cells is true of resistances; if a circuit is divided, the difference of potential between the terminals makes the e.m.f. of each shunt the same. Hence, *if a constant current is desired, connections should be made in series; but, if a constant e.m.f. is desired, connections should be made in parallel or multiple.*

38. Joint Resistance.—When two or more resistances are connected in parallel, the resistance of the combination is less than the resistance of either; this must be so, since the area of all the conductors is greater than the area of any one of them. Referring to Fig. 7, the resistance of the shunts considered as a single combination and called the **joint resistance** is equal to the reciprocal of the sum of their conductances. If the resistance of the upper wire and coil is 24 ohms, its conductance is $\frac{1}{24}$ mho; if the resistance of the lower wire and coil is 40 ohms, its conductance is $\frac{1}{40}$ mho; the sum is $\frac{1}{24} + \frac{1}{40} = \frac{8}{120} = \frac{1}{15}$ mho; and the joint resistance is $1 \div \frac{1}{15} = 15$ ohms. This method of finding the joint resistance may be applied to any number of shunts.

The joint resistance being known and the current dividing at A being 15 amperes, the voltage of the current in the shunts (due to the difference of potential between A and D) may be calculated by Ohm's law. The total current of 15 amp. unites at D, and since the joint resistance is 15 ohms, the voltage at A is $E = IR = 15 \times 15 = 225$ volts. Hence, the current going

through U is $I' = \frac{E}{R'} = \frac{225}{24} = 9.375$ amp., and the current through L is $I'' = \frac{E}{R''} = \frac{225}{40} = 5.625$ amp., the same result as previously obtained.

To make the foregoing a little clearer refer to Fig. 9. A and D are two parts of a water main that is divided into two branches, B and C. The pressure per square inch at m and n is the same as at A; hence, the specific pressure P at A is the moving force (motive force) at m and n that causes the water to flow through the shunts B and C; and this pressure corresponds in all respects to the e.m.f. at A in Fig. 7. Consequently, dividing or splitting

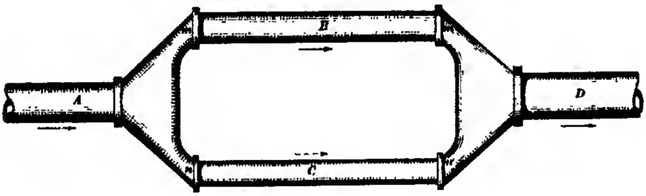

FIG. 9.

a current, thus making a multiple circuit, does not change the motive force; and what is true of water is, in this case, also true of an electric current. It makes no difference whether the current is divided or whether it is united, the motive force is unchanged. Thus, the motive force (specific pressure) at D, Fig. 9, is the same as at m' and n'; and in Fig. 8, the e.m.f. at m is equal to the e.m.f. at n plus the e.m.f. required to overcome the resistance between m and n, which is the e.m.f. of one cell.

Note that the joint resistance is less than the resistance of either shunt. This was to be expected; since, if one of the shunts were to be opened, *all* the current would then flow through the other shunt. Since $R = \frac{E}{I}$ and E is not changed, it follows that if I be increased, R must be decreased, and R is decreased by connecting the other shunt. The more paths (shunts) there are for the current to follow, the less the resistance, and the joint resistance is less than that of the shunt having the smallest resistance. Also, the sum of the currents in all the shunts must always equal that at the point where it divides and the point where it unites, and this, in turn, must equal the current at any point of the mains.

39. Applying Ohm's Law to Parts of a Circuit.—*Whenever a continuous current is flowing, Ohm's law may be applied to any section of the circuit, provided the resistance is taken as the resistance of that section only, and the voltage is taken as that due to the difference of potential between the two ends of the section.* For instance, in the last article, the voltage across the shunts was 225 volts, which was produced by the potential difference between A and D; the resistance in the upper shunt between A and D is 24 ohms, and in the lower shunt it is 40 ohms; hence, if it is desired to find the current in either shunt, the voltage and resistance for that shunt must be substituted for E and R in the formula expressing Ohm's law.

It is to be noted that in a multiple circuit if the resistance is high, only a small current will flow through that shunt; but if the resistance is low, a large current will flow through it. No matter how many shunts there are in a multiple circuit, nor how high the resistance of one of these shunts may be, the current flows through all the shunts, though only a very small current may flow through the shunt having the highest resistance. Electricity is like water in this respect: it does not follow one path or several paths; it follows *all* paths that form a closed circuit with the main. The strength of the current in any shunt, any part of a shunt, or any part of the circuit may be found by Ohm's law when the voltage and resistance of the section are known; or, if the current and resistance are known, as is usually the case, the voltage across the section may be calculated; or, finally, as is sometimes the case, if the strength of the current through the section and the voltage across the section are known, the resistance can be calculated by Ohm's law.

EXAMPLE 1.—Referring to Fig. 6, suppose the battery to be made up of 20 cells connected in series, and that each cell has an e.m.f. of 1.8 volts and an internal resistance of 1.45 ohms. The resistances of the coils C_1 and C_2 being 7 ohms and 56 ohms, respectively, the resistance of the wire being 5 ohms, and the current strength being .33026 ampere, what is the resistance of coil C_3?

SOLUTION.—Since the voltage is known only for the difference of potential between the terminals of the battery, it is necessary to calculate the total resistance, subtract from this the sum of the known resistances, and the remainder will be the resistance of the coil. The total voltage is $1.8 \times 20 = 36$ volts: the current is .33026 amperes; hence, the total resistance is $R = \frac{E}{I} = 36 \div .33026 = 109.00 +$ ohms. The resistance (internal) of the battery is $1.45 \times 20 = 29$ ohms; the sum of the external resistances, not

including coil C_2, is $5 + 7 + 56 = 68$ ohms; and $68 + 29 = 97$ ohms, the sum of the known resistances. Therefore, the resistance of coil C_1 is $109 - 97 = 12$ ohms. *Ans.*

EXAMPLE 2.—Referring to Fig. 10, suppose B to be a battery of 20 cells connected in multiple, and that the wires leading from the cells to the mains connect with the mains so near together that for practical purposes they may be said to connect at points, as m and n in Fig. 8. If each cell has an e.m.f. of 1.8 volts and the internal resistance of each cell is 1.44 ohms, what is the strength of the current through coils C_1 and C_2, assuming that the wire in shunts M and N is twice as long as in shunts P and Q?

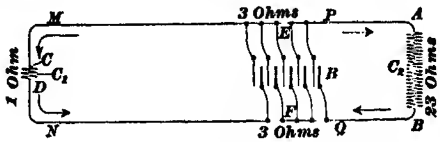

FIG. 10.

SOLUTION.—Since the cells are all connected in multiple, the e.m.f. of the circuit is 1.8 volts: and since there are 20 cells, the internal resistance R_i of the battery is $R_i = \frac{1.44}{20} = .072$ ohm. Considering E and F as the points where the wires from the battery connect with the mains, the length $ECDF$ is twice $EABF$, and the resistance of the former is twice that of the latter. (See Art. 27.) Since the total resistance of the wire is $3 + 3 = 6$ ohms, the resistance of $ECDF$ is $6 \times \frac{2}{2+1} = 4$ ohms, and of $EABF$ $6 \times \frac{1}{3} = 2$ ohms. The conductance of $ECDF$ is $\frac{1}{4+1} = .2$ mhos and of $EABF$ is $\frac{1}{23+2} = .04$ mhos; the total conductance of the external circuit is $.2 + .04 = .24$ mhos and its resistance is $\frac{1}{.24} = 4.167$ ohms.

Therefore, the total resistance of the multiple circuit is $4.167 + .072 = 4.239$ ohms; the strength of the current is $\frac{1.8}{4.239} = .4246$ amperes. The current flowing through coil C_1 is $.4246 \times \frac{\overset{10}{\cancel{.20}}}{\underset{12}{\cancel{.24}}} = .354$ amperes. *Ans.*

The current flowing through coil C_2 is $.4246 \times \frac{\cancel{.04}}{\underset{6}{\cancel{.24}}} = .0708$ amperes. *Ans*

Since the external resistance is 4.167 ohms and the internal resistance only .072 ohm, the latter may be neglected in practice, and the strength of the current is $\frac{1.8}{4.167} = .43$ amp. The results obtained above may be considered in practice as .35 amp. and .07 amp., and the sum is .42 amp.

39. *Applying Ohm's Law to Parts of a Circuit.*—*Whenever a continuous current is flowing, Ohm's law may be applied to any section of the circuit, provided the resistance is taken as the resistance of that section only, and the voltage is taken as that due to the difference of potential between the two ends of the section.* For instance, in the last article, the voltage across the shunts was 225 volts, which was produced by the potential difference between A and D; the resistance in the upper shunt between A and D is 24 ohms, and in the lower shunt it is 40 ohms; hence, if it is desired to find the current in either shunt, the voltage and resistance for that shunt must be substituted for E and R in the formula expressing Ohm's law.

It is to be noted that in a multiple circuit if the resistance is high, only a small current will flow through that shunt; but if the resistance is low, a large current will flow through it. No matter how many shunts there are in a multiple circuit, nor how high the resistance of one of these shunts may be, the current flows through all the shunts, though only a very small current may flow through the shunt having the highest resistance. Electricity is like water in this respect: it does not follow one path or several paths; it follows *all* paths that form a closed circuit with the main. The strength of the current in any shunt, any part of a shunt, or any part of the circuit may be found by Ohm's law when the voltage and resistance of the section are known; or, if the current and resistance are known, as is usually the case, the voltage across the section may be calculated; or, finally, as is sometimes the case, if the strength of the current through the section and the voltage across the section are known, the resistance can be calculated by Ohm's law.

EXAMPLE 1.—Referring to Fig. 6, suppose the battery to be made up of 20 cells connected in series, and that each cell has an e.m.f. of 1.8 volts and an internal resistance of 1.45 ohms. The resistances of the coils C_1 and C_2 being 7 ohms and 56 ohms, respectively, the resistance of the wire being 5 ohms, and the current strength being .33026 ampere, what is the resistance of coil C_3?

SOLUTION.—Since the voltage is known only for the difference of potential between the terminals of the battery, it is necessary to calculate the total resistance, subtract from this the sum of the known resistances, and the remainder will be the resistance of the coil. The total voltage is $1.8 \times 20 = 36$ volts: the current is .33026 amperes; hence, the total resistance is $R = \frac{E}{I} = 36 \div .33026 = 109.00 +$ ohms. The resistance (internal) of the battery is $1.45 \times 20 = 29$ ohms; the sum of the external resistances, not

resistance is $\frac{544}{25} = 21.76$ ohms. The internal resistance of the battery is $1.875 \times 24 = 45$ ohms; the external resistance is $6 + 21.76 + 9 = 36.76$ ohms; and the total resistance is $36.76 + 45 = 81.76$ ohms. By Ohm's law, the strength of the current flowing through the entire circuit is $I = \frac{E}{R} = \frac{36}{81.76} = .44031$ amp. The current in the upper shunt is $.44031 \times \frac{8}{25} = .1409$ amp., and in the lower shunt it is $.44031 \times \frac{17}{25} = .29941$ amp. By Ohm's law, $E = IR$; hence, since the resistance to A is $45 + 6 = 51$ ohms, the voltage necessary to carry the current to A is $.44031 \times 51 = 22.456$ volts; the resistance to D is $51 + 21.76 = 72.76$ ohms, and the voltage necessary to carry the current to D is $72.76 \times .44031 = 32.037$ volts; the voltage necessary to carry the current across the shunts is $32.037 - 22.456 = 9.581$ volts. *Ans.* Or, since the resistance of the shunts is 21.76 ohms and the current (total) across them is .44031 ampere, the e.m.f. required to carry the current across the shunts is $E = .44031 \times 21.76 = 9.581$ volts, the same result as before.

That the voltage required to send the current through the upper and lower shunts is the same and is equal to the result just obtained may be readily proved by multiplying the current in either shunt by the resistance of the shunt; thus, for upper shunt, $E = .1409 \times 68 = 9.581+$ volts, and for lower shunt, $E = .29941 \times 32 = 9.581+$ volts.

41. Referring to the example just given, the e.m.f. of the external circuit is highest where the current leaves the last cell and enters the wire; from this point on, it continually decreases, becoming zero where the wire connects with the anode of the first cell. For instance, referring to Fig. 11, the voltage required to force the current through the battery is $.44031 \times 45 = 19.814$ volts; the e.m.f. at m, where the current enters the wire is $36 - 19.814 = 16.186$ volts; at A, the e.m.f. is $36 - 22.456 = 13.544$ volts; at D, the e.m.f. is $36 - 32.037 = 3.963$ volts; and at n, the e.m.f. is $36 - 36 = 0$. This corresponds exactly to the flow of water through a pipe; as the distance from the supply increases, the force (pressure) causing the water to flow decreases. This loss in motive force, which occurs in every circuit, is called the **line drop**, and more will be said concerning this later. It may be here stated, however, that in well-designed electric circuits, the line drop is comparatively small. Since a loss of electromotive force implies (and is the result of) a loss of potential, the term **loss of potential** is frequently used instead of line drop. It is the loss of potential between any

two points that causes the current to flow between them, because this loss of potential equals the potential required to overcome the resistance between those two points.

In connection with the foregoing, it is to be noted that the e.m.f. referred to is the e.m.f. required to drive the current through the external circuit. The actual e.m.f. at the cathode must be greater than this by an amount sufficient to drive the current through the internal resistance, and this is the e.m.f at the anode. In the case of a dynamo, let E = total e.m.f. generated, R = total resistance, E' = e.m.f. required for external circuit, E'' = e.m.f. required for overcoming internal resistance, R'', of dynamo; then, $E = E' + E''$ = e.m.f. of current at point where it leaves the dynamo (the positive brush) and $E'' = E - E'$ = e.m.f. at point where current returns and enters the dynamo (the negative brush). The current remains constant throughout the circuit; because, as the e.m.f. decreases, the resistance to be overcome also decreases, and in the same proportion. When the e.m.f. drops to the value E'', the resistance is that of the internal circuit only, and $I = \frac{E''}{R''} = \frac{E}{R}$.

Referring again to the preceding example and to Fig. 11, if the resistance of the wire in the upper shunt be 2 ohms between A and a, 1 ohm between b and c, 3 ohms between d and e, and 2 ohms between f and D in the upper shunt (the total is $2 + 1 + 3 + 2 = 8$ ohms), what is the line drop between A and a, A and b, A and c, A and d, A and e, and A and f? The resistance between A and a is 2 ohms and the current is .1409 ampere; hence, the line drop between A and a is $E_a = .1409 \times 2 = .2818$ volt. Similarly, the line drop between A and b is $E_b = .1409 \times 12 = 1.6908$ volts; between A and c, the line drop is $E_c = .1409 \times 13 = 1.8317$ volts; between A and d, the line drop is $E_d = .1409 \times 33 = 4.6497$ volts; between A and e, the line drop is $E_e = .1409 \times 36 = 5.0724$ volts; between A and f, the line drop is $E_f = .1409 \times 66 = 9.2994$ volts.

It will be observed that when calculating the line drop between any two points, the strength of the current flowing between those points is multiplied by the total resistance between those points. Thus, the line drop between b and e is $.1409 \times (1 + 20 + 3) = .1409 \times 24 = 3.3816$ volts.

42. Use of Series and Parallel Connections.—A common use of the series circuit is a street lighting system in which the arc

(or incandescent) lamps are connected in series. The lamps are all alike, and each lamp offers the same resistance. Since in a series circuit, the strength of the current is the same throughout the circuit, the voltage across each lamp is the same also, neglecting the line drop in the conductor (which is comparatively small), because $E = IR$, and as I and R do not change, E remains constant. Therefore, the voltage of the dynamo generating the current is equal to the voltage of one lamp multiplied by the number of lamps, to which must be added the voltage necessary to send the current through the conductors.

In the case of houses, factories, mills, and public buildings, it is desirable to keep the e.m.f. constant, since the various incandescent lamps, motors, fans, electric irons, etc. are all designed for a certain voltage; the resistances and current strengths vary greatly, however, and it is therefore necessary to connect with the mains in multiple. By so doing, every lamp, motor, fan, or other piece of electrical apparatus is connected in parallel with each other, and every one has the same voltage at its terminals. The dynamo furnishing the current for such a circuit will have passing through it the sum of all the currents required to operate all the electrical apparatus in the circuit.

When lamps are connected in series, if one burns out, thus breaking the circuit, all the lamps in the circuit will go out. For this reason, lamps connected in series are fitted with automatic *cut-outs*, as they are called; then if a lamp burns out, the circuit is closed automatically, cutting out the lamp, and the remaining lamps continue to burn.

43. Fig. 12 shows in diagrammatic form how lamps are connected in multiple. Here AB and CD are the mains, or rather, sub-mains, since they lead from the street mains to the house or other building. The current flows from the dynamo to A and on to B, dividing at a, b, c, etc., flowing through the lamps, and then back through the other sub-main CD to the negative pole of the dynamo. Assuming that the resistances of all the lamps are equal, the total resistance of all the lamps (when all are burning) will be equal to the resistance of one lamp divided by the number of lamps, provided the conductors are of sufficient size to allow the line

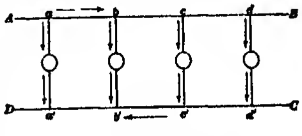

FIG. 12.

drop to be neglected. Suppose there are 50 lamps, the voltage of each being 110 volts and the resistance of each when hot (burning) being 220 ohms. When all the lamps are burning, the total resistance will be 220 ÷ 50 = 4.4 ohms. If the resistance of the conductors (mains) is .6 ohm, the resistance of the circuit is 4.4 + .6 = 5 ohms, and the strength of current required to light the lamps is I = 110 ÷ 5 = 22 amp.

If only 25 lamps are burning, the total resistance will be 220 ÷ 25 = 8.8 ohms; the line drop to the first lamp will be the same as before, .6 ohm, and the resistance of the circuit will be 8.8 + .6 = 9.4 ohms; and the current required for 25 lamps will be $I = \frac{110}{9.4} = 11.7$ amp.

That the current for 25 lamps is more than one-half that for 50 lamps is due to the fact that the resistance of the mains is the same (practically) in both cases. Therefore, there is less drop of pressure in the mains, with half the lamps out, and the applied e.m.f. at the lamp terminals is greater, thus forcing more current through them.

WORK, ENERGY, AND POWER

44. Work.—When a force overcomes a resistance and acts through a certain distance, **work** is done. The unit of work in English measures is the **foot-pound,** which means that a force of one pound has acted against a resistance of one pound through a distance of one foot. If a pound weight be raised vertically a distance of one foot, one foot-pound of work will be done, because the pound weight offers a resistance of one pound and this resistance is overcome through a distance of one foot. Consequently, to find the work done in overcoming any resistance whatever, multiply the resistance (expressed in pounds) by the distance through which the force acts (expressed in feet); the product will be the work in foot-pounds.

Work always implies a movement; if a body in motion be acted on by a force, it will either increase or decrease the velocity of the body (if it be an external force), and work will be done. If, however, the body be at rest and the force is not great enough to cause a movement in the body, no work will be done. For instance, a man may push with all his might against a stone wall;

he will thus exert considerable force on the wall, but no work will be done, unless the wall moves.

45. The practical electrical unit of work is the **volt-coulomb,** and is the work done by one coulomb of electricity when the e.m.f. of the current is one volt. Since an ampere is one coulomb per second and a coulomb is one ampere-second (see Art. **18**), the practical unit of electrical work may be defined as:

The practical unit of electrical work is the work done when a current of one ampere having an e.m.f. of one volt flows for one second.

Instead of the term volt-coulomb, the single word **joule** (named after James Prescott Joule) is used. Hence, *to find the work done by an electric current, multiply the current in amperes by the time in seconds and by the e.m.f. of the current in volts; the result will be the work in joules.* To express this as a formula,

Let W = the work in joules;
I = the strength of current in amperes;
E = electromotive force of current in volts;
t = time that current acts, in seconds;

then,
$$W = IEt$$

Example.—If a current of 11.5 amperes, having an e.m.f. of 110 volts, flows for 24 seconds, what is the work done?

Solution.—Substituting in the formula, $W = 11.5 \times 110 \times 24 = 30{,}360$ joules. *Ans.*

46. Energy.—A body has **energy** when it has capacity to do work. When the weight that drives the pile in the case of a pile driver, is raised to a certain height above the pile, it possesses energy; because, when released and allowed to fall on the pile it will drive the pile into the earth, thus overcoming a resistance through a distance and doing work. A gram of gunpowder possesses energy; because, when exploded, it drives a bullet a certain distance into a block of wood, say, thus overcoming the resistance of the wood through a certain distance. Energy is always measured by the work it can do; hence, it is measured in the same units as work, and the formula for work, given in the preceding article, may be used for finding the energy of an electric current.

Example.—If a current of .8 ampere flows for 15 seconds under an e.m.f. of 55 volts, what is its energy, that is, how much work can it do?

Solution.—The work that the current can do is $W = .8 \times 55 \times 15 = 660$ joules; therefore, the energy (effective) of the current is 660 joules. *Ans.*

47. Power.—Power is the rate of doing work. In order to compare the capacity of two machines, it is not sufficient to compare the work done by one with that done by the other, unless the time is the same, since one machine might do as much work in an hour as the other did in a month. To obtain a proper basis of comparison, it is necessary to find the work done by each machine in a certain specified time. The time is usually taken as one second; hence, dividing the work done by the time in seconds that it took to do it, gives the work per second, which is called the **power** of the machine or of whatever agent did the work. In English measures, the unit of work is one foot-pound; consequently, the unit of power is **one foot-pound per second.** Similarly, the unit of electrical power is **one joule per second,** since the unit of electrical work is one joule. Instead of the term joule per second, the single word **watt** (named after James Watt) is used. Hence, to find the power in watts, divide the number of joules by the time in seconds.

Representing the power by P and the energy or work by W,

$$P = \frac{W}{t} \qquad (1)$$

That is, the power in watts is equal to the work in joules divided by the time in seconds. But, by Art. **45**, $W = IEt$; hence, $\frac{W}{t} = \frac{IEt}{t} = IE$. Therefore, the power of an electric current in watts is equal to the strength of the current in amperes multiplied by the e.m.f. in volts, or

$$P = IE \qquad (2)$$

This expression is strictly true for direct current power only. In Art. **192** an expression for alternating current power is given. The difference is that the power factor of the current has to be taken into consideration. The student, however, need have no concern over this matter for the present.

By Ohm's law, $E = IR$*; substituting this value of* E in formula (2), $P = I \times IR = I^2R$, and

$$P = I^2R \qquad (3)$$

That is, the power of a current in watts is equal to the square of the current in amperes multiplied by the resistance in ohms.

EXAMPLE.—Under the conditions of Art. **43**, it was shown that a current of 22 amperes was required to light 50 lamps in multiple. The e.m.f. across each lamp was 110 volts and the total resistance was 5 ohms; what was the total power required in watts?

SOLUTION.—Using formula (2),

$$P = 22 \times 110 = 2420 \text{ watts. } \textit{Ans.}$$

Using formula (3), the resistance being 5 ohms,

$$P = 22^2 \times 5 = 2420 \text{ watts. } \textit{Ans.}$$

While both formulas give the same result, as they should, it may happen in practice that the voltage may not be known, in which case, formula (3) would be used; or the resistance may not be known, in which case formula (2) would be used. It is to be noted that the number of watts required for each lamp in the foregoing example is 2420 ÷ 50 = 48.4 watts.

48. The Kilowatt.—For measuring large amounts of power, the watt is too small a unit; hence, one thousand watts has been adapted as the unit of machines, unless they are very small, and this unit is called a **kilowatt.** To express watts as kilowatts (abbreviation K.W.), divide the number of watts by 1000 by moving the decimal point three places to the left; to express kilowatts in watts, multiply the number of kilowatts by 1000 by moving the decimal point three places to the right. Thus, 2420 watts = 2.420 K.W., and 15.7 K.W. = 15,700 watts.

Roughly speaking, 1 kilowatt equals $1\frac{1}{3}$ horsepower, and 1 horsepower equals $\frac{3}{4}$ kilowatt.

49. The Kilowatt-hour.—The watt or kilowatt is useful for comparing the capacities of machines, but it is useless in estimating costs in connection with the use of power. For example, suppose in a certain mill, the motors are operated by a current received from a large central generating station. Suppose the rated power of all the motors in the mill is, say, 300 K.W. If all the motors were operating all the time, a basis for computing the cost could be arrived at. Evidently, all the motors will not be working all the time; some of the motors will operate only a part of each 24 hours, and some may work only at odd times. Consequently, the power of a motor (which may not work even to full capacity when running) cannot be used as a fair basis for estimating the price to be paid for the use of current. If, however, the work done be known, this can be used in calculating costs, because a certain number of joules (or foot-pounds) of work represent a certain expenditure of energy, no matter how derived; and it makes no difference, in computing the cost of this energy, whether the energy was expended in one second or one week.

By formula (1), Art. **47,** $P = \frac{W}{t}$, from which, $W = P \times t =$

Pt. If, therefore, the power of a machine be known, the amount of work in joules that it does in a given time is equal to the power in watts multiplied by the time in seconds that the machine is running; that is, if the power of a machine is 1 watt and it runs for 1 second, the work done is 1 watt × 1 second = 1 watt-second = 1 joule. The watt-second is too small a unit for practical purposes, and the kilowatt-hour is used instead. Since 1 K.W. = 1000 watts and 1 hour = 60 × 60 = 3600 seconds, 1 kilowatt-hour = 1000 × 3600 = 3,600,000 watt-seconds = 3,600,000 joules.

The abbreviation for kilowatt-hour is usually written K.W.H.; hence, 30 K.W.H. = 30 × 3600000 = 108,000,000 joules. An instrument called a watt-hour meter (or kilowatt-hour meter), which will be described later, measures the current supplied in kilowatt-hours; and with such a meter in place, it makes no difference when or for how long the current is used, since the meter registers the amount of energy consumed, and it is paid for accordingly.

50. Forms of Energy.—Energy exists in a number of forms, but for present purposes, it will be considered only in connection with the application of an electric current. When the current drives a motor, the energy of the current is transformed from electric energy to mechanical energy. When an electric current is used to heat a flat iron, electric energy is transformed into heat, *i.e.*, heat energy. The electric current is also used to decompose a solution in order to obtain a chemical element or compound; thus, by passing a current through a certain brine solution, caustic soda and chlorine gas are obtained, and electric energy is transformed into chemical energy. It may here be stated that whenever a high resistance is offered to the passage of an electric current, heat is generated; this is what makes a lamp glow or an electric iron hot. When heat is the object sought, the energy thus expended is useful; but, in the large majority of cases, the heating of the conductors is a disadvantage, and the energy expended in heating is wasted.

51. Efficiency.—It is impossible for any machine or mechanism to give out as much energy or power as it receives, because it is not possible to operate it without friction, and the work done in overcoming friction is lost; there are usually other losses also besides that due to overcoming friction. The work or power that

is *supplied* to a machine is called the **input**, and the work or power that is *given out* by the machine is called the **output**; the quotient obtained by dividing the output by the input is called the **efficiency** of the machine.

Let W_i = the energy or work supplied (the input);
W_o = the energy or work given out (the output);
P_i = the power supplied (the input);
P_o = the power given out (the output);
e = the efficiency;

then,

$$e = \frac{W_o}{W_i} = \frac{P_o}{P_i} \qquad (1)$$

Efficiency is commonly expressed as a percentage, in which case, the value obtained for e in the above formula is multiplied by 100.

The efficiency of a *combination* of several machines or mechanisms *is the product of the efficiencies of all the machines or mechanisms that make up the combination.* For example, suppose a dynamo is driven by a belt that is, in turn, driven by an engine, and that the dynamo drives a motor. If the efficiency of the engine be e_1, of the belt e_2, of the dynamo e_3, of the conductor carrying the current from the dynamo to the motor e_4, and of the motor e_5, the efficiency of the entire combination is

$$e = e_1e_2e_3e_4e_5 \qquad (2)$$

EXAMPLE.—Suppose that in a combination like that just mentioned, the engine and dynamo have rated efficiencies of 87% and 92%, respectively; that the efficiency of the belt is 98%, and of the conductor carrying the current from the dynamo to the motor is 98.5%. If the motor receives 12.4 K.W. and gives out 11.2 K.W., what is (*a*) the efficiency of the motor? (*b*) what is the efficiency of the entire combination? (*c*) what power could be supplied by the engine if there were no losses?

SOLUTION.—(*a*) The input of the motor is 12.4 K.W. and the output is 11.2 K.W.; hence, the efficiency of the motor is, by formula (1), $\frac{11.2}{12.4}$ = .9032+ = 90.32%. *Ans.*

(*b*) The efficiency of the entire combination is, by formula (2), e = .87 × .98 × .92 × .985 × .9032 = .6978+, say 69.78%. *Ans.*

(*c*) The efficiency of the entire combination must equal the output of the motor divided by the power that could be supplied by the engine if there were no losses; let P_o = the former and P_i = the latter; then $e = \frac{P_o}{P_i}$, from which, $P_i = \frac{P_o}{e} = \frac{11.2}{.6978}$ = 16.05+ K.W. *Ans.*

The entire amount of power lost is 16.05 − 11.2 = 4.85 K.W.

Pt. If, therefore, the power of a machine be known, the amount of work in joules that it does in a given time is equal to the power in watts multiplied by the time in seconds that the machine is running; that is, if the power of a machine is 1 watt and it runs for 1 second, the work done is 1 watt $\times$ 1 second = 1 watt-second = 1 joule. The watt-second is too small a unit for practical purposes, and the kilowatt-hour is used instead. Since 1 K.W. = 1000 watts and 1 hour = 60 $\times$ 60 = 3600 seconds, 1 kilowatt-hour = 1000 $\times$ 3600 = 3,600,000 watt-seconds = 3,600,000 joules.

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53. Formulas Containing P = Power.—Formula (2), Art. **47**,

$$P = IE \qquad (1)$$

may be written in several ways, one of which has already been given, viz.,

$$P = I^2R \qquad (2)$$

By Ohm's law, $I = \frac{E}{R}$, which substituted in the first formula, gives

$$P = \frac{E^2}{R} \qquad (3)$$

From these three formulas, the following are derived:

$$I = \frac{P}{E} \qquad (4)$$

$$I = \sqrt{\frac{P}{R}} \qquad (5)$$

$$E = \frac{P}{I} \qquad (6)$$

$$E = \sqrt{PR} \qquad (7)$$

$$R = \frac{P}{I^2} \qquad (8)$$

$$R = \frac{E^2}{P} \qquad (9)$$

As mentioned in Art. **47**, these equations when applied to alternating currents, should be modified by the introduction of the "power factor," explained in Art. **192.** For small amounts of current, and in domestic use, the power factor is not considered, and these equations apply to alternating current and are strictly correct for all direct-current power.

Example 1.—If the e.m.f. of a lamp is 110 volts and the resistance of the lamp is 220 ohms, how many watts are required by the lamp?

Solution.—Since formula (3) includes E and R, both of which are known, apply it in this case, obtaining $P = \frac{110^2}{220} = 55$ watts. *Ans.*

Example 2.—If on a 110-volt circuit a 50-watt lamp be inserted, what must be the resistance of the lamp?

Solution.—Here E and P are known and R is required. Using formula (9), $R = \frac{110^2}{50} = 242$ ohms. *Ans.*

Example 3.—If an electric iron has a resistance of 48 ohms and uses 256 watts, what is the strength of the current?

SOLUTION.—Here R and P are known and I is required. By formula (5), $I = \sqrt{\frac{P}{R}}$. Substituting the values of P and R, $I = \sqrt{\frac{256}{48}} = 2.309$ amp. *Ans.*

EXAMPLE 4.—Referring to the last example, what is the e.m.f. of the current?

SOLUTION.—Since P and R are known, E may be found by formula (7), $E = \sqrt{PR} = \sqrt{256 \times 48} = 110.8+$ volts. *Ans.*

Or, since I was found to be 2.309 amp., E might have been found by Ohm's law; thus, $E = IR = 2.309 \times 48 = 110.8+$ volts, as before.

EXAMPLE 5.—Suppose that a 25-watt tungsten lamp is burned an average of 4¾ hours every night from September to May, inclusive. If the light company charge 10 cents per kilowatt hour, what is the average cost of burning this lamp per month. Also what is the cost per hour?

SOLUTION.—The total number of days is $30 + 31 + 30 + 31 + 31 + 28 + 31 + 30 + 31 = 273$; the total number of hours that the light burns is $273 \times 4\frac{3}{4} = 1296\frac{3}{4} = 1296.75$ hours; total number of watt-hours used $1296.75 \times 25 = 32419$ W.H., or 32.419 K.W.H.; total cost for the 9 months is $32.419 \times 10 = 324.19$ cents; and the average cost per month is $324.19 \div 9 = 36.02$, say 36 cents per month. *Ans.*

The cost per hour is $\frac{25 \times 1}{1000} \times 10 = .25$ cent $= \frac{1}{4}$ cent. *Ans.*

EXAMPLE 6.—Referring to example 3, what is the cost per hour of operating the electric iron at 10 cents per kilowatt hour?

SOLUTION.—The cost is $\frac{256 \times 1}{1000} \times 10 = 2.56$ cents per hour. *Ans.*

EXAMPLES

1. When and why is static electricity troublesome in the paper mill? How is it formed?

2. In what units are the following expressed: (*a*) quantity of electricity, (*b*) rate of flow of electricity, (*c*) electric potential, (*d*) electrical resistance?

3. What quantity of electricity will pass through a conductor if the e.m.f. is one volt and the resistance is one ohm?

4. If an electric bell circuit is furnished current from two cells in series each with an e.m.f. of 1.75 volts and an internal resistance of 1.1 ohms, and the current is .3 ampere, what is the resistance of the exterior circuit? *Ans.* 9.47 − ohms.

5. If this circuit were made up with the cells in multiple, what would be the strength of the current? *Ans.* .175 − amp.

6. A circuit is divided into 3 shunts whose resistances are 2, 5 and 9 ohms. What is the current in each of these parallel circuits if the current in the main is 15 amperes? *Ans.* 9.247 −; 3.699 −; 2.055 − amp.

7. If the resistance of the main in example 6 is 120 ohms, what is the e.m.f. for the whole external circuit? *Ans.* 1818.5 volts.

8. A 50-H.P. motor driving a beater receives 37.3 kilowatts but delivers only 40 H.P. What is the efficiency of the motor? *Ans.* 80%.

9. The basement of a paper mill is lighted by 10 75-watt lamps.

(*a*) How many horsepower-hours are consumed in 24 hours and (*b*) what is the cost per day at 1.2 cents per kilowatt hour?

Ans. (*a*) 24.12+ h.p.-hr. (*b*) 21.6 cents.

CONDUCTORS AND INSULATORS

54. Conductor Losses.—In Art. 9, it was stated that there was no such substance as a *perfect* non-conductor, that is, one that would under no circumstances conduct electricity. If the voltage (e.m.f.) is high enough, an appreciable amount of electricity will pass through any substance; but for relatively low voltages, some substances are such poor conductors that practically no current can flow through them. Such substances are called **insulators**; and if a conductor be entirely covered with an insulating substance, the current will be confined to the conductor, in the same manner as the wall of a pipe confines water.

If two "bare" wires (either or both carrying a current) touch each other, a *short circuit* is the result; that is, the current is divided between the two conductors in the same manner as in a multiple circuit, both wires become shunts, the resistance is decreased, and the current is greatly increased. To prevent this short circuiting, it is necessary to cover the wires with an insulating material; and when this is done, the covered wires may be placed in contact with each other without any danger of a short circuit being formed.

Various materials are used for the insulating covering, rubber being the principal one. The effectiveness of the insulation depends not only upon the material of which it is made but also upon its thickness, the thicker the covering the greater the resistance it offers to the passage of electricity through it. At the same time, it is not advisable to have it too thick, since this greatly increases the size and weight of the conductor. Consequently, in practice, the carrying capacity of wires is limited. For instance, an insulated wire intended for use on a 110- or 220-volt circuit would not be suitable at all for carrying 2200 volts; in the latter case, if two wires were to come in contact, the insulation would probably be "punctured" and a short circuit would result.

55. Whenever there is a leakage of current through the insulation, there is a loss in power, because the current is wasted and can do no useful work. It is important, therefore, to reduce this loss as much as possible by proper selection of the wire for any particular installation.

The loss by leakage through the insulation is generally quite small and in most installations may be neglected. The greatest source of loss is that due to the line drop, and this is one of the principal considerations in all power circuits. In an elementary text of this kind, it is not feasible to enter thoroughly into all the phases of the subject; but it may be pointed out that the line drop depends upon the size and length of the wire, the material of which it is made, and the temperature of the wire when the current is flowing. These elements will be considered separately.

56. Length of Conductor.—The resistance of a conductor having a uniform cross-sectional area varies directly as its length. Let l_1 be the length of a conductor having a resistance r_1, and let l_2 be the length of a conductor having a resistance r_2, both conductors having the same cross-sectional area throughout the entire lengths; then,

$$r_1 : r_2 = l_1 : l_2;$$

from which,

$$r_2 = \frac{l_2 \times r_1}{l_1}; \qquad (1)$$

and

$$l_2 = \frac{l_1 \times r_2}{r_1} \qquad (2)$$

By means of formula (1), the resistance of any conductor may be found if the resistance of a known length of the same conductor is given; and by formula (2), the length of a conductor having a known or required resistance may be found if the resistance of a known length of the same conductor is given. For example, the resistance of 1000 feet of No. 10, B & S gauge, copper wire is almost exactly 1 ohm; then, the resistance of 3480 ft. of the same wire is, by formula (1), $r_2 = \frac{3480 \times 1}{1000} = 3.48$ ohms. If it were required to find the length of the same wire to have a resistance of .87 ohm, apply formula (2), and $l_2 = \frac{1000 \times .87}{1} = 870$ ft. Both of these results might have been obtained by substituting in the proportion given above, instead of using the formulas.

57. Area of Conductor.—If two conductors have the same length and uniform (but different) cross-sectional areas, their resistances vary inversely as their cross-sectional areas. Let the resistance and area of one conductor be r_1 and a_1, and of the other r_2 and a_2; then,

$$r_1 : r_2 = a_2 : a_1;$$

from which,
$$r_2 = \frac{r_1 a_1}{a_2}; \qquad (1)$$

and
$$a_2 = \frac{r_1 a_1}{r_2} \qquad (2)$$

Formula (1) may be written $r_2 = r_1\left(\frac{a_1}{a_2}\right)$. If the conductor is a wire, its cross-section is almost invariably a circle, in which case, letting d_1 and d_2 be the diameters,

$$r_2 = \frac{\frac{1}{4}\pi d_1^2}{\frac{1}{4}\pi d_2^2} \times r_1$$

from which,
$$r_2 = r_1 \times \frac{d_1^2}{d_2^2} \qquad (3)$$

As an example, the diameter of a No. 10, B & S gauge wire is very nearly $\frac{1}{10}$ inch, and as stated in the last article, the resistance of 1000 ft. of copper wire of this gauge is very nearly 1 ohm; consequently, the resistance of 1000 ft. of copper wire having a diameter of .081 in. is, by formula (3), $r_2 = 1 \times \frac{.1^2}{.081^2} = 1.524$ ohms.

58. Mils and Circular Mils.—The unit of measure for the diameter of wires used for conductors of electricity is the mil, one mil being 1000th of an inch, just as one mil is 1000th of a dollar. Therefore, if the diameter is given in inches or fraction of an inch, the diameter in mils may be found by multiplying the diameter in inches by 1000; or, if the diameter is given in mils, it may be expressed in inches by dividing the diameter in mils by 1000. To multiply by 1000, move the decimal point three places to the right; and to divide by 1000, move the decimal point three places to the left. Thus, 534 mils = .534 in., 27.45 mils = .02745 in. Also, $\frac{1}{8}$ in. = .125 in. = .125 × 1000 = 125 mils, and .030214 in. = 30.214 mils.

The unit of cross-sectional area of a wire is generally taken as the circular mil, a circular mil being the area of a circle 1 mil in diameter. The area of a wire in *circular mils is thus equal to the square of the diameter in mils; for instance, if the diameter of* a wire is 345.8 mils, the area of its cross-section is $345.8^2 =$ 119,577.64 circular mils. It must be remembered that π has not been used, so the area is in **circular,** not **square,** units. If the area in square mils is desired, it is necessary to multiply the area in circular mils by $.7854 = \frac{\pi}{4}$; and if the area in square inches is desired, divide the area in square mils by 1,000,000, that is, move the decimal point 6 places to the *left*. This is evidently correct since 1 mil = .001 in., and 1 square mil = $.001^2 = .000001 = \frac{1}{1000000}$ sq. in.

Although the area in circular mils is not the true area, which is the area in square mils, nevertheless, it is convenient to use it in all wiring calculations, and it can be substituted directly in formula (2); it can also be used in all ratios and proportions. Wire tables give the gauge numbers, diameters in mils, areas in circular mils, and the resistance in ohms for 1000 feet.

59. The B & S. Wire Gauge.—All wire used in electrical work is made in certain standard sizes; and instead of specifying the different sizes of wire by the actual diameter in mils (or inches), the sizes are denoted by numbers. The size of any particular wire is measured by means of an instrument called a **wire gauge,** which is made in many forms, one of which is a circular disk having slots around the outside, the distance between the two sides of a particular slot being equal to the diameter of the wire having the same number as the slot. The term *wire gauge* is applied not only to the measuring instrument itself but also to the scale showing the relation between the numbers and the diameters of the wire. The gauge almost universally used by the electrical workers on this continent is that known as the Brown & Sharpe (B. & S.) or American wire gauge. The following table gives the gauge number, diameter in mils, area in circular mils, and the resistance in ohms of 1000 ft. of annealed copper wire.

RESISTANCE OF ANNEALED COPPER WIRE

Gauge No.	Diameter in mils d	Area in circular mils d^2	Ohms per 1000 ft. at 20°C. or 68°F.	Gauge No.	Diameter in mils d	Area in circular mils d^2	Ohms per 1000 ft. at 20°C. or 68°F.
0000	400.00	211,600	.04893	16	35.890	1288.1	8.038
000	409.64	167,800	.06170	20	31.961	1021.5	10.14
00	364.86	133,080	.07780	21	28.462	810.09	12.78
0	324.86	105,530	.09811	22	25.347	642.47	16.12
1	289.30	83,694	.1237	23	22.571	509.45	20.32
2	257.63	00,373	.1560	24	20.100	404.01	25.63
3	229.42	52,033	.1967	25	17.900	326.41	32.31
4	204.31	41,742	.2480	26	15.640	254.08	46.75
5	181.94	33,102	.3128	27	14.195	201.50	51.38
6	162.02	26,250	.3944	28	12.641	156.79	64.79
7	144.28	20,817	.4973	29	11.257	126.72	81.70
8	128.46	16,510	.6271	30	10.025	100.50	103.0
9	114.43	13,094	.7908	31	8.928	79.71	129.9
10	161.89	10,382	.6672	32	7.650	63.26	163.8
11	96.742	8,234.1	1.257	33	7.080	50.13	206.6
12	80.808	6,529.6	1.586	34	6.365	39.75	260.5
13	71.962	5,178.5	1.999	35	5.615	31.53	328.4
14	64.084	4,106.8	2.521	36	5.000	25.00	414.2
15	57.068	3,256.8	3.176	37	4.453	19.83	522.2
16	50.820	2,582.7	4.009	38	3.965	15.72	658.5
17	45.257	2,048.2	5 055	39	3.531	12.47	830.4
18	40.303	1,624.3	6.374	40	3.145	9.86	1047.6

60. Referring to the table just given, note that the diameter of a No. 10 wire is 101.89 mils = .10189 in., or .1 = 1/10 in. quite closely; and when exact values are not required, this value for the diameter may be used in practice. The resistance of 1000 ft. of No. 10 wire is .9972 ohm, which is almost exactly 1 ohm; in fact, in practice, it would be best to take this resistance as 1 ohm, because the quality of the wire is not uniform, and the real resistance is very likely to be higher than that given in the table.

A further examination of the table shows that whenever 3 is *added* to the gauge number, the area of the wire having this number is *half* and the resistance is *double* that of the former. Thus, adding 3 to 10 makes 13; the area of No. 13 wire is 5.178, which is very nearly one-half of 10,381, and the resistance is 1.999, which is very nearly twice .9972, the area and resistance of No. 10 wire. Similarly, *subtracting* 3 from the gauge number,

gives the number of a wire that has twice the area and half the resistance. Thus, the area and resistance of No. 8 wire is 16,510 circular mils and .6271 ohm; multiplying the first value by 2 and dividing the second by 2 gives 33,020 circular mils and .31355 ohm, which correspond very closely with the values given in the table for No. 5 (8 − 3 = 5) wire. Therefore, the resistance of two No. 8 wires connected in parallel will be the same as that of one No. 5 wire of the same length, and the resistance of 4 No. 8 wires in a divided circuit will be the same as that of 1 No. 2 wire of same length.

61. When Length and Area both Vary.—Since the resistance varies directly as the length and inversely as the area of the cross-section the following proportion will express this fact:

$$r_1 : r_2 = \frac{l_1}{a_1} : \frac{l_2}{a_2}$$

from which,

$$r_2 = \frac{r_1 a_1 l_2}{a_2 l_1} \qquad (1)$$

If the conductor is a round wire, a_1 and a_2 may be replaced by d_1^2 and d_2^2, and

$$r_2 = \frac{r_1 d_1^2 l_2}{d_2^2 l_1} \qquad (2)$$

It is to be noted that formula (2) can be applied only to wires having a circular cross-section, while formula (1) can be applied to any conductor, provided its cross-section is uniform throughout its entire length. It is also well to note what is meant by *length*. Regardless of its shape, the length of a conductor is always measured in the direction in which the current flows. Thus, Fig. 13 represents an iron block having the dimensions given. If the current flows in the direction of the arrow A, the length is 6 in. and the cross-sectional area is 2 × 8 = 16 square inches; if the current flows in the direction of the arrow B, the length is 8 in. and the cross-sectional area is 2 × 6 = 12 sq. in.; if the current flows in the direction of the arrow C, the length is 2 in. and the cross-sectional area is 6 × 8 = 48 sq. in.

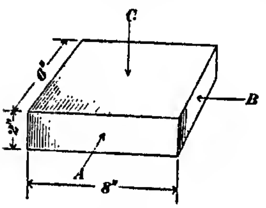


Fig. 13.

Example.—Knowing that the resistance of 1000 ft. of No. 16 copper wire is 4.009 ohms, calculate (*a*) the resistance of one mile of No. 12 wire; also (*b*) the resistance of 1000 ft. of No. 12 wire.

SOLUTION.—(*a*) The diameter of No. 16 wire is, from table, 50.82 mils, and the diameter of No. 12 wire is 80.808 mils. Substituting in formula (2),

$$(a)\qquad r_2 = \frac{4.009 \times 50.82^2 \times 5280}{80.808^2 \times 1000} = 8.372 \text{ ohms. } \textit{Ans.}$$

$$(b)\qquad r_2 = \frac{4.009 \times 50.82^2 \times 1000}{80.808^2 \times 1000} = 1.5856 \text{ ohms. } \textit{Ans.}$$

Or, using formula (1), $a_1 = 2582.7$ circ. mils, $a_2 = 6529.9$ circ. mils (from the table), and

$$(a)\qquad r_2 = \frac{4.009 \times 2582.7 \times 5280}{6529.9 \times 1000} = 8.372 \text{ ohms. } \textit{Ans.}$$

$$(b)\qquad r_2 = \frac{4.009 \times 2582.7 \times 1000}{6529.9 \times 1000} = 1.5856 \text{ ohms. } \textit{Ans.}$$

Note that the answer to (*b*) agrees with the value given in the table for the resistance of 1000 ft. of No. 12 wire.

62. Megohm and Microhm.—When *meg* or *mega* is prefixed to the name of a unit, it denotes a unit 1,000,000 times as large; thus, 1 megohm = 1,000,000 ohms. Hence, to express ohms in megohms, divide the number of ohms by 1,000,000, which is conveniently done by moving the decimal point 6 places to the *left;* and to express megohms in ohms, move the decimal point 6 places to the *right.* For example, 914,060,000 ohms = 914.06 megohms; and 15.63 megohms = 15,630,000 ohms. The megohm is used for measuring very high resistances.

When *micr* or *micro* is prefixed to the name of a unit, it denotes a unit only one-millionth ($\frac{1}{1000000}$th) as large; thus, 1 microhm = .000001 ohm. Hence, to express ohms in microhms, multiply the number of ohms by 1,000,000, which is conveniently done by moving the decimal point 6 places to the *right;* to express microhms in ohms, divide the number of microhms by 1,000,000 by moving the decimal point 6 places to the left. Thus, .002145 ohm = 2145 microhms, and 703 microhms = .000703 ohm. The microhm is used in measuring very low resistances.

EXAMPLE.—What is the resistance of a rectangular sheet of copper 24 ft. long, 3 ft. wide, and $\frac{1}{16}$ in. thick, the current flowing in the direction of the longest side?

SOLUTION.—Referring to the table in Art. **59**, note that the diameter of No. 36 wire is exactly 5 mils = .005 in. The area in square mils is $.7854 \times 5^2 = .7854 \times 25 = 19.635$ sq. mils. The resistance per thousand feet is 414.2 ohms, and the resistance in microhms per foot is $414{,}200{,}000 \div 1000 = 414{,}200$ microhms. The cross-sectional area of the copper sheet is $3 \times 12 \times \frac{1}{16} = 2.25$ sq. in. = 2,250,000 sq. mils. Substituting in formula (1), Art. **61**, $l_1 = 1$ ft., $l_2 = 24$ ft., and

$$r_2 = \frac{414{,}200 \times 19.635 \times 24}{2{,}250{,}000 \times 1} = 86.75 \text{ microhms.} \qquad \textit{Ans.}$$

63. Conducting Material.—The resistance of a conductor depends also upon the material of which it is made. The best conducting material is annealed silver, and the next best is annealed copper; hard drawn silver or copper has a considerable higher resistance than when annealed.

The materials most used for conducting wires are copper, aluminum, and iron, the latter being used in telegraphy, the current carried being quite small. Iron may also be used when it is desired to obtain a high resistance, as when it is desired to convert the current into heat. The resistance of aluminum is about 1.8 times as great as copper; but since a cubic inch of copper weighs nearly 3½ times as much as a cubic inch of aluminum, the same weight of aluminum wire will offer only about one-half the resistance of copper wire, because $3.5 \div 1.8 = 1.9+$, say 2. Annealed, pure iron wire has about 6 times the resistance of copper, and German silver wire has about 13 times the resistance of annealed copper wire. The resistance of commercial iron wire may be taken as 7 times that of copper.

Consequently, if it is desired to find the resistance of aluminum, iron, or German silver wire, find the resistance of copper wire of the same length and diameter and then multiply by 1.8, 7, or 13, respectively.

64. Temperature.—For most metals, particularly all those used as conductors, an increase in temperature increases the resistance; but, for carbon and liquids (water, electrolytes, etc.), the resistance decreases as the temperature increases. The increase in resistance due to an increase in temperature is not uniform, but varies with the material of which the conductor is made; it also varies for the same material for different temperatures. For practical purposes, however, and for temperatures between 32°F. and 212°F., the increase in resistance may be taken as about $\frac{1}{400}$th for every degree rise in temperature for copper and aluminum wire. Let R_1 be the resistance at some known temperature, t_1 and R_2 the resistance at some higher temperature t_2; then,

$$R_2 = R_1\left(1 + \frac{t_2 - t_1}{400}\right)$$

For example, the resistance of No. 16 copper wire at 68°F. is 4.009 ohms per 1000 feet; the resistance at 150°F. is about

$$R_2 = 4.009\left(1 + \frac{150 - 68}{400}\right) = 4.831- \text{ ohms per 1000 ft.}$$

A practical application of this effect is the resistance pyrometer. A fine platinum wire, attached to conductors of very low resistance, in a suitable mounting is exposed to the temperature to be measured. A current of known voltage is passed and if the resistance of the platinum wire is different from that at which the instrument is standardized, the current will vary and can be measured by an ammeter. The scale on the ammeter can be made to show the temperature.

65. In the case of wires for lines, coils, motors, etc., the increase of resistance due to an increase in temperature means that the voltage (line) drop also increases with the temperature, thus increasing the power loss. If a wire is allowed to carry more current than it should, the temperature rises higher and higher and the power (watts) loss becomes greater and greater. If the current is excessive, the conductor becomes so hot that it will damage the rubber or other insulation, and the conductor will be ruined. If this excess of current be increased still more, the conductor may become heated so greatly that it will melt or fuse. A larger wire will offer less resistance, and the heating effect will likewise be less. The current carrying capacity of conductors is thus seen to be limited; and this fact is extremely important when deciding on the size of wire for carrying the loads of dynamos, motors, lighting circuits, etc.

When a machine is running, the bearings heat on account of the friction between the rubbing surfaces. By making the bearings as smooth as possible and making them of sufficient size to keep the pressure per square inch low, the heating effect is kept within reasonable limits. The friction in this case corresponds to the resistance R that opposes the flow of an electric current, and the resistance in both cases results in heat. By enlarging the conductor or the bearing, the heating effect is reduced. The following table gives the safe carrying capacity of copper wires for interior conductors, according to the standard adopted by the American Institute of Electrical Engineers for rubber insulations. The table does not take into account the effect of drop, the currents specified being such as to prevent gradual deterioration of the insulation by the heat of the wires. For insulated aluminum wire, the safe carrying capacity is 84 per cent. of that given in the table for copper wire of the same size.

CARRYING CAPACITY OF COPPER WIRES

B. & S. Gauge No.	Amperes	B. & S. Gauge No.	Amperes
0000	225	5	55
000	175	6	50
00	150	8	35
0	125	10	25
1	100	12	20
2	90	14	15
3	80	16	6
4	70	18	3

66. Fuses.—In many cases, it is desirable that the temperature shall not rise above a certain point; this is especially true of interior wiring circuits, where a high temperature might result in fires. To accomplish this result, a short strip (usually an alloy of lead and tin) having a high resistance and a low melting point, and called a **fuse**, is inserted in the circuit. Suppose it is desired to restrict the strength of the current to 10 amperes; with a fuse of the proper size, a current of this strength will have no effect on it; but if the current be increased to say, 20 amperes, the fuse will melt and the circuit will be broken. The use of fuses thus keeps lighting circuits, motors, etc. from carrying more current than they are designed for.

67. The Incandescent Lamp.—The heating of a conductor by a current passing through it is the principle of the incandescent electric lamp. When a 110-volt lamp is connected in a circuit having an e.m.f. of only, say, 40 volts, the current flowing will be too small to heat the filament high enough to make it glow brightly; as the voltage increases, the current increases, in accordance with Ohm's law (the resistance being practically constant), and the increase in current increases the temperature of the filament. At 100 volts, the filament will become quite bright, and at 110 volts, the lamp will give out its full, rated candlepower. If the voltage be still further increased, the lamp will burn (glow) with intense brilliancy, but its life will be shortened, and if too high a voltage be used, the filament will become hot enough to melt, or "burn out." Even when the current is kept at the proper voltage the filament is very hot; it does not melt, because its melting point is extremely high; and it does not burn away, because the bulb contains no oxygen or other supporter of combustion, the inside of the bulb being generally a

practically perfect vacuum. If the bulb contained air (and, consequently, oxygen), the filament would burn up at a voltage much less than the normal value of 110 volts.

LINE DROP AND LINE LOSS

68. Line Drop.—The term *line drop* was defined in Art. **41.** It is evident from what has preceded that there will be more or less of a drop in voltage for every foot of length of the circuit from where the current leaves the dynamo or other source of current to where it returns. Strictly speaking, the term line drop is used to refer to the loss of voltage in the conductors alone, whence its name; it has no relation to the drop in voltage due to the operation of motors, lamps, or other electric apparatus.

Line drop is often expressed as a percentage of the voltage supplied to the circuit by the dynamo. Thus, suppose the dynamo supplies a current of 20 amperes at an e.m.f. of 110 volts to drive a motor; if the resistance of the wire leading to the motor is .1 ohm and the resistance of the return wire from the motor to the dynamo is also .1 ohm, the total resistance in the line is $.1 + .1 = .2$ ohm. The drop in line voltage due to this resistance is, by Ohm's law, $E = IR = 20 \times .2 = 4$ volts; the voltage across the motor is $110 - 4 = 106$ volts. The line drop in this case is $4 \div 110 = .0364 = 3.64$ per cent. of the voltage at the dynamo. If it were desired to supply the current to the motor at a voltage of 110 volts, it would be necessary for the dynamo to supply current to the line at $110 + 4 = 114$ volts. In the latter case, the percentage drop in line would be $4 \div 114 \times 100 = 3.51$ per cent. Although the percentage of line drop is less, the actual value is the same as before, or 4 volts; and since the e.m.f. of the line is greater, the power required to drive the dynamo is also greater.

69. Line Loss.—Since it requires power to keep a current flowing, a certain amount of power is required to keep the current flowing through the line wires of any system. The power consumed for this purpose is wasted, and this is called the **line loss.** Knowing the total resistance of the line and the strength of the current, the line loss in watts is found by formula (2), Art. **53,** $P = I^2R$. Thus, referring to the last article, the current is 20 amperes and the resistance is .2 ohm; hence, the line loss is 20^2

$\times .2 = 80$ watts. The line loss might have been calculated by formula (1), since $P = IE = 20 \times 4 = 80$ watts, as before. It is usually more convenient, however, to calculate the line loss by formula (2), since the current and the resistance of the circuit are the two factors most generally known; consequently, the line loss is frequently called the I^2R loss.

Ordinarily, the actual drop in commercial electric circuits is several per cent.; in long transmission lines, it may be 10, 12, or 15 per cent. or even higher for very long lines. Evidently, excessive drops should be avoided wherever possible.

70. Relation of Line Voltage to Line Drop and Line Loss.—The line drop and, as a consequence, the line loss may frequently be lessened by a different arrangement of the machines or lights in a circuit. Suppose it is desired to use 10 110-volt

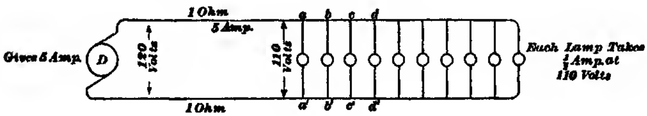


Fig. 14.

lamps, each taking ½ ampere, and that the lamps are arranged in multiple, as shown in the diagram, Fig. 14. Suppose further that the lamps are 2000 ft. from the dynamo, and that No. 7 copper wire is used for conductors. Calling the resistance of 1000 ft. of No. 10 wire 1 ohm, the resistance of 1000 ft. of No. 7 wire is only one-half as great (see Art. 60, second paragraph), or ½ ohm. The total length of the circuit is $2 \times 2000 = 4000 = 4 \times 1000$ ft., and the total resistance of the line is $.5 \times 4 = 2$ ohms, or 1 ohm in each wire.

Since the current is divided among 10 lamps and each lamp takes .5 ampere, the total current is $10 \times .5 = 5$ amperes. The volts drop in line is $E = IR = 5 \times 2 = 10$ volts, and the line loss is $P = I^2R = 5^2 \times 2 = 50$ watts. The power used by the lamps is $P = IE = 5 \times 110 = 550$ watts. Thus, in order to get 550 watts to the lamps, it was necessary to waste 50 watts in the line, or $50 \div (550 + 50) \times 100 = 8\frac{1}{3}$ per cent.

Now suppose that instead of the arrangement of lamps shown in Fig. 14, the lamps were connected as shown in Fig. 15. Here there are 5 sets of two lamps in series, the sets being connected in parallel with the mains. The e.m.f. between a and a', b and

b', etc. is now $110 \times 2 = 220$ volts instead of 110 volts, as in Fig. 14, but the current in each shunt remains the same as before, or $\frac{1}{2}$ ampere. As there are 5 shunts, the total current will be $.5 \times 5 = 2.5$ amperes. The resistance in line will also be the same as before, or 2 ohms, and the line drop will be $2.5 \times 2 =$ 5 volts, or one-half as much as before. The line loss will be $2.5^2 \times 2 = 12.5$ watts, and the percentage of line loss will be $12.5 \div (550 + 12.5) \times 100 = 2\frac{2}{9}$ per cent. The total watts consumed by the 10 lamps is the same in both cases, because the number of watts taken by each shunt in Fig. 15 is twice the number taken in Fig. 14, since the current is now divided among 5 shunts, each getting $2.5 \div 5 = .5$ ampere (the same as before); but the e.m.f. across the shunts is now 220 volts, instead of 110 volts, the num-

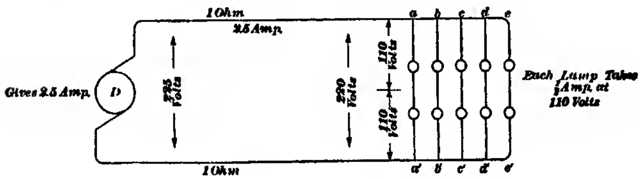


FIG. 15.

ber of watts taken by one shunt is $P = IE = .5 \times 220 = 110$ watts, and the number taken by the 5 shunts (10 lamps) is $110 \times 5 = 550$ watts. The same power has been transmitted to the same lamps, with a line loss of only 12.5 watts instead of 50 watts, the line loss being $2\frac{2}{9}$ per cent. or only $\frac{12.5}{50} = \frac{1}{4}$th that of the first arrangement.

The reason for the decrease in line loss is easily found. In the formula $P = IE$, P(the power) is the same in both cases for the lamps; but for the line loss, P is not the same; the line drop has been halved and the current has been halved, thus making P only $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$th as great. This fact is also indicated by the formula $P = I^2R$; if R remains the same and the current is halved, the value of P will be only $(\frac{1}{2})^2 = \frac{1}{4}$th as great. It will be noted that in order to reduce the current one-half, the e.m.f. was doubled; hence, doubling the e.m.f. decreased the line loss to one-fourth. In general, if e' be the e.m.f. before the change and e'' be the e.m.f. after the change, and $k = \frac{e'}{e''}$, then

the power loss after the change will be k^2 times that before the change. Thus, suppose the e.m.f. of a certain circuit is 125 volts, and after certain changes have been made in the connections, the e.m.f. is 300 volts, the load, being the same as before. Then, the line loss is $\left(\frac{125}{300}\right)^2 = \left(\frac{5}{12}\right)^2 = k^2 = \frac{1}{2.4^2} = \frac{1}{5.76}$th of that before the change. This shows the advantage of a high voltage in the line, and explains the use of voltages as high as 110,000 or 220,000 volts for long distance transmission. The power generated is the same, but less is wasted in transit.

PRIMARY AND SECONDARY CELLS

71. Primary Cells.—A **primary cell** is one that generates its own current of electricity; it always consists of two *different* conductors that are in contact with a conducting solution called the electrolyte, the conductors being connected by a wire to complete the circuit.

The simplest form of cell is that described in Art. **10** and illustrated in Fig. 1. The positive plate of any primary cell is almost invariably zinc, as it is comparatively cheap and is readily acted upon by different electrolytes. The negative plate is usually copper or carbon, but may be of lead, nickel, silver, etc. The one necessary condition to be fulfilled by any primary cell is that there must be two different conductors, one of which must be chemically acted upon by the electrolyte when the circuit is closed.

Primary cells are made in many forms, and many combinations of plates and electrolytes are used. The e.m.f. of the current generated varies between (roughly) ½ volt and 2 volts, according to the type of cell employed. The strength of the current likewise varies greatly, depending upon the e.m.f. and the internal resistance. As stated in Art. **37**, the e.m.f. of any cell does not depend upon the size of the cell, the e.m.f. of a small cell being exactly the same as that of a large cell of the same kind, but the large cell will produce a greater current than the small cell.

72. Wet and Dry Cells.—The cells used for bells, annunciators, etc. are classified as wet cells and dry cells. Both make use of practically the same materials: zinc for the anode, carbon for the cathode, and sal ammoniac (ammonium chloride, NH_4Cl) for

the electrolyte; other materials may be added to the latter to improve the action.

In the wet cell, a small amount of sal ammoniac is dissolved in water and poured into a glass jar. The carbon is usually put in a porous cup and placed in the center of the jar. The zinc, in the form of a rod or of a hollow cylinder, is kept away from the carbon by the cover of the jar and by the porous cup. The current in passing through the electrolyte passes through the porous cup also.

In the case of the dry cell, the sal ammoniac solution is made into a paste, being mixed with crushed coke and other materials, and the paste is put into a zinc jar, which acts as the zinc plate. The carbon is then placed in the center of the paste, and is kept away from the zinc by the paste and the insulating cover. As will be seen, the dry cell is not actually dry, the fluid being held by the paste.

72. In the case of both the wet and the dry cell, the zinc it consumed by the chemical action of the sal ammoniac, both of which gradually disappear. To replace these in the wet cell, more fluid is added and another zinc plate is put in. But since the zinc forms the container in the dry cell, the entire cell is thrown away when the zinc is exhausted. There is not much loss in this, since the cost of a dry cell is very small. The fact that dry cells can be placed in any position and are readily conveyed from place to place makes them very convenient for many purposes. Both the wet and the dry cell here described has an e.m.f. of about 1.5 volts.

73. Consumption of Electrodes.—Strictly speaking, the words anode, cathode, and electrode refer to the *point* or *place* where the circuit wires are connected to the plates or conductors, that is, to the points P and N, Fig. 1. However, it is quite customary to denote an entire plate by these names, and there is no objection to this when there is no possibility of misunderstanding. Hence, when the cell is generating a current, the zinc plate may be called the negative electrode or anode, and the copper plate may be called the positive electrode or cathode. It must not be forgotten, however, that the zinc plate is positive and the copper plate negative in relation to the electrolyte.

The chemical action of the electrolyte in a cell consumes the zinc, the electrolye uniting with it and forming a chemical com-

pound of little or no commercial value. This result is similar, from a chemical standpoint, to the burning of coal in a furnace, where the oxygen of the air unites with the carbon of the coal. Zinc may thus be considered as a fuel, and its consumption by the electrolyte may be regarded as very slow combustion. The cost of zinc is very much higher than that of coal; hence, the cost of energy obtained by using zinc in a battery of cells is very much higher than the cost of energy obtained by burning coal under a boiler, and the use of primary batteries is thus limited to places and circumstances where very small amounts of power are required, and, particularly, at irregular intervals.

74. Secondary, or Storage, Cells.—When the positive plate of a primary cell is practically all consumed, it is replaced with another similar plate. If, however, instead of replacing the worn out plate, a current from a D.C. dynamo or other outside source is sent through the cell in the reverse direction, by connecting the copper plate to the positive main and the zinc plate to the negative main, the zinc will be deposited on the zinc plate until it has been restored to its former condition. It requires just as much electricity to restore the plate as was originally obtained from it when it was being consumed, and it is evident that if the action (direction) of the current be again reversed, as much current will be obtained as was supplied to the cell by the dynamo or other source. The process of sending a current of electricity through a cell from an outside source is called **charging**; and any cell that is charged in this manner is called a **secondary cell**, a **storage cell**, or an **accumulator**, the most common name being **storage battery**, although the word battery implies more than one cell. When a storage cell is delivering current, it is said to be **discharging**; when receiving current, it is said to be **charging.**

Note carefully that a storage cell does not store *electricity;* it stores chemical energy. In charging, electrical energy is transformed into chemical energy and stored in the cell; in discharging, this chemical energy is changed back again into electrical energy.

As in every other case of transformation of energy, there are losses, and the efficiency of the transformation is always less than 100 per cent. For one thing, the voltage of the current in charging must be higher than that of the current obtained when discharging; this is partly due to the resistance of the electrolyte in the cell, which must be overcome. When charging a cell, the

current must be sent in against the e.m.f. of the cell and the resistance of the cell; on discharging, a part of the e.m.f. of the cell must be used to overcome this internal resistance when forcing the current through it in the other direction; therefore, only a part of the total e.m.f. of the cell is available at the cathode. An electric storage cell never gives out as much energy as is put into it, the best types giving out about 75 per cent. of the energy required to charge them; the efficiency is thus about 75 per cent.

75. Lead Accumulators.—One of the most successful forms of storage cells or accumulators uses lead and lead peroxide (PbO_2), the lead for the negative plate or grid and the lead peroxide for the positive plate or grid. The electrolyte is a solution of sulphuric acid and water. When the cell is discharging, lead sulphate is formed on both grids; when a reverse current charges the cell, the sulphate on the anode becomes spongy lead and that on the cathode lead peroxide (or dioxide, PbO_2).

If a lead type of storage cell is left for any length of time, in a partially or fully discharged condition, the lead sulphate is acted upon by the sulphuric acid and is changed into an insoluble sulphate. A cell in this condition is said to have become "sulphated." If the cell is left too long a film of this insoluble sulphate, forms all over the grids and the cell cannot be recharged, because this substance will not change back to sponge lead (Pb) or lead peroxide (PbO_2). If the cell is only slightly "sulphated" the film may be dislodged by charging for a few minutes with a current three or four times the normal charging current. The "sulphate" then drops to the bottom of the jar, where there is usually room for it, and a new surface of the grid is exposed. The e.m.f. of the lead accumulator, as it is called, is about 2 volts. The internal resistance is very low, thus allowing a small cell, comparatively speaking, to deliver a large current. By putting enough cells in series, any desired voltage may be obtained.

76. Uses of Storage Batteries.—In power plants furnishing current for lighting and other purposes, the demand for current varies greatly from day to day and at various periods during the day. For example, in a large city, the power required for lighting will be very much greater in winter than in summer; it will also be very much greater at certain hours of the day than at other times. The heaviest load will usually be from 5:00 p.m.

to 8:00 p.m. during the long winter nights. Storage batteries can be used to good advantage in such plants; they are charged when the load is light and discharged when the load is heavy, the current received from them being added to the current generated by the dynamos.

Storage batteries are used for supplying power to the motors of electric automobiles; they are used in city telephone central offices, for ignition, starting, and lighting of gasoline automobiles, and for many other purposes.

EXAMPLES

1. A paper mill is 6 miles from its electric power plant. Current at 2200 volts is carried by a No. 0 copper wire. What is the resistance in ohms and the percentage loss in volts for each conductor if the current is 100 amperes? *Ans.* 3.108 ohms; 14.14 per cent.

2. (*a*) Referring to example 1, what must the voltage be at the generator if there are two conductors and if 2200 volts are to be delivered at the mill? (*b*) What is the total line loss in horsepower?

Ans. (*a*) 2821.6; (*b*) 83.32+

3. If a house has 10 25-watt lamps, 6 40-watt lamps, and a 5-ampere electric iron all connected in parallel on a 110-volt circuit, what will be the current strength and what size copper wire should be used for the main circuit? *Ans.* 9.45 amp.; No. 14 wire.

4. Referring to examples 1 and 2, what would be the size and resistance of aluminum wire that would give about the same or less resistance?

Ans. No. 0000; .0881 ohm per 1000 ft.

5. Referring to example 4, how much money would be saved or lost per day by the change, assuming current to cost 1 cent per kilowatt-hour and the current is used throughout the entire day of 24 hrs?

Ans. Saved $1.52.

6. If a generator operates at 90 per cent. efficiency, what horsepower will it require to maintain a current of 100 amperes in a circuit having a resistance of 30 ohms? *Ans.* 446.8 H.P.

7. (*a*) How many lead accumulators (storage cells) would be required to give an e.m.f. of 110 volts? (*b*) How would the cells be connected?

Ans. (*a*) 55 cells

ELEMENTS OF ELECTRICITY

(PART 1)

EXAMINATION QUESTIONS

(1) Mention (*a*) some of the advantages of using electricity. (*b*) What is meant by the term "generating electricity?" (*c*) What is the essential difference between static electricity and dynamic electricity?

(2) What is (*a*) the essential difference between positive electricity and negative electricity? (*b*) If a body is charged with electricity, how may this be ascertained, and how can it be determined whether the charge is positive or negative?

(3) Define the following terms: (*a*) electrode; (*b*) pole; (*c*) external circuit; (*d*) open circuit; (*e*) cell; (*f*) battery; (*g*) storage cell; (*h*) what is the difference between a primary cell and a secondary cell?

(4) How many (*a*) coulombs of electricity are required to deposit 3.24 grams of silver from a standard silver nitrate solution? (*b*) If the silver is deposited in 21 minutes, what is the strength of the current?

Ans. (*a*) 2898 coulombs. (*b*) 2.3 amperes.

(5) The internal resistance of a certain cell is 1.72 ohms; the external circuit is made up of 115 feet of No. 16 B. & S. copper wire. If the cell operates a buzzer that has a resistance of 12 ohms, and the e.m.f. of the cell is 1.56 volts, what is the strength of the current? *Ans.* 0.11 ampere.

(6) What is (*a*) the resistance of a 25-watt lamp on a 110-volt circuit? (*b*) The strength of the current flowing through the lamp?

Ans. (*a*) 484 ohms. (*b*) 0.2273 ampere.

(7) What is (*a*) a shunt? (*b*) a short circuit? (*c*) a multiple circuit? (*d*) Suppose a circuit is made up of a wire leading from

a dynamo to a motor and then back to the dynamo; suppose further that there is a defect in the insulation, so that the bare wire comes in contact with a metal rod or post, one end of which is buried in the earth; what happens?

(8) Referring to the last question, what will be the effect on the motor (*a*) if the bare spot is on the wire leading to the motor? (*b*) if on the return wire? Give reasons for your answer.

(9) Suppose a multiple circuit to be made up of four shunts A, B, C and D having resistances as follows: resistance of A = 12.4 ohms, of B = 10.8 ohms, of C = 9.3 ohms, and of D = 13.7 ohms. What is (*a*) the joint resistance? (*b*) if the strength of the current where it divides is 11.6 amperes, what is the current in each shunt? (*c*) what is the e.m.f. in each shunt?

Ans. (*a*) 2.8268 ohms.
(*b*) A = 2.6444 amp., B = 3.0362 amp., C = 3.5259 amp., D = 2.3935 amp.
(*c*) 32.791 volts.

(10) In a direct-current circuit, the e.m.f. at the dynamo is 240 volts, the strength of the current is 20 amperes, and No. 0 B. & S. copper wire is used for the entire circuit. The current is used to drive two motors connected in parallel and situated 8850 feet from the dynamo. If the power taken by one of the motors is 1.6 k.w., (*a*) how much is left to drive the other motor? (*b*) What per cent of the power is lost in transmission?

Ans. (*a*) 2.5054 k.w.
(*b*) 14.47%.

(11) Referring to the last example, the power delivered by the dynamo remaining the same, what power will be available to the second motor (*a*) if the e.m.f. be doubled? (*b*) if the e.m.f. be halved? (*c*) what do these results indicate in connection with long-distance transmission?

Ans. (*a*) 3.0263 k.w.
(*b*) 0.4215 k.w.

(12) A 150-horsepower dynamo drives 3 10-k.w. and 2 15-k.w. motors. If the efficiency of each of the smaller motors is 80%, of the larger motors 85%, of the dynamo 88%, and of the transmission 97%, (*a*) how many 60-watt lamps can be burned when all the motors are running at their rated capacity? (*b*) What is the efficiency of the entire combination?

Ans. (*a*) 212 lamps.
(*b*) 76.42%.

(13) What is (*a*) the resistance of a 15-watt lamp on a 110-volt circuit? (*b*) of a 100-watt lamp on the same circuit? (*c*) what strength of current is required for the 15-watt lamp? (*d*) for the 100-watt lamp?

Ans. (*a*) 806⅔ ohms. (*b*) 121 ohms. (*c*) 0.1364 ampere. (*d*) 0.9091 ampere.

(14) Assuming that an electric iron requires a current of 2.4 amperes at 110 volts, what is the cost of operating it for 6½ hours at (*a*) 8 cents per kilowatt-hour? (*b*) at 10 cents per kilowatt-hour?

Ans. (*a*) 13.73 cents. (*b*) 17.16 cents.

(15) If the diameter of a certain copper wire is 40.303 mils, (*a*) what is its area in circular mils? (*b*) in square mils? (*c*) what is the resistance of 650 feet of this wire at 20°C.? (*d*) what would be the diameter of an aluminum wire that has half this resistance at the same temperature?

Ans. (*a*) 1624.33 circ. mils. (*b*) 1275.75 sq. mils. (*c*) 4.143 ohms. (*d*) 76.47 mils.

(16) What is the resistance of the copper wire in the last example at (*a*) 96° F.? (*b*) at 40° F.?

Ans. (*a*) 4.433 ohms. (*b*) 3.853 ohms.

(17) How would a battery of 20 cells be connected up to get (*a*) the maximum current? (*b*) the maximum voltage? Give reasons for your answer.

(18) How many (*a*) kilowatts are used by a grinder that takes 240 horsepower to operate it? (*b*) If the bearings are allowed to get out of order so that the friction losses are increased 5%, how many kilowatts must then be furnished to the grinder?

Ans. (*a*) 179 k.w. (*b*) 188 k.w.

ELEMENTS OF ELECTRICITY

(PART 2)

MAGNETISM

PROPERTIES OF MAGNETS

77. Natural Magnets.—The ancients found in Magnesia, Asia Minor, a certain kind of stone or ore that had the peculiar property of attracting to it pieces of iron. It was also found that if a bar-shaped piece of this stone be suspended at its middle from a thread, one end of the bar always pointed toward the north, and that this end always pointed north without regard to how the ends of the bar pointed when first suspended. For instance, if the end pointing north were marked and if the bar were originally placed so as to point east and west, it would swing around, the marked end pointing north and the other south. For this reason, the stone from which the bar was made was called **lodestone,** which means *leading stone.* Anything that possesses this property of always pointing in a north and south direction when suspended from a thread, resting on a pivot, or floating in a liquid, and which will attract and lift iron particles, is called a **magnet;** the entire arrangement is called a **compass,** and the magnet itself is called the **compass needle.** Hence, a piece of lodestone is a magnet; and because it is found in a free state in nature, it is called a **natural magnet.** Chemically, lodestone is an iron ore, oxide of iron (Fe_3O_4), and the ore is called *magnetite.*

78. Artificial Magnets.—If a natural magnet be drawn over a bar of hardened steel from one end to the other several times, the movement being always in the same direction, see Fig. 16, it will be found that the steel bar has the same properties as the natural magnet. It is not necessary to exert any degree of pressure; simply see that the surface of the natural magnet touches that of the steel bar. When the end of the bar is reached, lift

the natural magnet and carry it to the other end of the bar through the air. After the steel bar has been **magnetized,** as the process is called, it becomes what is known as an **artificial magnet;** and if the steel has been previously hardened, it will retain its magnetism indefinitely, being then called a **permanent magnet.** Any permanent magnet, whether natural or artificial, may be used to make other magnets in the same manner as that just described for making an artificial magnet.

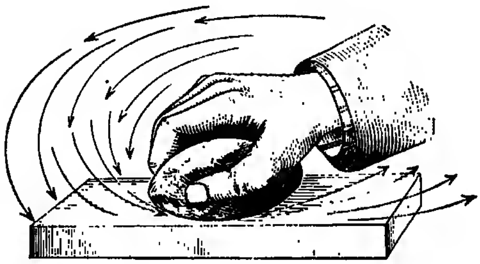


Fig. 16.

79. Magnet Poles.—The ends of a magnet are called its poles. The end that points *north* when the magnet is free to swing in a horizontal plane (as when balanced on a pivot or suspended from a thread) is called the **north pole,** and the other end is called the **south pole.** Whatever its shape, every magnet has a north and a south pole; and if the magnet has the approximate shape of a bar, these poles are opposite each other. The straight line joining the poles and passing through the magnet is called the **axis** of the magnet. A straight bar magnet may be bent into the shape of a letter U or a horseshoe, in which case it is called a U-magnet or a horseshoe-magnet, and the axis will then have a similar shape. The poles will then no longer be opposite each other, but alongside each other; but there will still be two poles, one north and the other south.

If a magnet be broken in two pieces between the poles, each piece will be a magnet, and each will have its north and south poles; in fact, no matter how many pieces may be made of the original magnet, nor how they are taken, every piece will be a magnet and will have its own north and south poles. It is impossible to have a magnet with one pole.

80. Determining the Poles of a Magnet.—In Fig. 17 is shown a compass. The needle, which has been magnetized, is a thin piece of steel having the shape of two equal isosceles triangles joined at their bases. The needle swings in a horizontal plane on a pivot. That half of the needle which has the end that points north is usually blue and the other end is white; the blue end is, therefore, the north pole and the white end is the south pole. If the north pole of another compass be brought near the north pole of the first one, the two needles will swing apart; the same thing will happen if two south poles are brought near each other. But if the north pole of one compass be brought near the

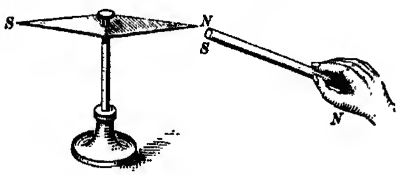


Fig. 17.

south pole of the other, the two needles will swing until they either touch or their axes are parallel. This is also true of magnets; if the poles are *alike*, it may require considerable force (depending upon the strength of the magnets) to make the two surfaces touch, while if they are *unlike* poles, it requires considerable force to separate them. Whence, the law:

Like poles repel each other; unlike poles attract each other.

If, therefore, it is desired to find which of the two poles of a magnet is the north pole and which is the south pole, all that is required is to bring one end of the magnet near, say the north end of the compass, see Fig. 17, and note what happens. If the compass end swings toward the magnet, that end of the magnet is the *south* pole (since *unlike* poles attract each other); but, if the needle swings away from the magnet, then that end is the north pole.

81. Distribution of Magnetism.—Magnetism is not equally distributed over the entire length of the magnet between the poles; this is to be expected, since, because the poles are opposite in character, there must be a place between them where there is apparently no magnetism. Suppose a straight bar magnet be rolled in iron filings; on being lifted out, the appearance of the

magnet will be somewhat as shown in Fig. 18. A large mass of filings will be collected at each end of the bar, and if the bar is cylindrical, these masses will be pear-shaped. The number of particles of iron in the masses will be greatest at the ends, and in the middle of the bar, there will be practically none. The particles form masses all around the bar, except at the middle line *abcd*, which is called the **neutral line.** If, however, the bar be cut in two at the neutral line, two magnets are instantly formed, and the neutral line will be shifted to halfway between the ends of each piece, as in the original bar.

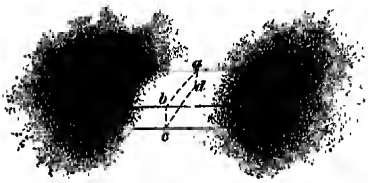


FIG. 18.

82. Magnetic Induction.—If one end of a magnet be touched to a piece of soft iron, the iron immediately becomes a magnet, the end nearest the magnet being of opposite polarity; that is, if the end of the magnet is a south pole, the end of the iron that touches it will be a north pole, and vice versa. If the magnet be lifted and the piece of iron is not too heavy, the force of magnetic attraction will lift the iron also. If the free end of the iron be touched to another piece of soft iron, that also will immediately become a magnet, the end touching the first piece of iron having a pole of opposite polarity. In this manner, several pieces of iron may be lifted as shown at (*a*), Fig. 19. It is to be noted, however, that as soon as one of the pieces of iron is removed from the vicinity of the magnet, it ceases to be a magnet. Such is not the case when the pieces lifted are steel; in this instance, some of the magnetism remains in the steel after removal from the magnet, and the same is true to a certain extent if the iron be not perfectly pure.

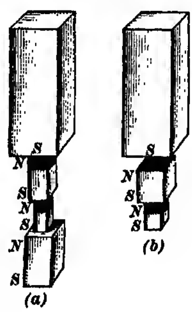


FIG. 19.

Referring to Fig. 19 (*b*), suppose the first piece of iron is brought quite near to the end of the magnet, but is not allowed to touch it; then, as before, the iron will become a magnet, its

poles being of opposite polarity to those of the magnet, and it may even lift a second piece, as indicated in the figure. This action will occur, *even though the piece of iron be separated* from the magnet by a sheet of glass, paper, or any substance whatever, whether magnetic or not. The magnetic properties that are thus imparted to the iron are said to be due to **magnetic induction,** and magnetism is said to be **induced** in the iron. Whenever a piece of iron or steel is magnetized without coming in contact with a magnet, it is magnetized by induction.

83. Magnetic and Non-magnetic Substances.—When a piece of soft iron is magnetized, it is called a **temporary magnet,** because it is a magnet only so long as it is under magnetizing influence. Any substance that can be made into a temporary or permanent magnet is called a **magnetic substance,** and all others are called **non-magnetic substances.** In addition to iron and steel, nickel, cobalt, chromium, cerium, and oxygen are slightly magnetic, but the force of attraction existing between them and a magnet is very small as compared with iron and steel; hence, iron and steel are the only substances used in practice for magnetizing purposes.

It is to be noted that when a magnet attracts a piece of iron (or steel), the iron attracts the magnet with exactly the same force that the magnet exerts on the iron. The effect is similar to the action and reaction of forces; when a weight rests on a table, it presses against the table, and the table reacts and presses against the weight. There is, however, this difference; the force that can be exerted by a magnet diminishes gradually from the poles to the neutral line, where it is zero; but any point or place on the surface of a temporary magnet may be one of the poles, and that pole will attract the magnet always with the same intensity, if the area in contact with the pole of the magnet is the same.

Although non-magnetic substances cannot be magnetized and, therefore, cannot be attracted or repelled by a magnet, they cannot prevent magnetic induction from taking place through them. This was shown in connection with the experiment illustrated in Fig. 19.

LINES OF FORCE

84. Definition.—Strictly speaking, a **pole** is a *point;* hence, when it is desired to refer to an area about a pole, it is better, usually, to employ the term **pole-piece.**

If the pole-pieces of two magnets having opposite polarity are brought near each other, as in Fig. 20, the two magnets attract each other with a certain force. Assume, now, that the two pole-pieces are similar and equal prisms, that the planes of their sides coincide, and that their ends are flat and parallel. Now, if the ends of the pole-pieces are comparatively small, the force exerted between the two poles may be considered as distributed uniformly over the entire surface of the ends, and if the ends be considered as divided into a very large number of little squares, the magnetic force exerted between the poles may also be considered as divided into the same number of equal parts. The effect produced will be the same as though every one of the small forces acted in a line that passed through the centers of gravity of two opposite small squares. For this reason, these small forces are called **lines of force;** and the sum of all the lines of force will evidently be the total force exerted between the two poles. There is, of course, really no such thing as a line of force; it is, however, an extremely convenient conception and is universally used in all electro-magnetic calculations. A line of force when considered as a force and not as a line or path is sometimes called a **maxwell** (named after James Clerk-Maxwell). Hence, instead of the expression 25,000 lines of force, 25,000 maxwells is equally proper.

FIG. 20.

85. Direction of Lines of Force.—With the pole-pieces close together and arranged as described in connection with Fig. 20, the lines of force may be regarded as right lines. In reality, however, the lines are curved, as may easily be proved by direct experiment. Place a bar magnet on a table, and on top of the magnet lay a sheet of paper (or a pane of window glass), supporting the edges so the sheet will lie flat in a horizontal plane. On top of the sheet, sprinkle a thin layer of fine iron filings. Each iron particle immediately becomes a temporary magnet, by

induction, and the filings arrange themselves as shown in Fig. 21, the north pole of one touching the south pole of the preceding particle, one after another, and all forming distinct curved lines extending from pole to pole. If the magnet be turned so as to bring another side against the sheet, the same result will be obtained, and it is therefore inferred (and with truth) that these lines show the direction of the lines of force, and that they (the lines of force) completely fill the space surrounding the magnet. It is assumed that they issue from the north pole

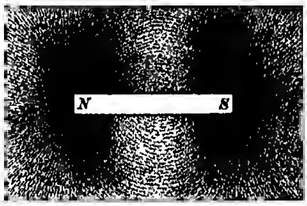


FIG. 21.

of the magnet, make a complete circuit through the surrounding medium, which is usually air, re-enter the magnet at the south pole, and then pass through the magnet to the north pole. The path, which is thus closed and complete, is called the **magnetic circuit**, and every line of force is a closed curve and is in itself

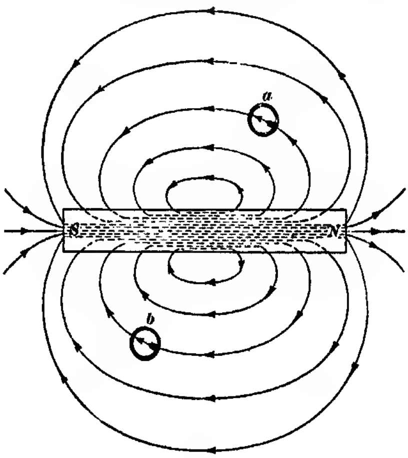


FIG. 22.

a complete magnetic circuit. The space surrounding a magnet and penetrated by lines of force is called a **magnetic field**. Note that the magnetic field includes only that space that is penetrated by lines of force.

That the direction of the lines of force is from the north pole to the south pole is readily established by placing a small compass in the magnetic field. The compass needle will invariably place itself so that its axis will lie in a line of force, and it will point away from the north pole toward the south pole, as indicated in Fig. 22.

86. Reference to Figs. 21 and 22 makes clear the fact that the greatest density of lines of force is at the poles, where they are very closely aggregated. As the distance from the poles increases, whether in the direction of the axis of the magnet or at right angles to it, the number of lines per unit of area decreases. If a right section be taken through a magnetic field, the number of lines of force passing through a square inch or a square centimeter of the section is called the **strength of the field** or **field density.** It is desirable to have a uniform method of measuring the field density. This is expressed as the total number of lines of force passing out of (or into) a pole-piece divided by the projected area (area of a right section) of the pole-piece. Thus, suppose 140,000 lines of force are passing through the north pole-piece of a magnet and that the projected area of the pole-piece is 3.5 square inches; then, the field density = strength of field = $140,000 \div 3.5 = 40,000$ lines of force per square inch = 40,000 maxwells per square inch.

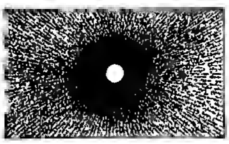


FIG. 23.

87. If, instead of placing the sheet of paper on the side of the magnet, as in Fig. 21, it is placed on one of the pole-pieces, as in Fig. 23, the filings will arrange themselves in radial lines, extending outward from the center of the pole-piece. The greatest number of particles, and, consequently, the greatest density, will be at the center of the pole-piece, as shown.

88. Lines of Force Cannot Intersect.—Lines of force can never intersect, or cut, one another; this fact may also be shown by means of iron filings. Thus, if two bar magnets be arranged as in Fig. 24 (*a*), covered with a sheet of paper, and iron filings are sprinkled over the paper, then when unlike poles face each other, as at (*a*), the lines of force coalesce and extend from the north pole of one magnet into the south pole of the other. If like poles face each other, as at (*b*), the lines of force curve away

from one another, as though they were pushing the poles apart. A similar action occurs when like poles are placed so that their axes make an angle with each other, as at (c).

The imaginary lines or paths formed by the filings are caused by the magnetic force of the magnet, which by induction, makes each little particle of iron a temporary magnet, and which immediately arranges itself so that its axis lies in (coincides with) the line of force passing through it. Also each particle attracts another particle, and thus makes a continuous chain that outlines the line of force.

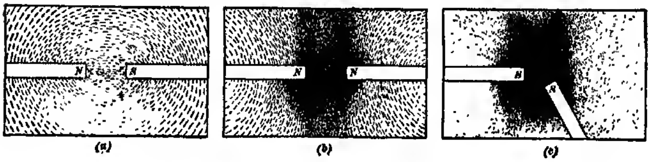


FIG. 24.

89. Nature of Magnetism.—Just what magnetism is, is not known; but the relation between magnetism and electricity is very close, in fact they may be different forms of the same thing. It is certain that magnetism is not a fluid, though some of its properties resemble those of fluids; and because of this, field density is frequently called **flux density**, the word flux meaning *flow*. The term **magnetic flux** means all the lines of force in the field, while flux density means the number of lines of force per square unit (square inch or square centimeter) at the point of the field considered, the value being different for different parts of the field.

Magnetism is not a material substance; for, if a magnet be used to make another magnet, as described in Art. 78, the original magnet retains all its magnetism; in other words, nothing material passes from the first magnet to the second. In fact, the same magnet may be used to magnetize any number of magnets without losing *any of its own magnetism.*

90. Permanency of Magnets.—All permanent magnets will, in time, lose a part of their magnetism unless they are protected by an armature or keeper. The **armature** is a piece of soft iron laid across the pole-pieces of a U-shaped or a horseshoe magnet, as shown in Fig. 25. The lines of force pass from the north pole

of the magnet into the armature, through the armature to the south pole, and then through the body of the magnet to the north pole again. A magnet thus protected by an armature will retain its magnetism at full strength indefinitely.

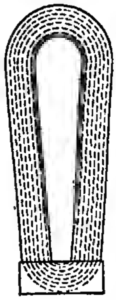


FIG. 25.

If a piece of soft iron be placed anywhere in a magnetic field, the lines of force tend to crowd together and pass through the iron, because the iron offers very much less resistance than air or any other non-magnetic substance, and the iron becomes a temporary magnet by induction. But, if the iron be free from impurities, it will lose its magnetic properties as soon as the magnetic stream or flow ceases or the iron is removed from the magnetic field. The iron tends to hold the magnetic flux in the same manner that a pipe holds water that is flowing through it.

ELECTROMAGNETISM

91. Magnetic Field Around Conductor.—Suppose a wire conductor, which may be either bare or insulated, be passed vertically through a sheet of paper that is kept in a horizontal position, as indicated in Fig. 26, where the white dot represents a cross-section of the conductor, and that iron filings are sprinkled over the paper. If, now, an electric current be sent through the conductor, the iron filings will arrange themselves about the conductor in concentric circles; in other words, the space about the conductor becomes a magnetic field, and the current induces magnetism in the filings. If a number of sheets of paper are strung along the wire, iron filings being sprinkled on each, the same result will be observed on all of the sheets, as indicated in (*a*), Fig. 27. To prove that a magnetic field exists, place a compass near the conductor, and the needle will be deflected, the needle pointing in the direction of the lines of force, the axis of the

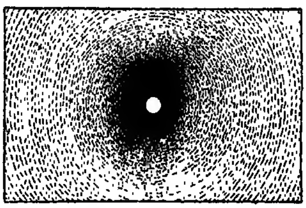


FIG. 26.

needle lying in a line tangent to one of the circles. If the current in the conductor flows as shown by the arrow from m toward n, the compass indicates that the lines of force have the same direction as the motion of the *hands of a watch;* but if the current is reversed, flowing from n toward m, the direction of the lines of force is also reversed, and their direction is *opposite to that of the hands of a watch.* In the first case, the direction of the lines of force is called **clockwise or right-hand rotation,** and in the second case, **counterclockwise** or **left-hand rotation.**

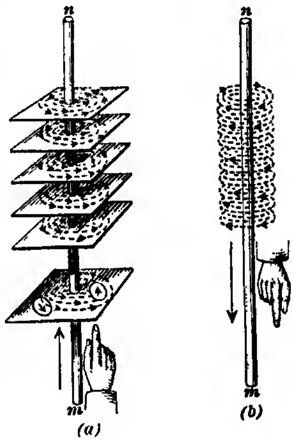


FIG. 27.

The field density is greatest at the surface of the conductor and decreases as the distance from the conductor increases. The lines of force form concentric circles about the conductor, and throughout its whole length, as indicated in (*b*), Fig. 27; as in the case of magnets, they do not (cannot) intersect.

92. Direction of Lines of Force.—It is important to know the direction of the lines of force around a conductor, and this may always be determined by the following rule:

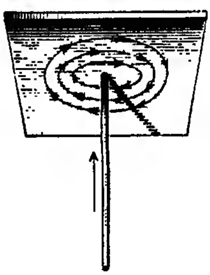


FIG. 28.

Rule.—*Place the index finger parallel to the conductor and point it in the direction that the current is flowing; the lines of force will then be clockwise around the conductor.*

Thus, in (*a*), Fig. 27, the current is flowing from m toward n; pointing the index finger in this direction, as shown, the lines of force are clockwise, as indicated by the compasses and arrowheads. In (*b*), the direction of the current is from n toward m; the lines of force are clockwise relative to the direction in which the finger is pointing, but they are counterclockwise relative to (*a*). If the two conductors be considered as lying in a horizontal

position and the observer be considered as looking from m toward n in both cases, then, when the current is flowing away from the observer, the direction of the lines of force are clockwise; but, if the current is flowing toward the observer, the direction of the lines of force is counterclockwise.

The watch or clock is always supposed to be *behind* the lines of force; it is for this reason that the arrowheads in Fig. 27 (*a*) are reversed, the direction in which the reader is looking at the picture being down instead of upward, in the direction of the arrow. The correct representation would be as shown in Fig. 28.

93. Attraction and Repulsion between Conductors.—If two conductors, both carrying a current, are placed near each other, the conductors being parallel (or nearly parallel), they will either attract or repel each other, according to whether the currents are flowing in the same or in opposite directions. In (*a*), Fig. 29, the currents are flowing in the same direction. As the lines of force spread out from the conductors, they meet and blend (coalesce); they also tend to shorten, and this produces a pull that tends to bring the conductors together; that is, they attract each other. In (*b*), the currents are in opposite directions; the lines of force are also in opposite directions, being clockwise about one conductor and counterclockwise about the other; they therefore cannot blend or coalesce, and since they cannot intersect, they are bent out of their natural position; the force required thus to distort them tends to push the conductors apart, and they therefore repel each other.

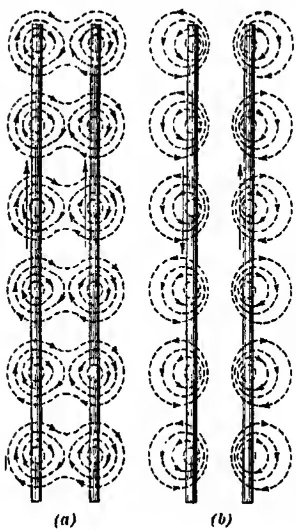


FIG. 29.

94. Direction of Current in a Conductor.—Suppose a compass to be placed so as to point north and south, that is, the ends of the needle lie directly over the N and S marks on the dial. If,

now, a conductor carrying a current be held *over* and parallel with the needle, and the direction of the current be *from south* to north, the north pole of the needle will be deflected toward the *west*. The reason for this will be clear after studying Fig. 30. Before deflection, the needle is pointing in the same direction as the wire, and the lines of force about the wire are at right angles to the lines of force passing out of the north pole and into the south pole of the needle, which is a magnet. As a consequence, the needle swings so that

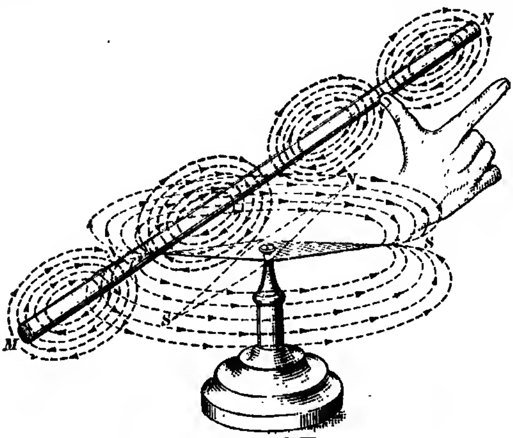


Fig. 30.

the lines of force in its magnetic field will blend or coalesce with those about the conductor. As shown in the figure it must necessarily swing toward the west, since if it swung the other way (toward the east), the lines of force in one field would oppose those in the other. Notice that when the conductor is over the needle, the *bottom* parts of the circles representing the lines of force about the wire coalesce with those *above* the needle. If, however, the compass be held above the conductor, the upper parts of the circles representing the lines of force about the wire coalesce with the lines of force under the needle, and this makes the north pole of the needle swing toward the *east*. If the direction of the current in the conductor be reversed, flowing from *n* toward *m*, the direction in which the needle points will also be reversed in the two cases.

If, therefore, the conductor is *over* and parallel to the needle and the right hand be held so the index finger points north (parallel with the conductor) and the thumb is placed at right angles to the conductor, as shown in the figure, with the back of the hand up, then, when the current is flowing from south to north (from *m* to *n*), the needle will point in the same direction as thumb; but if the current is flowing the other way, from *n* to *m*, the needle will point in the opposite direction. Hence, to find the direction of the current, place the conductor so that it will lie in a general north and south direction, place a compass under it, and in front of the observer; then if the needle points toward the west the current is *away* from the observer; but if the needle points toward the east, the current is flowing *toward* the observer. When speaking of the needle "pointing," it is always understood to mean the direction in which the north pole points.

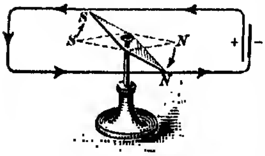


Fig. 31.

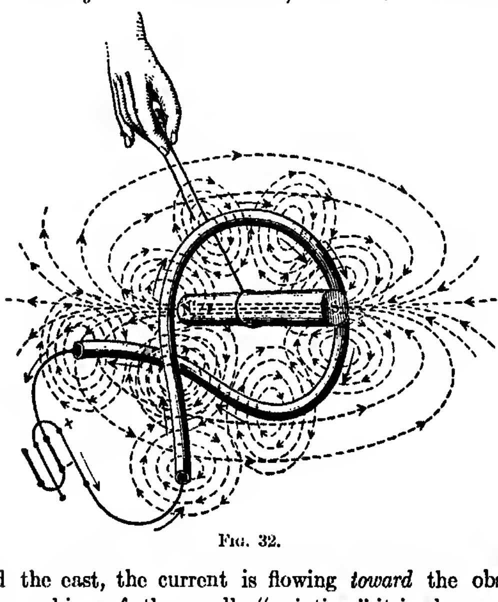


Fig. 32.

95. If the conductor form a loop so it can pass over and under the compass needle, as shown in Fig. 31, the direction of the cur-

rent in the upper wire will be opposite to that in the lower wire, and both wires will act to turn the needle in the same direction. In the figure, the current is produced by the cell shown at the right, and the direction is indicated by the arrowheads. The needle will evidently turn in the direction of the arrows.

96. The Solenoid.—If a conductor carrying a current be bent into a loop and placed in, say, a vertical position, as in Fig. 32, and a piece of soft iron be suspended from a string attached to the iron at its center of gravity, the iron will turn until its axis coincides with or is parallel with the axis of the loop. The reason for this is that the lines of force about the conductor all try to pass through the iron, which is much more *permeable*, as it is termed, than the air; they do not all pass through, but many

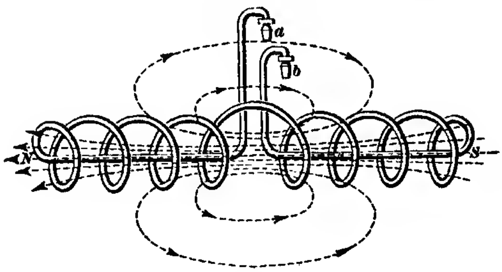


FIG. 33.

of them do, with the result that they are distorted and are no longer circles, but take the shape of the lines of force passing through a magnet. The piece of iron then becomes a temporary magnet, having a north and south pole.

This effect is better shown by bending the wire into a number of loops or coils, all having the same diameter, the general shape being that of a helical spring. By returning the ends of the wires through the helix and out at the middle, as shown in Fig. 33, and suspending from pivots at the ends, without breaking the circuit, the helix will be found to have the properties of a magnet; it will have a north and south pole, a neutral line, and it will swing so as to point in a north and south direction, the same as a compass; it will also attract and repel similar coils or magnets. Whenever a conductor is coiled into the form of a helix, the helical part is called a **solenoid**.

97. Poles of the Solenoid.—Referring to Fig. 32, note that the loop is wound like one turn of a right-hand helix or screw thread. The direction of the lines of force around the conductor is indicated by the arrowheads. The piece of iron distorts and bends these lines, causing them to enter the iron at the end marked S and leave at the end marked N. The same thing happens when the wire is bent into a number of coils of the same kind, as in Fig. 33, except that the lines of force in the adjacent coils coalesce, making numerous very long lines and increasing the magnetic properties of the solenoid. If, however, the conductor be reversed, so that the coil or coils form a left-hand helix (corresponding to a left-hand screw thread), the direction of the current in the conductor will be opposite to that in the above case, the direction of the lines of force will be reversed, and what was the north pole of the solenoid then be the south pole. Therefore, if the direction of the current be known, the poles of the solenoid may be identified by the following rule:

Rule.—*Looking through the solenoid from the end at which the current enters, if the helix winds clockwise away from the observer, the conductor is wound into a right-hand helix, the current flows around in the direction of the hands of a watch, and the end nearest the eye is a south pole. But, if the helix winds counterclockwise away from the observer, the conductor forms a left-hand helix, the current flows around in a counterclockwise direction, and the end nearest the eye is a north pole.*

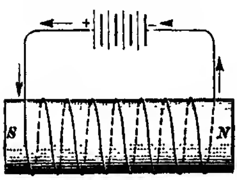


FIG. 34.

In practice, the best way to determine the polarity of a solenoid is to hold a compass near one end; if the needle is repelled, that end of the solenoid is of the *same* kind as the end of the needle that is repelled (since like poles repel each other); otherwise, it is of opposite polarity.

98. The Electromagnet.—If the conductor be wound around a cylindrical iron or steel bar, as shown in Fig. 34, the bar, which is called a **core,** attracts the lines of force and greatly increases the magnetic properties of the solenoid. The bar becomes a magnet, and if made of steel and the current is continued for any length of time, it will become a permanent magnet. If made of iron, the strength of the magnet will be greater than if made of

steel, but it will lose practically all its magnetism as soon as the current ceases to flow. Magnets made in this manner are called **electromagnets.**

The strength of an electromagnet depends upon the strength of the electric current, upon the number of turns of wire in the coil, and upon area of cross section of the core. In practice, the coils are made up of a very large number of turns of very fine insulated wire, preferably, copper wire. The wire is insulated to prevent a short-circuiting of the current; but *the insulation does not insulate the lines of force*—it insulates the current only. The wire is small in order to increase the voltage of the current. A small wire offers greater resistance than a larger wire; and since by Ohm's law, $E = IR$, it follows that if the strength of the current remains the same, increasing the resistance increases the e.m.f. of the circuit.

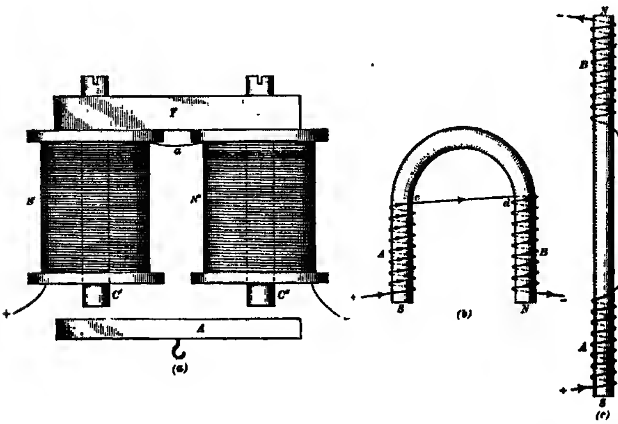
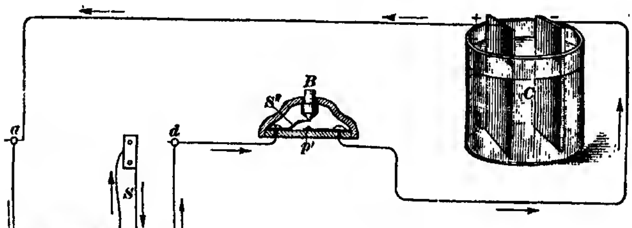


Fig. 35.

99. Electromagnets are made in many forms, one of which is shown at (*a*), Fig. 35. Here S' and S'' are spools or bobbins made up of a very great number of turns of fine, insulated copper wire. C' and C'' are soft iron cores, which pass through the spools and are held in place by the yoke Y. A is a soft iron armature, or keeper, to which is attached a hook, from which weights or loads may be hung. The wire is continuous, and after being wound around one spool, is continued around the other, the connection between the two being at a. The manner in which

the winding is done is shown at (*b*), which represents a bar bent into the form of a U. The wire is wound around *A*, passes over the top at *c*, then under *B*, at *d*, and around down to the end in the *opposite* direction. Observe that both coils are right-hand helixes, as they must be since the bar is a continuous one; if straightened out, it would look, with its winding, as shown at (*c*). According to the rule of Art. **97**, the end *A* is the south pole and the other end is the north pole. Had the winding been in the form of a left-hand helix, the poles would have been reversed. Note that the arrowheads on the two coils point toward one another; they indicate the direction of the current.

Contact of the armature with the poles of the magnet greatly increases the strength of the magnet, because practically all the lines of force pass through the armature and, consequently, through the poles. If, however, the armature is separated from



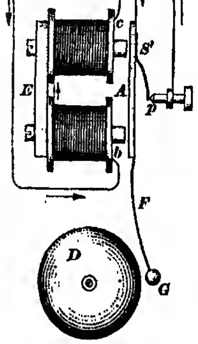


Fig. 36.

the poles by an *air gap*, many of the lines of force are diverted from the armature, and the number passing through the poles is smaller, which lessens considerably the strength of the magnet.

100. The Electric Bell.—One of the practical uses of the electromagnet is the electric bell. Referring to Fig. 36, *C* represents a cell, either wet or dry, but preferably a dry cell. *E* is an electromagnet, whose armature *A* is attached to a spring *S* that ordinarily keeps the armature from contact with the poles of the magnet. The armature carries a spring *S'*, which presses against the pivot *p*. The push button *B* is kept from contact with the pivot *p'* by the spring *S''*, thus leaving the circuit open. When

the button B is pushed down, the circuit is closed; the current flows from the positive electrode of the cell to the binding post a; enters the magnet at b and leaves at c; flows along the spring S and armature A to pivot p, to binding post d, and thence back to the negative electrode of the cell. But as soon as the current passes through the magnet coils, the poles of the magnet draw the armature A into contact with them; this takes the spring S' away from the pivot p and breaks the circuit. The coils then cease to be a magnet, and the spring S draws the armature away to its former position, spring S' comes into contact with p, the circuit is again closed, and the armature is again drawn to the poles. The armature carries a clapper F, the end G of which strikes the bell every time the armature is drawn to the poles. Therefore, the bell will ring as long as the push button B is held down. On releasing it, the circuit is open, and the bell will no longer ring.

101. Magnetic Leakage.—All the lines of force induced in an electromagnet do not follow a single path; some of them stray and take shorter paths, even though they have to pass through

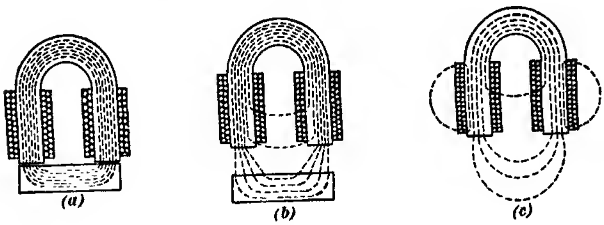


FIG. 37.

the air. If, however, the armature is in contact with the poles, practically all the lines of force are confined to the core and armature, as shown diagrammatically in Fig. 37 at (*a*). If the armature is at some distance from the poles, as shown at (*b*), some of the lines will fail to cross the air gap to the armature, as indicated. The total number of lines of force, the flux, will be less also. Assuming that the same magnet is used in both cases, the lifting or attractive power of the magnet in the second case is less than in the first case. If there is no armature, the result is shown diagrammatically at (*c*). Here there are more "stray" lines of force and a much smaller number of lines of force in the core; conse-

quently, the magnet in the third case is much weaker than in either of the two other cases.

Those lines of force that do not pass through the poles have no effect on the lifting or attractive power of the magnet; they constitute what is called the **magnetic leakage.** The magnetic leakage depends upon the magnetic substance composing the core of the magnet, the uniformity of the material (freedom from and distribution of impurities), whether or not there is an armature, whether or not it is in contact with the poles, and if not, upon the length of the air gap between the armature and the poles. The shorter the air gap the less is the magnetic leakage, and the greater the strength of the magnet. This confirms what was stated in Art. 99.

ELECTRICAL MEASURING INSTRUMENTS

MEASURING CURRENT

102. Galvanometers.—Any instrument that measures electric currents by the effects produced by the electromagnetic action is called a **galvanometer.** The word means *galvanic* measurer, and

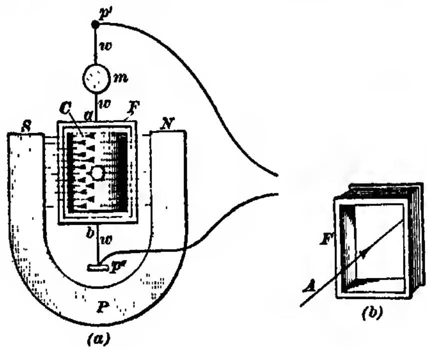


FIG. 38.

was named after Alvisio Galvani. There are many kinds and makes of galvanometers, but only the D'Arsonval galvanometer named after its inventor, will be described here. The instrument is constructed as follows:

Referring to Fig. 38, at (*a*) is shown a permanent magnet *P*, which stands in a vertical position. Between the poles of the magnet is suspended from a fine silver wire a light, hollow rectangular frame, shown in detail at (*b*); the thickness of the frame is quite small as compared with its height and breadth. The frame is wound with many turns of fine, insulated wire, thus making it a solenoid. The wire suspension *w* is pulled taut between the supports p' and p'', so as to support the solenoid and leave it free to turn. The solenoid is arranged relative to the poles of the magnet as shown at (*a*), with the axis *A* of the solenoid making an angle with the direction of the lines of force between the north and south poles of the magnet. Within the solenoid, is a soft-iron, cylindrical core *C*, which acts as an armature to attract lines of force and makes a strong, uniform magnetic field. The silver suspension wire is connected to one end of the coil at *a* and to the other end at *b*, and it is also connected to the circuit at p' and p''; thus when a current is flowing through the circuit, it passes through the coil of the solenoid. Since the lines of force through the solenoid make an angle with those passing between the poles of the magnet, the two sets of lines of force tend to coalesce, thus causing the solenoid to turn and twist the suspension wire *w*. When the current is shut off (by opening the circuit), the wire untwists, and the solenoid returns to its former position. Since the force required to twist the wire increases with the amount of twist, and since the force causing the twist increases with the strength of the current, it is evident that the angle turned through by the solenoid increases or decreases as the current increases or decreases. To measure the angle, a pointer or small mirror *m* is attached to the suspension wire. A beam of light thrown on the mirror from some point in a line perpendicular to the plane of the mirror will be reflected back to the point from which it came; but, as the solenoid turns, the mirror turns also, and the beam of light is reflected to some other point. Knowing the strength of the current at different times and marking the points to which the light reflects, a scale can be constructed that will measure the strength of an unknown current.

Instead of a mirror, a pointer may be attached to the wire, and a scale for measuring unknown currents may be constructed in a similar manner.

103. As previously stated, galvanometers are made in many forms. While any instrument used to measure currents by

means of its electromagnetic action is a galvanometer, the term is usually restricted to instruments used in laboratories and in precise measurements. As will presently be shown, a galvanometer can be so constructed that it will measure the e.m.f. of a current in volts, in which case, it is called a **voltmeter**; or, it can be so constructed that it will measure the strength of a current in amperes, in which case, it is called an *ampere meter* or **ammeter**, the latter being the name in commercial use.

104. The Weston Ammeter.—The Weston ammeter is a special form of the D'Arsonval galvanometer; its construction is shown in Fig. 39. A perspective view of the entire instrument is shown

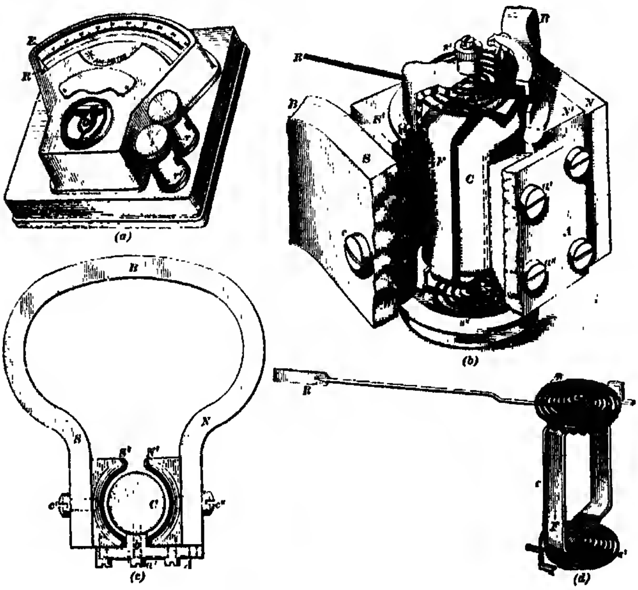


FIG. 39.

at (*a*); a perspective of the working parts, with a portion removed, is shown at (*b*); a plan view showing the magnet and soft-iron core is given in (*c*); and (*d*) is a perspective showing the solenoid, springs, and pointer. Referring to (*b*) and (*c*), B is a permanent magnet, which has attached to its poles N and S soft-iron pole pieces N' and S', the inner surfaces of which are curved to a circular arc. A brass plate A is screwed to the outer ends

of the pole pieces and carries a lug b to which the round iron core C is attached by the screws a' and a''. The core C fits inside the solenoid F, shown at (d), in the manner illustrated in (b). The spiral springs s' and s'', at the top and bottom of the solenoid, tend to prevent the solenoid from turning. To the upper part of the solenoid is attached a light aluminum pointer R that moves over the graduated scale E, shown in (a). When the circuit is open, the springs bring the pointer to 0, the left-hand end of the scale which indicates zero; but when the circuit is closed, the coil (solenoid) turns, as explained in Art. **102**, carries the pointer with it, against the resistance of the springs, and the reading of the scale shows the strength of the current in amperes. The action is exactly the same as in the D'Arsonval galvanometer. The current enters the instrument at the binding post r, which is marked + on the instrument, flows through the solenoid, and leaves the instrument at the binding post r'.

The distinguishing feature of any ammeter is its very low internal resistance; this is necessary, in order that the full strength of the current may pass through it. A Weston ammeter of the kind just described, and which will indicate up to 15 amperes, has an internal resistance of only .0022 ohm; hence, when indicating to full capacity, the drop in voltage between the two binding posts is, by Ohm's law ($E = IR$) only $15 \times .0022 = .033$ volt, and the loss in power is only ($P = I^2R$) $15^2 \times .0022 = .495$ watt, say half a watt. For lower values of I, the loss is much less; thus, for 5 amperes, the drop in voltage is only $5 \times .0022 = .011$ volts, and the loss in watts is $5^2 \times .0022 = .055$ watt. Note, however, that the entire current goes through the instrument.

105. Ammeter Connections.—All the current to be measured must flow through the ammeter; hence, the ammeter must be connected in *series* with the apparatus receiving the current to be measured, preferably *between* the apparatus and the source from which the current comes. Suppose, for example, that it were desired to measure the current that flows through the lamp L in (a), Fig. 40. If C is the conductor carrying the current from the source to the lamp, the ammeter must be placed on this wire, the binding post marked + being connected to that part coming from the source and the other binding post to that part leading to the lamp. All the current received by the lamp then passes through the ammeter.

If the connections at the binding posts were reversed, the pointer would tend to turn in the opposite direction, and the ammeter might either be damaged or destroyed.

Suppose that by mistake the ammeter were connected in parallel with the lamp, as shown at (*b*), Fig. 40. If the e.m.f. of the circuit were 110 volts and the internal resistance of the ammeter were .0022, the strength of the current passing through the instrument would be, by Ohm's law, $I = \frac{E}{R} = \frac{110}{.0022} = 50{,}000$ amperes, which would completely destroy the ammeter by melting the resistance.

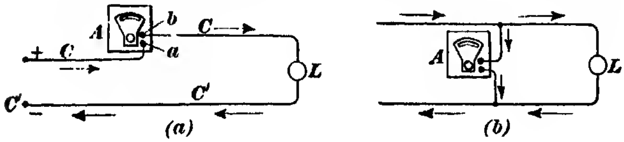


FIG. 40.

106. The Weston Voltmeter.—The distinguishing feature of a voltmeter is the extremely high internal resistance, which is necessary in order to make the current flowing through it exceedingly small. The resistance of a voltmeter that will record up to 150 volts is 18,000 ohms. Therefore, by Ohm's law, when registering 110 volts, the strength of the current that flows through it is only $I = \frac{E}{R} = \frac{110}{18{,}000} = .00611+$ ampere, and the power it absorbs in watts is $P = \frac{E^2}{R} = \frac{110^2}{18{,}000} = .672+$ watt.

The Weston voltmeter, which is illustrated in Fig. 41, resembles very closely the Weston ammeter. The chief difference in external appearance is in the position of the binding posts, one being placed on either side of the magnet poles instead of both on one side, as in the ammeter. The solenoid is wound with finer wire, and there are many more turns. The current enters the instrument at the binding post r', which is marked +, and then passes to a high resistance R by the wire a; from R, it goes to the solenoid F by wire b, and leaves the solenoid and the instrument by wire c, which connects with the other binding post r''. The instrument frequently has a switch S, in the form of a push button, so that the instrument records only when the button is pushed, thus closing the circuit. The principal dif-

ference between the Weston voltmeter and the ammeter is the resistance R and the fact that the solenoid is wound with many more turns of finer wire; otherwise, the two instruments are practically alike in their mechanical construction.

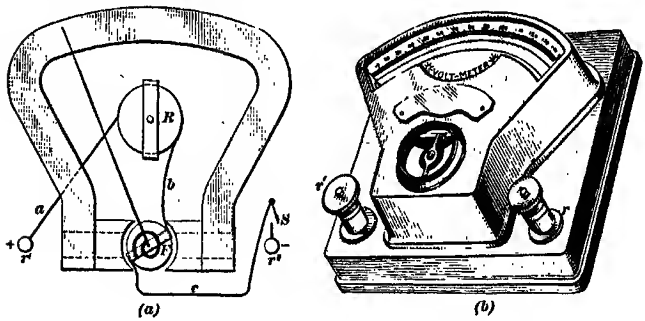


FIG. 41.

107. Voltmeter Connections.—A voltmeter must always be connected in parallel (multiple) with the apparatus whose e.m.f. it is desired to measure; the e.m.f. of the voltmeter will then be the same as that of the apparatus (Art. **37**). Thus, referring to Fig. 42, suppose it were desired to find the voltage of the lamp L. At a, a point near to where the current enters the lamp, a connection is made to the binding post marked +; at b, a point near to where the current leaves the lamp, a connection is made to the other binding post. The current thus divides at a and unites at b, and the *voltage* of the instrument is the same as that of the

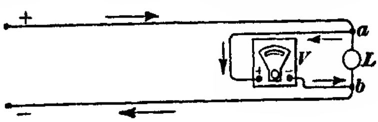


FIG. 42.

lamp. Observe that in the case of the ammeter, both connections are to the same (the +) wire; but, in the case of the voltmeter, the connections are made to both (the + and the −) wires of the circuit.

108. Watt Meters and Watt-Hour Meters.—Knowing the current in amperes and the e.m.f. in volts, the watts (electric power) can be found by multiplication, since $P = IE$, Art.

47. Insofar as the individual user of electric power is concerned, he does not wish to know the power for a particular time, but at any time; for this reason, instruments called **watt meters** are manufactured. The details of their construction and operation are somewhat complicated and will not be described here, further than to state that a watt meter does not have a permanent magnet, but two sets of coils (solenoids), one being connected to the circuit in series (like an ammeter) and the other being connected in multiple (like a voltmeter). One of the coils is free to turn, while the other is fixed and takes the place of a magnet. The effect is such that the pointer, as it moves over the scale, indicates watts instead of volts or amperes. For measuring the electrical energy supplied to houses, offices, etc., what are called **watt-hour meters** are employed. In these instruments a set of gears is caused to rotate; and since each revolution is proportional to a certain number of watt-hours, the number consumed in a given time is recorded in much the same manner as in a gas meter. As previously stated, the watt-hour indicates a certain number of units of work or energy, and is the basis on which the consumption of electricity is bought and paid for.

109. Detector Galvanometer.—When a galvanometer is so made that it merely detects the presence of a current and indicates its direction, but does not measure its value in amperes or

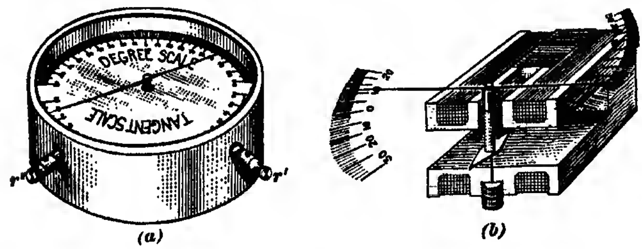


FIG. 43.

volts, it is called a **detector galvanometer**. One form of such an instrument is shown in Fig. 43. A perspective view is shown at (*a*), *r*′ and *r*″ being the binding posts. At (*b*) is shown one-half of the solenoid, which consists of two coils, both wound in the same direction, and formed of a large number of turns of very fine, insulated wire. The middle part of the frame on which the coils are wound is cut out, to permit the insertion of a magnetic

needle and its staff. The staff carries a long aluminum pointer at its upper end, the pointer being placed at right angles to the axis of the needle. The dial has two scales, one being divided into degrees and the other into parts that correspond to the tangents of the angles indicated by the degrees. One of the halves of the pointer moves over the degree scale and the other over the tangent scale. When no current is flowing through the coils, the pointer rests over the zero mark in the middle of both scales; but, when a current is flowing, the needle is deflected to the right or left of the zero mark, according to its direction (see Art. 95), a certain amount, the value of which depends upon the strength of the current. The amount of the deflection from zero does not vary in direct proportion to the angle turned through when measured in degrees, but is in proportion to the tangent of the angle.

To use the instrument, turn it until the pointer rests over the two marks indicated by 0; the needle then points north and south. Now make connection with the circuit, and if a current is flowing the pointer will move. Assuming that the coils are wound into a right-hand helix, a movement to the left of 0 indicates that the current is flowing away from the observer, while a movement to the right indicates that the current is flowing toward the observer.

The instrument may be used to *compare* the strengths of two different currents. Thus, suppose that on being connected to one circuit, the needle deflects 21 degrees; this corresponds to .38 on the tangent scale. Suppose, further, that on being connected to another circuit, the needle deflects 36 degrees; this corresponds to .73 on the tangent scale. The strength of the current in the second circuit is then $\frac{.73}{.38} = 1.92$ times that in the first circuit. If the strength of the currents in the two circuits be denoted by I' and I'' and the readings on the tangent scale by T' and T'', respectively, $I' : I'' = T' : T''$, or $I'' = I'\left(\frac{T''}{T'}\right)$ Since the value of I' is not generally known, the above proportion is best expressed by writing it in the form $\frac{I''}{I'} = \frac{T''}{T'}$; and since T' and T'' are read directly on the scale, this will give the ratio of the currents in the two circuits.

In one make of this instrument, the coil is wound with No. 30

B. & S. wire of such length as to give a resistance of about 30 ohms; it is so sensitive that a current of .00001 ampere will deflect the needle about 1 degree. These instruments should be handled very carefully.

MEASURING RESISTANCE

110. Rheostats.—It is frequently desirable to increase or decrease the strength of the current flowing through a circuit or a shunt or to obtain a current of some particular strength. From Ohm's law, $I = \frac{E}{R}$, and if E, the e.m.f. of the circuit or shunt, remains the same, the value of I, the strength of the current in amperes, can be changed by changing the resistance R. Any device for changing R without opening the circuit is called a **rheostat,** which is derived from two Greek words—rheo (to flow) and statos (standing, stop); the word therefore literally means flow-stopper. An *adjustable* resistance or an apparatus for varying the resistance is a rheostat; it slows down the current by absorbing electrical energy, which heats the resistance and is thus prevented from doing useful work.

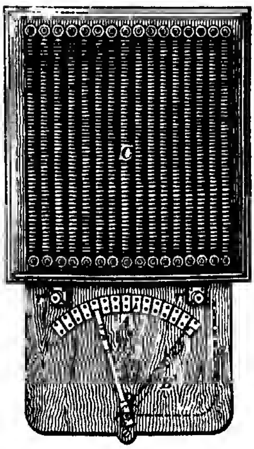


Fig. 44.

111. A form of **sliding contact rheostat,** to be placed against a wall, is shown in Fig. 44. There are 16 coils, the resistances of which are known, and each coil is connected to one of the contact pieces A, which are arranged in an arc of a circle. The first two (left-hand) coils are connected to the left-hand contact piece, the third coil is connected to the next contact piece, and so on, all the coils being connected in series. The current enters at the binding post r', passes through the wire a to the end b of the swinging arm B, then through the arm to the contact piece on which it rests, thence to the coil connected to the contact piece and all the other coils to the left of it; the first coil on the left connects to the other binding post r'', which is connected with the circuit.

With the arm in the position B′, the current has to pass through all the coils; but, as the arm is moved to the left, the coils are gradually cut out. When the arm is in the position B, only five of the contact pieces are in series, that is, the current passes through only $5 + 1 = 6$ coils.

An adjustable resistance of a nature similar to the foregoing is frequently called a **resistance box.** Since the resistance of each coil is known, the strength of the current can be varied almost at will.

112. The Wheatstone Bridge.—A special form of rheostat that is used for measuring an unknown resistance is known as the **Wheatstone bridge.** To understand the principle of its operation, consider the diagram, Fig. 45. The current is supplied by a battery B; it flows from B to a, where it divides, a part going through the upper shunt U–X and the remainder through the lower shunt L–A. Both shunts are considered as being divided into two parts, the upper into U and X and the lower into L and A. U is called the **upper balance arm,** L the **lower balance arm,** A is called the **adjustable resistance,** and X is the **unknown resistance,** which is to be measured. The e.m.f. in both shunts is the same (see Art. **37**), the resistance in the arms U, L, and A is known, and the resistance X can be found by proportion, as will now be shown.

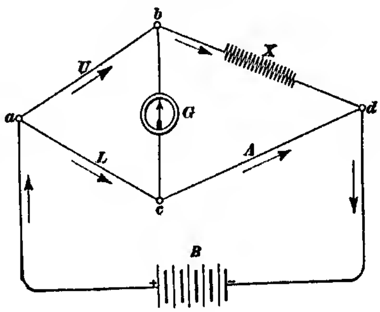


FIG. 45.

The fall of potential between the point a, where the current divides, and d, where it unites, is the same for both shunts, and for the same *proportionate distance* from a, the fall of potential is the same in both shunts. Denote the fall between a and b by U, between b and d by X, between a and c by L, and between c and d by A; then, when $U : X = L : A$, the fall of potential at b is equal to the fall of potential at c. Connecting a sensitive galvanometer at b and c, as shown, the current will divide at b whenever there is a difference of potential between b and c; but, if there is no such difference, no current will flow between b

and c. Therefore, when the current passes from the battery to a, there will usually be a movement of the galvanometer needle; but, by adding to or cutting out resistance in the adjustable arm A, the needle can be made to point to 0, in which case, the e.m.f. at b will be the same as at c, no current flows through the galvanometer, and $U : X = L : A$, from which $U \times A = L \times X$, or

$$X = A \times \frac{U}{L}$$

Thus, if the resistance $U = 100$ ohms, $L = 10$ ohms, and $A = 547$ ohms, the unknown resistance is $X = 547 \times \frac{100}{10} = 5470$ ohms. Again, if the resistance $U = 10$ ohms, $L = 100$ ohms, and $A = 206$ ohms, $X = 206 \times \frac{10}{100} = 20.6$ ohms.

113. In practice, the resistances are made up of standard coils of known resistance and wound non-inductively. If a wire (insulated) be folded in the middle and both parts wound together into a coil, the current induces lines of force that circulate around the two parts in opposite directions, thus preventing any magnetic action; in other words, the coils will not be solenoids, and they are then said to be wound **non-inductively.** The contact pieces, which form the arms, have a rectangular cross-section and are of sufficient area to offer very slight resistance. Connection is made by plugs p, as indicated in Fig. 46. One end of the coils is connected to one contact piece and the other to the next adjacent one, as indicated by the top view in Fig. 47. When a plug is *in*, the current flows through it instead of through the coil, the resistance of which is much higher; but, when a plug is *out*, the current must flow through the coil to complete the circuit through the arm. Hence, a resistance is thrown into the circuit only when a plug is *out*. Referring now to Fig. 47, the arms of the bridge are arranged somewhat in the form of a letter M. The wire from the battery connects at a, where the current divides, a part going to b, where connection is made with the galvanometer and the unknown resistance, and a part going to e, where connection is made with the galvanometer and the adjustable resistance. Evidently, ab corresponds to the upper arm, Fig. 45, ac corresponds to the lower arm, and *cefghid*

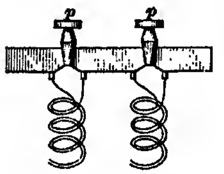


FIG. 46.

corresponds to the adjustable arm. For convenience, the upper and lower arms are also made adjustable, so as to allow for a wide variation in the unknown resistance, there being three resistances in each of 10, 100, and 1000 ohms respectively. The coils in the adjustable arm have resistances of 1, 2, 2, 5; 10, 20, 20, 50; 100, 200, 200, 500; and 1000, 2000, 2000, 5000, 10,000 ohms. The sum of these resistances is 21,110 ohms, and the plugs may be so set as to throw into the circuit from the adjustable arm any resistance expressed by an integer from 1 ohm to 21,110 ohms, and the range of the instrument is from .01 ohm to 2,111,000 ohms. Thus, if the 10 plug is out in the upper arm, the 1000 plug in the lower arm, and the 1 plug in the adjustable arm, $X = A\left(\frac{U}{L}\right) = 1 \times \frac{10}{1000} = .01$ ohm; or, if the 1000 plug is out in the upper arm, the 10 plug in the lower arm, and all the plugs are out of the adjustable arm, $X = 21{,}110 \times \frac{1000}{10} = 2{,}111{,}000$ ohms.

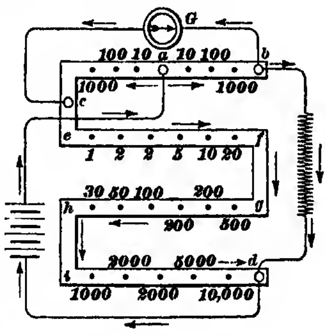


Fig. 47.

114. Measuring Resistance with an Ammeter and Voltmeter. The resistance of any part of a circuit may be determined with a voltmeter and ammeter and an application of Ohm's law. Thus, let the diagram, Fig. 48, represent a circuit, S being the source of the current (battery or dynamo), and suppose it is desired to measure the resistance between a and b, the resistance consisting of one or more lamps, a motor, a rheostat, or anything else that offers resistance. Connect the ammeter in series with the resistance, so that all the current that goes through the resistance goes through the ammeter; then connect the volt-meter in parallel with the resistance, the current being divided at a and united at b. Be sure that the current enters both instruments at the binding

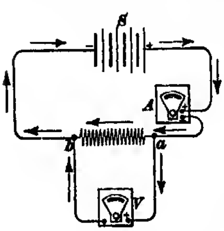


Fig. 48.

posts marked +. Read both instruments at as nearly the same instant as possible. Suppose that the voltmeter reads 216 volts and the ammeter reads 4.8 amperes; then, by Ohm's law, $R = \frac{E}{I} = \frac{216}{4.8} = 45$ ohms, the resistance between a and b.

EXAMPLE 1.—Referring to Fig. 47, suppose the 10-ohm plug between a and b, the 100-ohm plug between a and c, and the following plugs 1, 5, 50, 200, 500, 1000, and 2000, between c and d are out; what is the value of the unknown resistance?

SOLUTION.—The total resistance represented by the plugs that are out in the adjustable arm is $2000 + 1000 + 500 + 200 + 50 + 5 + 1 = 3756$; hence, by Art. **112**, $X = 3756 \times \frac{10}{100} = 375.6$ ohms. *Ans.*

EXAMPLE 2.—Suppose a voltmeter to be connected in parallel with the upper and lower carbons of an arc lamp and that an ammeter is connected in series with the same lamp. If, when the lamp is burning, the voltmeter reads 44 volts and the ammeter reads 9.9 amperes, what is the resistance of the lamp when hot?

SOLUTION.—The e.m.f. of the current flowing through the carbons is 44 volts, and the strength of the current is 9.9 amperes; hence, by Ohm's law, $R = \frac{E}{I} = \frac{44}{9.9} = 4.44+$ ohms. *Ans.*

The result just obtained in the last example is called the **resistance hot** of the lamp. The resistance when the carbons are cold is considerably higher, because the resistance of carbon decreases as the temperature increases.

115. Ohmmeters.—What are called **ohmmeters** are instruments made on much the same principle as voltmeters. The scales, however, read ohms instead of volts. The connections for an ohmmeter are the same as for a voltmeter, that is, the instrument is connected in parallel with the resistance to be measured.

EXAMPLES

1. Show by a sketch how you would make an electro-magnet, indicating direction of current and the north and south poles.

2. Why does a solenoid behave like a magnet?

3. What is the purpose of a voltmeter? Of an ammeter? How is each connected in a circuit? What would happen to a voltmeter if it were connected like an ammeter?

4. How much work is required to run a grinder six days of 24 hours each if driven by a motor that takes an average of 460 amperes at 550 volts? Express the answer in kilowatts-hours and horsepower-hours.

Ans. 36,432 k.w.h.; 48,836 + h.p.h.

ELECTROMAGNETIC INDUCTION

116. Inducing a Current.—Let M, Fig. 49, be a magnet (either a permanent magnet or an electromagnet), and let AB be a conductor, the ends of which are connected to a sensitive galvanometer G, thus making a complete circuit consisting of the conductor AB, the wires Br' and $r''A$, and the galvanometer. Then suppose that the conductor be moved suddenly downward past and near one of the poles of the magnet, in this case, the north pole. The galvanometer needle will be seen to deflect, say to the right, thus indicating that a current has passed through the circuit. If the conductor be moved suddenly upward, the needle will deflect again, but in the opposite direction, showing that a current has passed through the circuit in a direction opposite to that which flowed through it when the conductor passed downward. When an electric current is created in this manner, it is said to be created by *induction*, and it is called an **induced current**, the entire process being called **electromagnetic induction.**

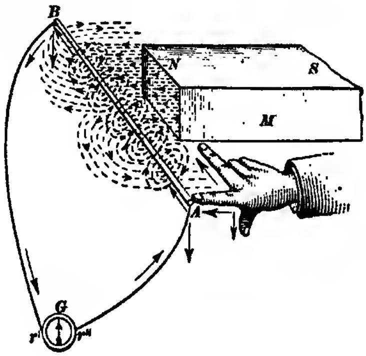


FIG. 49.

The reason for the flow of current is because the conductor as it passes through the magnetic lines of force issuing from the pole of the magnet has set up around it magnetic whorls, which correspond in every respect with those set up around a conductor when a current is flowing through it. If the circuit is complete, any conductor having magnetic whorls around it must have a current flowing through it. When the whorls are caused by lines of force coming from a magnet or electromagnet, as in the present case, the current is an induced current.

117. To determine the direction of the current, suppose the lines of force to be capable of being stretched and bent around. As the conductor moves down, the *under* side presses against the lines of force, stretching them and tending to bend them

around the conductor in the direction of the hands of a clock hence, the lines of force in the whorls will have a clockwise direction around the conductor, assuming that the observer be looking from A toward B, and the current will flow from A to B. (See Art. **92.**) When the conductor is moving upward, the *top* side presses against the lines of force, bending them around the conductor in a direction opposite to the hands of a clock, the lines of force in the whorls have a counterclockwise direction around the conductor, assuming the observer to be looking from A toward B, and the current flows from B to A. It may be considered that the lines of force in the whorls are the same as those of the magnet that were touched by the conductor, that they were stretched until they broke in two places, the ends uniting to make the lines of force in the whorls.

Another way to determine the direction of the current is to use Fleming's rule, usually called the *rule of the right hand.* Assume a bar magnet grasped at the north end, with the right hand, as shown in Fig. 50, with the index finger pointing in the direction of the lines of force, the thumb at right angles to the index finger, and the middle finger pointing downward at right angles to the thumb and index finger. If a conductor C, parallel or approximately parallel to the middle finger, be moved across the magnet in front of its pole and in the direction of the thumb, the direction of the current will be downward from A to B, in the direction pointed by the middle finger. If the direction of movement of the conductor be opposite to that here indicated, turn the hand and magnet over, so the palm will be upward and the thumb will point in the direction the conductor is moving; the middle finger will then point upward, showing that the current is moving from B to A. Hence, the rule:

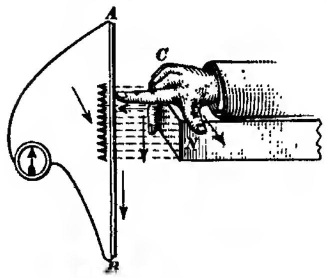


FIG. 50.

Rule.—*To determine the direction of the induced current, place the thumb and index and middle fingers of the right hand at right angles to one another; hold the hand so the index finger will point in the direction of the lines of force, the thumb in the direction of*

motion of the conductor, and the direction pointed to by the middle finger will be the direction of the current.

As an example, note the position of the fingers in Fig. 49. Applying the first method to Fig. 50, assume the observer is at *B;* the lines of force are bent around the conductor counter-clockwise; hence, the current is flowing toward the observer, that is, from *A* to *B*. The observer is supposed to be looking along the conductor from *B* toward *A*.

118. The creation of magnetic whorls by the breaking of the lines of force is called **cutting lines of force;** the more whorls there are about the conductor, the stronger will be the current passing through it; therefore, the greater the number of lines of force that are cut in a given time the stronger will be the current. If the circuit is not closed, as in the case of a wire moving across a magnetic field, no current can flow, but an electrical stress will be produced in the conductor; in other words, an e.m.f. will be generated. Consequently, the voltage, or e.m.f., also depends upon the number of lines force cut per unit of time, say per second.

119. Referring to Fig. 51, let *abcd* be a U-shaped frame made of some conducting material and supported between the poles of a magnet in such a manner that it will not cut lines of force; let *C* be a conductor resting on this framework, and cutting lines of force as it moves from *b* to *a* or from *a* to *b*. So long as *C* is stationary, no current will be generated, since it does not cut lines of force; but, when *C* moves, say, in the direction of the arrow toward *a*, it cuts lines of force, and according to the rule of the right hand, the current will flow from *f* toward *e*, thence to *b*, *c*, and *f*, again. That part of the conductor that cuts lines of force and is represented by *ef* is called the **internal circuit,** and the part *cbcf* of the frame through which the current flows is called the **external circuit.**

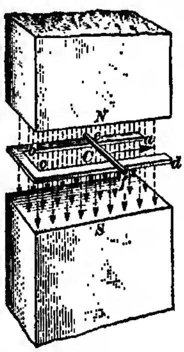


FIG. 51.

The number of lines of force cut by any movement of the conductor depends upon the flux density (see Art. 89), the length of the internal circuit, the velocity of the conductor through the

field, and the direction in which the conductor cuts the lines of force. Thus, suppose that $be = cf = 6$ inches, and that the points *e*, *b*, *c*, and *f* all lie in a plane that is parallel to the planes of the pole faces. Suppose further that the lines of force are parallel straight lines. A movement of the conductor from *b* to *e* will then cut a greater number of lines of force than for any other position of the frame, because, the distance *bc* remaining the same, if the frame be tipped so *bc* lies lower than *cf*, the rectangle *ebcf* will enclose a smaller number of lines of force than before; and the same will be true if *cf* be lower than *be*. Therefore, the conductor will cut the greatest number of lines of force for any particular distance traveled when it cuts them at right angles. Evidently, also, the longer the internal circuit the greater will be the number of lines of force cut; the greater the flux density the greater the number of lines of force cut; and the greater the velocity of the conductor the greater the distance it will travel in a unit of time, and the greater will be the number of lines of force cut in a unit of time.

120. It will be evident from the foregoing that an induced current is the result of motion; it does not matter whether the conductor is moved or whether the magnet is moved, so long as lines of force are cut. If either move so that lines of force are not cut, no whorls will be generated and no current or e.m.f. will result. For example, suppose the conductor be moved in the direction of its axis, that is perpendicular to the plane of the paper; no lines of force are cut and there will be no induced current or induced e.m.f. The same result will be obtained if the magnet be moved in the same direction, the conductor being stationary.

121. Inducing Current in Closed Coil.—Let *ABCD*, Fig 52, be a continuous conductor forming a rectangular-shaped coil; if passed between the poles of a magnet in the direction of the arrow so one side can cut lines of force, magnetic whorls will be set up around the conductor, and a current will flow in the direction of the arrows, Fig. 52. Suppose, however, that the length $CB = DA$ is small compared with the width of the pole faces, the result being that *BA* and *CD* are in the magnetic field at the same time. The whorls around *BA* are produced by the pressure of the *outside* of *BA* against the lines of force and are clockwise; the whorls around *CD* are produced by the pressure of the *inside*

of CD against the lines of force and are clockwise also. Consequently, the current in CD will be from D toward C, and since the current in BA is from A toward B, the two currents oppose each other. Therefore, if the flux density is uniform across the pole faces, making the flux density of the entire field uniform, there will be no current in the coil as long as the sides BA and CD are both in the field. If the field is not uniform, then that side which is in the part of the field of greater density will have the greater e.m.f., there will be a difference of potential, and the current will flow from that side having the greater e.m.f. to that side having the smaller e.m.f.

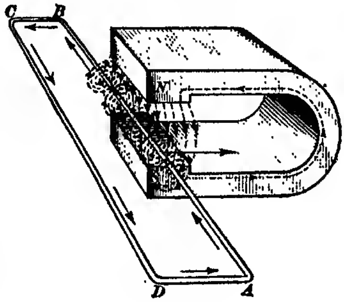


FIG. 52.

122. Suppose a circular coil to be placed in a uniform magnetic field, as indicated in Fig. 53, and suppose the coil to turn on the diameter ab, the half E moving down and the other half moving up, ab being horizontal. When the coil is in position (a), with the plane of its face parallel to the lines of force, none of them pass through the coil, and the slightest movement of the coil cuts them at right angles, thus generating an e.m.f. As the coil revolves, see (b), lines of force pass through the coil, and for the same amount of movement of the coil, the number of lines of force cut decreases; hence, the e.m.f. also decreases. The number of lines of force cut for a given arc moved through by the coil, and consequently, the e.m.f. generated, continue to decrease until the coil reaches the position (c), where the greatest number of lines of force pass through the coil and none are being cut for a slight movement of the coil. As the coil continues to turn, the number of lines of force being cut, and, consequently, the e.m.f. generated increase until they become a maximum when the coil again

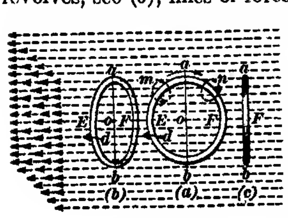


FIG. 53.

reaches the position indicated in (*a*), with its plane parallel to the lines of force, and cutting them at right angles. Since the number of lines of force cut is continually changing, the e.m.f. is also continually changing, and varies from 0, in position (*c*) to a maximum in position (*a*). Since there is a continual change in the e.m.f., a current will flow through the coil, but the strength of the current will not be uniform. If the coil turn about an axis passing through its center *o* and perpendicular to the face of the coil, there will be no change in the number of lines of force passing through the coil, no lines of force will be cut, no e.m.f. will be generated, and there will be no current.

123. Mechanical Work Necessary to Induce a Current.— Every electric current possesses energy, and this energy must be supplied from some outside source in some way. Every induced current produced by electromagnetic induction is the result of motion; and since motion implies the action of a force through a distance, it follows that work is done in producing the motion, it being stored up as electric energy. To do work, a force must act, and it is desired to ascertain the nature of the force in this case.

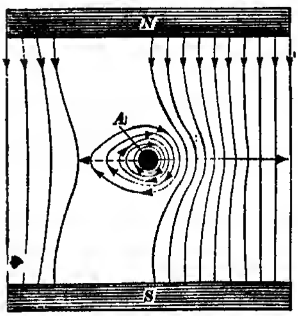


FIG. 54.

Referring to Fig. 54, *A* is a cross-section of a conductor moving through a magnetic field at right angles to the lines of force. As the conductor moves, it bends the lines of force, and it continues to bend them until they finally break and become whorls around the conductor. This action in forming the whorls can be produced only by the action of a force; in other words, the lines of force object to being broken and having their shape changed, and they react on the conductor with a force directly opposite to that which moves it. The full arrow indicates the force that moves the conductor and the dotted arrow indicates the force that opposes the motion—the resisting force. This force multiplied by the distance traveled by the conductor is the work done by the conductor and equals the energy of the current.

From the foregoing, it will be evident that to produce an elec-

tric current by induction, in the manner heretofore described, mechanical work must be done. In the case of a dynamo, the mechanical work is performed by the engine, waterwheel, or other prime mover that turns the armature or field, whichever revolves.

124. As another illustration, consider a circular closed coil, Fig. 55, and let a magnet M be moved into the coil, as shown. The coil will cut the lines of force coming out of the north pole of the magnet, magnetic whorls will be set up around the coil, moving counterclockwise, and induce an e.m.f. A current will flow in the direction indicated, because as the magnet passes through the coil, the flux density decreases, being greatest at the

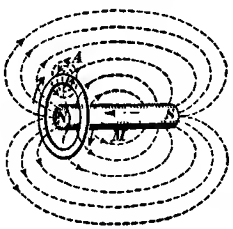


Fig. 55.

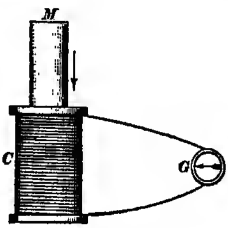


Fig. 56.

end, thus changing the e.m.f. and producing a movement of the current. Now referring to Fig. 32, it will be noticed that when a piece of soft iron is brought to the coil, the current must flow in the opposite direction around the coil in order that the iron may become a temporary magnet, with its poles the same as in Fig. 55. The direction of the current is therefore opposed to the magnetism, and tends to stop the movement of the magnet into the coil.

If the movement of the magnet be stopped, the current will also stop, since no lines of force are being cut. If the magnet be pulled out of the coil, the current will reverse in direction, it will act with the magnetism, and a force must be exerted to pull the magnet out. These effects are more strikingly evident when a solenoid is used instead of a coil. Thus, referring to Fig. 56, let C be a solenoid, the ends of the wires being connected to the galvanometer G. On inserting the magnet M, the galvanometer needle will deflect, say to the right; when the magnet stops moving, the needle moves back to 0; and when the magnet is

pulled out of the solenoid, the needle deflects in the other direction, to the left. The action is exactly the same as with the coil and magnet in Fig. 55, the coils that form the solenoid cutting the lines of force. If the magnet be reversed, with the other end entering the solenoid, the deflections of the needle will also be reversed.

125. Referring again to Fig. 55, consider the sides of the coil that would be touched by flat plates laid against them, as the poles of a magnet. When the magnet is entering the coil as shown in the figure, the result of the induced current is to make the right-hand side a north pole and the left-hand side a south pole; but, when the magnet is pulled out of the coil, these poles are reversed. Since like poles repel each other and unlike poles attract, a repelling force tends to resist the magnet when entering the coil, and an attractive force tends to resist it when leaving the coil. The same effect is produced in the solenoid, Fig. 56.

The whole matter is summed up in what is called Lenz's law. *The direction of an induced current is such that its magnetic field opposes the motion of the magnet that produces the field.*

126. Self-Induction.—What is called **self-induction** occurs whenever there is a sudden change in the current flowing through a circuit. For instance, suppose a steady current is flowing through a coil, the current being derived from a battery, dynamo, or other outside source; this current sets up a magnetic field about the coil and causes lines of force to pass through it. If the circuit be suddenly opened by, say, throwing a switch, the result is practically the same as withdrawing the magnet in Fig. 55, and a momentary current is induced in the circuit. The same effect, though not so marked, will be observed if a resistance is suddenly cut out of the circuit. Or, if the current be suddenly increased, as by throwing the switch so as to close the circuit, an induced current will result, in the same manner as when a magnet is suddenly inserted into a coil. These induced currents are said to be caused by *self-induction;* they oppose the original current when the current is increased, and they add to the strength of the original current when it is decreased. It therefore requires a small amount of time for a current to increase to its maximum or decrease to its minimum value on account of this self-induction, though the time is very short—only a very small fraction of a second.

127. Mutual Induction.—When two coils, one carrying a current, are so situated relatively to each other that the magnetic field of the coil carrying the current encloses the other coil, a current will be induced in the coil not carrying a current by what is called **mutual induction.** The coil carrying the current is called the **primary coil,** and the other coil is called the **secondary coil.** The secondary coil is usually placed so as to enclose the primary coil within it, but it may be placed outside of the primary, both having the same size, as in Fig. 57, where p and s are the primary and secondary coils, respectively. The current, in this case, is supplied by the battery B, and it flows through the coil p (which is a right-hand helix) as indicated by the arrows, that is, from the battery, through a, through the coil, and then through b. By the rule of Art. **97,** the end of the helix marked S is the south pole, and the right-hand end of the soft-iron core that passes through both coils is a north pole. The lines of force therefore have the direction indicated by the arrowheads, and some of them spread out so as to enter the other coil. The flux density is, of course, greatest at the poles S and N. Now if the current be suddenly "made" (turned on) by closing the switch k (pressing the key), the effect will be the same as though the secondary coil s had been suddenly moved along the core nearer the south pole S of the primary coil, since it then moves into a *denser* field, and the number of lines of force passing through the secondary coil is suddenly increased also, thus inducing an e.m.f. in the secondary coil. If the external circuit cd of the secondary be closed, a current will flow through it. It is important to observe that the current flows only during the short period following the "make" or "break" of the circuit and ceases to flow as soon as the stability of the circuit is established.

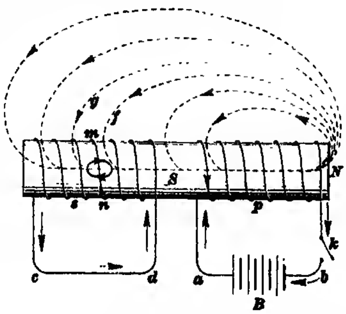


Fig. 57.

To determine the direction of the current, consider the turn mn of the coil and the line of force f. As mn moves to the right, it presses against f and bends it so that the direction of f around mn

will be clockwise; hence, by the rule of Art. **92**, the current is flowing away from the observer, or from n toward m, as indicated by the arrowheads. The direction may also be determined by the right-hand rule, the hand being held over the coil, the index finger pointing down (the direction of f), the thumb pointing to the right toward N (the direction of movement of the secondary coil), and the middle finger pointing away from the observer (the direction of the current). The arrows indicate the direction of the current in the external circuit and coil of the secondary.

Note that the direction of the current in the secondary is *opposite* to that in the primary. But, if the circuit be suddenly opened, by releasing the key k, the number of lines of force passing through the secondary will be suddenly decreased; the effect will be the same as though the secondary coil had been suddenly moved along the core to the left; mn will then press against the line of force f, which will wind around it counterclockwise, the direction of the current in the secondary will be reversed, and will be the *same* as that in the primary. Therefore, when the e.m.f. is *increasing*, the current in the secondary tends to move in a direction *opposite* to that in the primary; but when the e.m.f. is *decreasing*, the current in the secondary tends to move in the *same* direction as that in the primary.

128. From the foregoing, it is seen that an induced current may be generated in three ways: by electromagnetic induction, by self induction, and by mutual induction. When generated by electromagnetic induction, it is the result of motion; when generated by self induction or mutual induction, it is the result of a sudden change, usually by making or breaking the circuit, but will occur whenever there is a sudden change in the number of lines of force passing through the circuit.

It is likewise clear that: (*a*) whenever an electrical conductor cuts magnetic lines of force, an e.m.f. (electromotive force) is induced in it; (*b*) this e.m.f. is greater as the number of lines of force cut per second is greater; (*c*) the direction of the magnetic field produced by the induced current always *opposes the motion that produces the field*, and the e.m.f. produced is electrical energy derived from mechanical work.

ELEMENTS OF ELECTRICITY

(PART 2)

MAGNETISM

PROPERTIES OF MAGNETS

77. Natural Magnets.—The ancients found in Magnesia, Asia Minor, a certain kind of stone or ore that had the peculiar property of attracting to it pieces of iron. It was also found that if a bar-shaped piece of this stone be suspended at its middle from a thread, one end of the bar always pointed toward the north, and that this end always pointed north without regard to how the ends of the bar pointed when first suspended. For instance, if the end pointing north were marked and if the bar were originally placed so as to point east and west, it would swing around, the marked end pointing north and the other south. For this reason, the stone from which the bar was made was called **lodestone,** which means *leading stone.* Anything that possesses this property of always pointing in a north and south direction when suspended from a thread, resting on a pivot, or floating in a liquid, and which will attract and lift iron particles, is called a **magnet;** the entire arrangement is called a **compass,** and the magnet itself is called the **compass needle.** Hence, a piece of lodestone is a magnet; and because it is found in a free state in nature, it is called a **natural magnet.** Chemically, lodestone is an iron ore, oxide of iron (Fe_3O_4), and the ore is called *magnetite.*

78. Artificial Magnets.—If a natural magnet be drawn over a bar of hardened steel from one end to the other several times, the movement being always in the same direction, see Fig. 16, it will be found that the steel bar has the same properties as the natural magnet. It is not necessary to exert any degree of pressure; simply see that the surface of the natural magnet touches that of the steel bar. When the end of the bar is reached, lift

the natural magnet and carry it to the other end of the bar through the air. After the steel bar has been **magnetized,** as the process is called, it becomes what is known as an **artificial magnet;** and if the steel has been previously hardened, it will retain its magnetism indefinitely, being then called a **permanent magnet.** Any permanent magnet, whether natural or artificial, may be used to make other magnets in the same manner as that just described for making an artificial magnet.

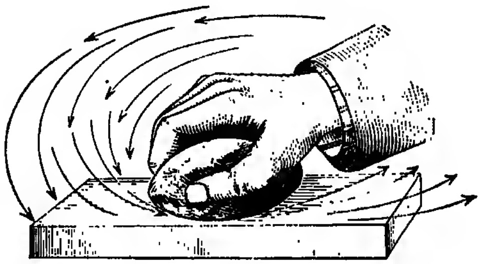


Fig. 16.

79. Magnet Poles.—The ends of a magnet are called its poles. The end that points *north* when the magnet is free to swing in a horizontal plane (as when balanced on a pivot or suspended from a thread) is called the **north pole,** and the other end is called the **south pole.** Whatever its shape, every magnet has a north and a south pole; and if the magnet has the approximate shape of a bar, these poles are opposite each other. The straight line joining the poles and passing through the magnet is called the **axis** of the magnet. A straight bar magnet may be bent into the shape of a letter U or a horseshoe, in which case it is called a U-magnet or a horseshoe-magnet, and the axis will then have a similar shape. The poles will then no longer be opposite each other, but alongside each other; but there will still be two poles, one north and the other south.

If a magnet be broken in two pieces between the poles, each piece will be a magnet, and each will have its north and south poles; in fact, no matter how many pieces may be made of the original magnet, nor how they are taken, every piece will be a magnet and will have its own north and south poles. It is impossible to have a magnet with one pole.

80. Determining the Poles of a Magnet.—In Fig. 17 is shown a compass. The needle, which has been magnetized, is a thin piece of steel having the shape of two equal isosceles triangles joined at their bases. The needle swings in a horizontal plane on a pivot. That half of the needle which has the end that points north is usually blue and the other end is white; the blue end is, therefore, the north pole and the white end is the south pole. If the north pole of another compass be brought near the north pole of the first one, the two needles will swing apart; the same thing will happen if two south poles are brought near each other. But if the north pole of one compass be brought near the

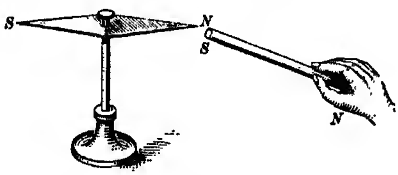


FIG. 17.

south pole of the other, the two needles will swing until they either touch or their axes are parallel. This is also true of magnets; if the poles are *alike*, it may require considerable force (depending upon the strength of the magnets) to make the two surfaces touch, while if they are *unlike* poles, it requires considerable force to separate them. Whence, the law:

Like poles repel each other; unlike poles attract each other.

If, therefore, it is desired to find which of the two poles of a magnet is the north pole and which is the south pole, all that is required is to bring one end of the magnet near, say the north end of the compass, see Fig. 17, and note what happens. If the compass end swings toward the magnet, that end of the magnet is the *south* pole (since *unlike* poles attract each other); but, if the needle swings away from the magnet, then that end is the north pole.

81. Distribution of Magnetism.—Magnetism is not equally distributed over the entire length of the magnet between the poles; this is to be expected, since, because the poles are opposite in character, there must be a place between them where there is apparently no magnetism. Suppose a straight bar magnet be rolled in iron filings; on being lifted out, the appearance of the

magnet will be somewhat as shown in Fig. 18. **A large mass** of *filings will be collected at each end of the bar, and if the bar is cylindrical, these masses will be pear-shaped. The number of particles of iron in the masses will be greatest at the ends, and in the middle of the bar, there will be practically none.* The particles form masses *all around the bar, except at the middle line abcd, which is called the* **neutral line.** If, however, the bar be cut in two at the neutral line, two magnets are instantly formed, and the neutral line will be shifted to halfway between the ends of each piece, as in the original bar.

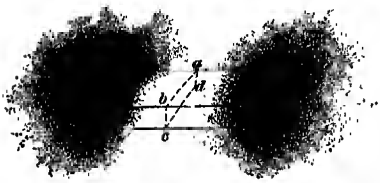


Fig. 18.

82. Magnetic Induction.—If one end of a magnet be touched to a piece of soft iron, the iron immediately becomes a magnet, the end nearest the magnet being of opposite polarity; that is, if the end of the magnet is a south pole, the end of the iron that touches it will be a north pole, and vice versa. If the magnet be lifted and the piece of iron is not too heavy, the force of magnetic attraction will lift the iron also. If the free end of the iron be touched to another piece of soft iron, that also will immediately become a magnet, the end touching the first piece of iron having a pole of opposite polarity. In this manner, several pieces of iron may be lifted as shown at (*a*), Fig. 19. It is to be noted, however, that as soon as one of the pieces of iron is removed from the vicinity of the magnet, it ceases to be a magnet. Such is not the case when the pieces lifted are steel; in this instance, some of the magnetism remains in the steel after removal from the magnet, and the same is true to a certain extent if the iron be not perfectly pure.

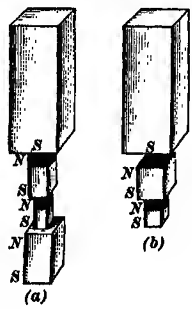


Fig. 19.

Referring to Fig. 19 (*b*), suppose the first piece of iron is brought quite near to the end of the magnet, but is not allowed to touch it; then, as before, the iron will become a magnet, its

poles being of opposite polarity to those of the magnet, and it may even lift a second piece, as indicated in the figure. This action will occur, *even though the piece of iron be separated* from the magnet by a sheet of glass, paper, or any substance whatever, whether magnetic or not. The magnetic properties that are thus imparted to the iron are said to be due to **magnetic induction**, and magnetism is said to be **induced** in the iron. Whenever a piece of iron or steel is magnetized without coming in contact with a magnet, it is magnetized by induction.

83. Magnetic and Non-magnetic Substances.—When a piece of soft iron is magnetized, it is called a **temporary magnet**, because it is a magnet only so long as it is under magnetizing influence. Any substance that can be made into a temporary or permanent magnet is called a **magnetic substance**, and all others are called **non-magnetic substances**. In addition to iron and steel, nickel, cobalt, chromium, cerium, and oxygen are slightly magnetic, but the force of attraction existing between them and a magnet is very small as compared with iron and steel; hence, iron and steel are the only substances used in practice for magnetizing purposes.

It is to be noted that when a magnet attracts a piece of iron (or steel), the iron attracts the magnet with exactly the same force that the magnet exerts on the iron. The effect is similar to the action and reaction of forces; when a weight rests on a table, it presses against the table, and the table reacts and presses against the weight. There is, however, this difference; the force that can be exerted by a magnet diminishes gradually from the poles to the neutral line, where it is zero; but any point or place on the surface of a temporary magnet may be one of the poles, and that pole will attract the magnet always with the same intensity, if the area in contact with the pole of the magnet is the same.

Although non-magnetic substances cannot be magnetized and, therefore, cannot be attracted or repelled by a magnet, they cannot prevent magnetic induction from taking place through them. This was shown in connection with the experiment illustrated in Fig. 19.

LINES OF FORCE

84. Definition.—Strictly speaking, a **pole** is a *point;* hence, when it is desired to refer to an area about a pole, it is better, usually, to employ the term **pole-piece.**

If the pole-pieces of two magnets having opposite polarity are brought near each other, as in Fig. 20, the two magnets attract each other with a certain force. Assume, now, that the two pole-pieces are similar and equal prisms, that the planes of their sides coincide, and that their ends are flat and parallel. Now, if the ends of the pole-pieces are comparatively small, the force exerted between the two poles may be considered as distributed uniformly over the entire surface of the ends, and if the ends be considered as divided into a very large number of little squares, the magnetic force exerted between the poles may also be considered as divided into the same number of equal parts. The effect produced will be the same as though every one of the small forces acted in a line that passed through the centers of gravity of two opposite small squares. For this reason, these small forces are called **lines of force;** and the sum of all the lines of force will evidently be the total force exerted between the two poles. There is, of course, really no such thing as a line of force; it is, however, an extremely convenient conception and is universally used in all electro-magnetic calculations. A line of force when considered as a force and not as a line or path is sometimes called a **maxwell** (named after James Clerk-Maxwell). Hence, instead of the expression 25,000 lines of force, 25,000 maxwells is equally proper.

FIG. 20.

85. Direction of Lines of Force.—With the pole-pieces close together and arranged as described in connection with Fig. 20, the lines of force may be regarded as right lines. In reality, however, the lines are curved, as may easily be proved by direct experiment. Place a bar magnet on a table, and on top of the magnet lay a sheet of paper (or a pane of window glass), supporting the edges so the sheet will lie flat in a horizontal plane. On top of the sheet, sprinkle a thin layer of fine iron filings. Each iron particle immediately becomes a temporary magnet, by

induction, and the filings arrange themselves as shown in Fig. 21, the north pole of one touching the south pole of the preceding particle, one after another, and all forming distinct curved lines extending from pole to pole. If the magnet be turned so as to bring another side against the sheet, the same result will be obtained, and it is therefore inferred (and with truth) that these lines show the direction of the lines of force, and that they (the lines of force) completely fill the space surrounding the magnet. It is assumed that they issue from the north pole of the magnet, make a complete circuit through the surrounding medium, which is usually air, re-enter the magnet at the south pole, and then pass through the magnet to the north pole. The path, which is thus closed and complete, is called the **magnetic circuit**, and every line of force is a closed curve and is in itself a complete magnetic circuit. The space surrounding a magnet and penetrated by lines of force is called a **magnetic field.** Note that the magnetic field includes only that space that is penetrated by lines of force.

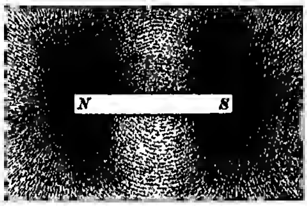


FIG. 21.

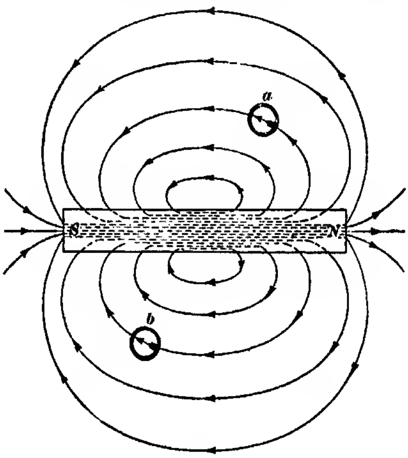


FIG. 22.

That the direction of the lines of force is from the north pole to the south pole is readily established by placing a small compass in the magnetic field. The compass needle will invariably place itself so that its axis will lie in a line of force, and it will point away from the north pole toward the south pole, as indicated in Fig. 22.

86. Reference to Figs. 21 and 22 makes clear the fact that the greatest density of lines of force is at the poles, where they are very closely aggregated. As the distance from the poles increases, whether in the direction of the axis of the magnet or at right angles to it, the number of lines per unit of area decreases. If a right section be taken through a magnetic field, the number of lines of force passing through a square inch or a square centimeter of the section is called the **strength of the field** or **field density.** It is desirable to have a uniform method of measuring the field density. This is expressed as the total number of lines of force passing out of (or into) a pole-piece divided by the projected area (area of a right section) of the pole-piece. Thus, suppose 140,000 lines of force are passing through the north pole-piece of a magnet and that the projected area of the pole-piece is 3.5 square inches; then, the field density = strength of field = 140,000 ÷ 3.5 = 40,000 lines of force per square inch = 40,000 maxwells per square inch.

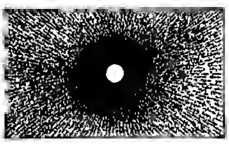


FIG. 23.

87. If, instead of placing the sheet of paper on the side of the magnet, as in Fig. 21, it is placed on one of the pole-pieces, as in Fig. 23, the filings will arrange themselves in radial lines, extending outward from the center of the pole-piece. The greatest number of particles, and, consequently, the greatest density, will be at the center of the pole-piece, as shown.

88. Lines of Force Cannot Intersect.—Lines of force can never intersect, or cut, one another; this fact may also be shown by means of iron filings. Thus, if two bar magnets be arranged as in Fig. 24 (*a*), covered with a sheet of paper, and iron filings are sprinkled over the paper, then when unlike poles face each other, as at (*a*), the lines of force coalesce and extend from the north pole of one magnet into the south pole of the other. If like poles face each other, as at (*b*), the lines of force curve away

from one another, as though they were pushing the poles apart. A similar action occurs when like poles are placed so that their axes make an angle with each other, as at (c).

The imaginary lines or paths formed by the filings are caused by the magnetic force of the magnet, which by induction, makes each little particle of iron a temporary magnet, and which immediately arranges itself so that its axis lies in (coincides with) the line of force passing through it. Also each particle attracts another particle, and thus makes a continuous chain that outlines the line of force.

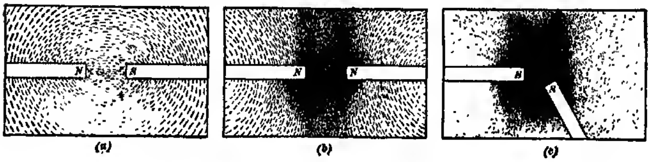


FIG. 24.

89. Nature of Magnetism.—Just what magnetism is, is not known; but the relation between magnetism and electricity is very close, in fact they may be different forms of the same thing. It is certain that magnetism is not a fluid, though some of its properties resemble those of fluids; and because of this, field density is frequently called **flux density**, the word flux meaning *flow*. The term **magnetic flux** means all the lines of force in the field, while flux density means the number of lines of force per square unit (square inch or square centimeter) at the point of the field considered, the value being different for different parts of the field.

Magnetism is not a material substance; for, if a magnet be used to make another magnet, as described in Art. **78**, the original magnet retains all its magnetism; in other words, nothing material passes from the first magnet to the second. In fact, the same magnet may be used to magnetize any number of magnets without losing *any of its own magnetism.*

90. Permanency of Magnets.—All permanent magnets will, in time, lose a part of their magnetism unless they are protected by an armature or keeper. The **armature** is a piece of soft iron laid across the pole-pieces of a U-shaped or a horseshoe magnet, as shown in Fig. 25. The lines of force pass from the north pole

of the magnet into the armature, through the armature to the south pole, and then through the body of the magnet to the north pole again. A magnet thus protected by an armature will retain its magnetism at full strength indefinitely.

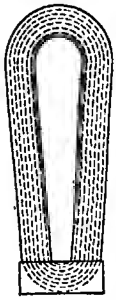


FIG. 25.

If a piece of soft iron be placed anywhere in a magnetic field, the lines of force tend to crowd together and pass through the iron, because the iron offers very much less resistance than air or any other non-magnetic substance, and the iron becomes a temporary magnet by induction. But, if the iron be free from impurities, it will lose its magnetic properties as soon as the magnetic stream or flow ceases or the iron is removed from the magnetic field. The iron tends to hold the magnetic flux in the same manner that a pipe holds water that is flowing through it.

ELECTROMAGNETISM

91. Magnetic Field Around Conductor.—Suppose a wire conductor, which may be either bare or insulated, be passed vertically through a sheet of paper that is kept in a horizontal position, as indicated in Fig. 26, where the white dot represents a cross-section of the conductor, and that iron filings are sprinkled over the paper. If, now, an electric current be sent through the conductor, the iron filings will arrange themselves about the conductor in concentric circles; in other words, the space about the conductor becomes a magnetic field, and the current induces magnetism in the filings. If a number of sheets of paper are strung along the wire, iron filings being sprinkled on each, the same result will be observed on all of the sheets, as indicated in (*a*), Fig. 27. To prove that a magnetic field exists, place a compass near the conductor, and the needle will be deflected, the needle pointing in the direction of the lines of force, the axis of the

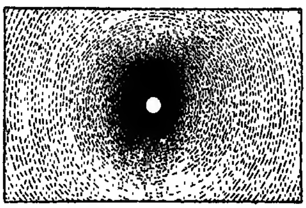


FIG. 26.

needle lying in a line tangent to one of the circles. If the current in the conductor flows as shown by the arrow from m toward n, the compass indicates that the lines of force have the same direction as the motion of the *hands of a watch;* but if the current is reversed, flowing from n toward m, the direction of the lines of force is also reversed, and their direction is *opposite to that of the hands of a watch.* In the first case, the direction of the lines of force is called **clockwise** or **right-hand rotation,** and in the second case, **counterclockwise** or **left-hand rotation.**

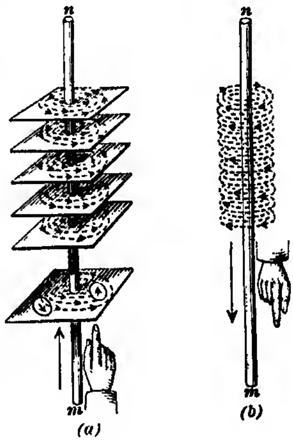


FIG. 27.

The field density is greatest at the surface of the conductor and decreases as the distance from the conductor increases. The lines of force form concentric circles about the conductor, and throughout its whole length, as indicated in (*b*), Fig. 27; as in the case of magnets, they do not (cannot) intersect.

92. Direction of Lines of Force.—It is important to know the direction of the lines of force around a conductor, and this may always be determined by the following rule:

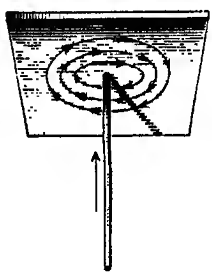


FIG. 28.

Rule.—*Place the index finger parallel to the conductor and point it in the direction that the current is flowing; the lines of force will then be clockwise around the conductor.*

Thus, in (*a*), Fig. 27, the current is flowing from m toward n; pointing the index finger in this direction, as shown, the lines of force are clockwise, as indicated by the compasses and arrowheads. In (*b*), the direction of the current is from n toward m; the lines of force are clockwise relative to the direction in which the finger is pointing, but they are counterclockwise relative to (*a*). If the two conductors be considered as lying in a horizontal

position and the observer be considered as looking from m toward n in both cases, then, when the current is flowing away from the observer, the direction of the lines of force are clockwise; but, if the current is flowing toward the observer, the direction of the lines of force is counterclockwise.

The watch or clock is always supposed to be *behind* the lines of force; it is for this reason that the arrowheads in Fig. 27 (*a*) are reversed, the direction in which the reader is looking at the picture being down instead of upward, in the direction of the arrow. The correct representation would be as shown in Fig. 28.

93. Attraction and Repulsion between Conductors.—If two conductors, both carrying a current, are placed near each other, the conductors being parallel (or nearly parallel), they will either attract or repel each other, according to whether the currents are flowing in the same or in opposite directions. In (*a*), Fig. 29, the currents are flowing in the same direction. As the lines of force spread out from the conductors, they meet and blend (coalesce); they also tend to shorten, and this produces a pull that tends to bring the conductors together; that is, they attract each other. In (*b*), the currents are in opposite directions; the lines of force are also in opposite directions, being clockwise about one conductor and counterclockwise about the other; they therefore cannot blend or coalesce, and since they cannot intersect, they are bent out of their natural position; the force required thus to distort them tends to push the conductors apart, and they therefore repel each other.

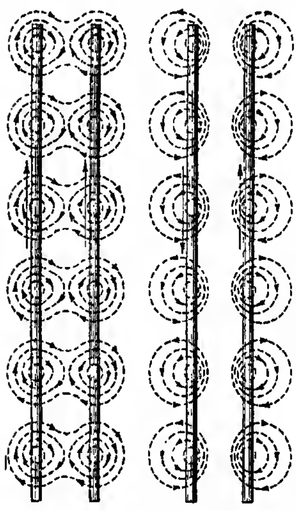


FIG. 29.

94. Direction of Current in a Conductor.—Suppose a compass to be placed so as to point north and south, that is, the ends of the needle lie directly over the N and S marks on the dial. If,

now, a conductor carrying a current be held *over* and parallel with the needle, and the direction of the current be *from south* to north, the north pole of *the needle will* be deflected toward the *west*. The reason for this will be clear after studying Fig. 30. Before deflection, the needle is pointing in the same direction as the wire, and the lines of force about the wire are at right angles to the lines of force passing out of the north pole and into the south pole of the needle, which is a magnet. As a consequence, the needle swings so that

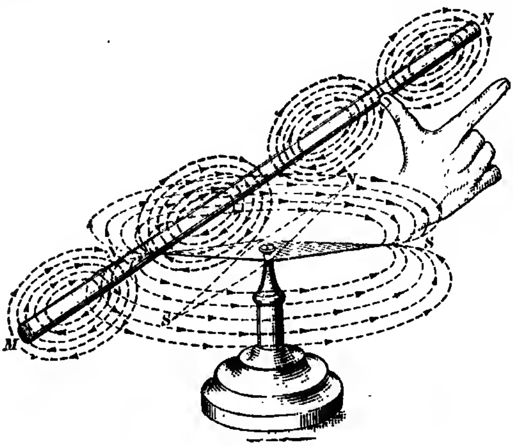


FIG. 30.

the lines of force in its magnetic field will blend or coalesce with those about the conductor. As shown in the figure it must necessarily swing toward the west, since if it swung the other way (toward the east), the lines of force in one field would oppose those in the other. Notice that when the conductor is over the needle, the *bottom* parts of the circles representing the lines of force about the wire coalesce with those *above* the needle. If, however, the compass be held above the conductor, the upper parts of the circles representing the lines of force about the wire coalesce with the lines of force under the needle, and this makes the north pole of the needle swing toward the *east*. If the direction of the current in the conductor be reversed, flowing from *n* toward *m*, the direction in which the needle points will also be reversed in the two cases.

If, therefore, the conductor is *over* and parallel to the needle and the right hand be held so the index finger points north (parallel with the conductor) and the thumb is placed at right angles to the conductor, as shown in the figure, with the back of the hand up, then, when the current is flowing from south to north (from m to n), the needle will point in the same direction as thumb; but if the current is flowing the other way, from n to m, the needle will point in the opposite direction. Hence, to find the direction of the current, place the conductor so that it will lie in a general north and south direction, place a compass under it, and in front of the observer; then if the needle points toward the west the current is *away* from the observer; but if the needle points toward the east, the current is flowing *toward* the observer. When speaking of the needle "pointing," it is always understood to mean the direction in which the north pole points.

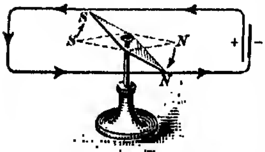


FIG. 31.

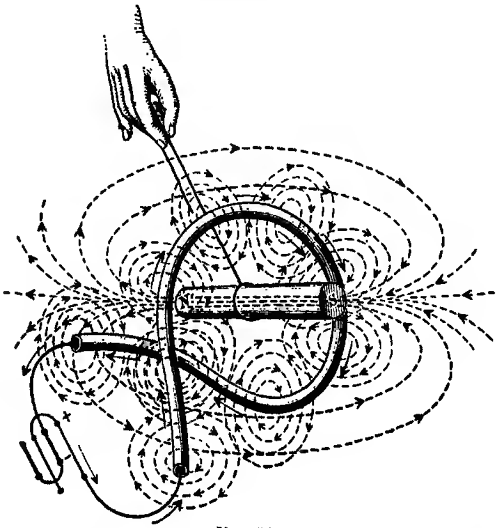


FIG. 32.

95. If the conductor form a loop so it can pass over and under the compass needle, as shown in Fig. 31, the direction of the cur-

rent in the upper wire will be opposite to that in the lower wire, and both wires will act to turn the needle in the same direction. In the figure, the current is produced by the cell shown at the right, and the direction is indicated by the arrowheads. The needle will evidently turn in the direction of the arrows.

96. The Solenoid.—If a conductor carrying a current be bent into a loop and placed in, say, a vertical position, as in Fig. 32, and a piece of soft iron be suspended from a string attached to the iron at its center of gravity, the iron will turn until its axis coincides with or is parallel with the axis of the loop. The reason for this is that the lines of force about the conductor all try to pass through the iron, which is much more *permeable*, as it is termed, than the air; they do not all pass through, but many

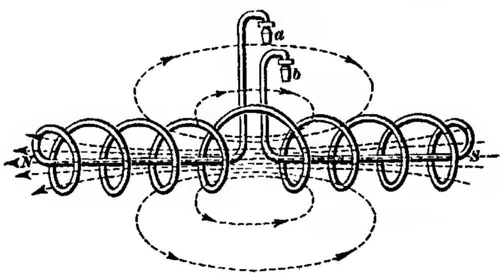


Fig. 33.

of them do, with the result that they are distorted and are no longer circles, but take the shape of the lines of force passing through a magnet. The piece of iron then becomes a temporary magnet, having a north and south pole.

This effect is better shown by bending the wire into a number of loops or coils, all having the same diameter, the general shape being that of a helical spring. By returning the ends of the wires through the helix and out at the middle, as shown in Fig. 33, and suspending from pivots at the ends, without breaking the circuit, the helix will be found to have the properties of a magnet; it will have a north and south pole, a neutral line, and it will swing so as to point in a north and south direction, the same as a compass; it will also attract and repel similar coils or magnets. Whenever a conductor is coiled into the form of a helix, the helical part is called a **solenoid.**

97. Poles of the Solenoid.—Referring to Fig. 32, note that the loop is wound like one turn of a right-hand helix or screw thread. The direction of the lines of force around the conductor is indicated by the arrowheads. The piece of iron distorts and bends these lines, causing them to enter the iron at the end marked S and leave at the end marked N. The same thing happens when the wire is bent into a number of coils of the same kind, as in Fig. 33, except that the lines of force in the adjacent coils coalesce, making numerous very long lines and increasing the magnetic properties of the solenoid. If, however, the conductor be reversed, so that the coil or coils form a left-hand helix (corresponding to a left-hand screw thread), the direction of the current in the conductor will be opposite to that in the above case, the direction of the lines of force will be reversed, and what was the north pole of the solenoid then be the south pole. Therefore, if the direction of the current be known, the poles of the solenoid may be identified by the following rule:

Rule.—*Looking through the solenoid from the end at which the current enters, if the helix winds clockwise away from the observer, the conductor is wound into a right-hand helix, the current flows around in the direction of the hands of a watch, and the end nearest the eye is a south pole. But, if the helix winds counterclockwise away from the observer, the conductor forms a left-hand helix, the current flows around in a counterclockwise direction, and the end nearest the eye is a north pole.*

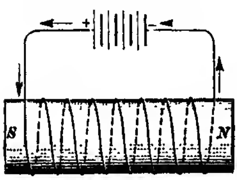


Fig. 34.

In practice, the best way to determine the polarity of a solenoid is to hold a compass near one end; if the needle is repelled, that end of the solenoid is of the *same* kind as the end of the needle that is repelled (since like poles repel each other); otherwise, it is of opposite polarity.

98. The Electromagnet.—If the conductor be wound around a cylindrical iron or steel bar, as shown in Fig. 34, the bar, which is called a **core**, attracts the lines of force and greatly increases the magnetic properties of the solenoid. The bar becomes a magnet, and if made of steel and the current is continued for any length of time, it will become a permanent magnet. If made of iron, the strength of the magnet will be greater than if made of

steel, but it will lose practically all its magnetism as soon as the current ceases to flow. Magnets made in this manner are called **electromagnets.**

The strength of an electromagnet depends upon the strength of the electric current, upon the number of turns of wire in the coil, and upon area of cross section of the core. In practice, the coils are made up of a very large number of turns of very fine insulated wire, preferably, copper wire. The wire is insulated to prevent a short-circuiting of the current; but *the insulation does not insulate the lines of force*—it insulates the current only. The wire is small in order to increase the voltage of the current. A small wire offers greater resistance than a larger wire; and since by Ohm's law, $E = IR$, it follows that if the strength of the current remains the same, increasing the resistance increases the e.m.f. of the circuit.

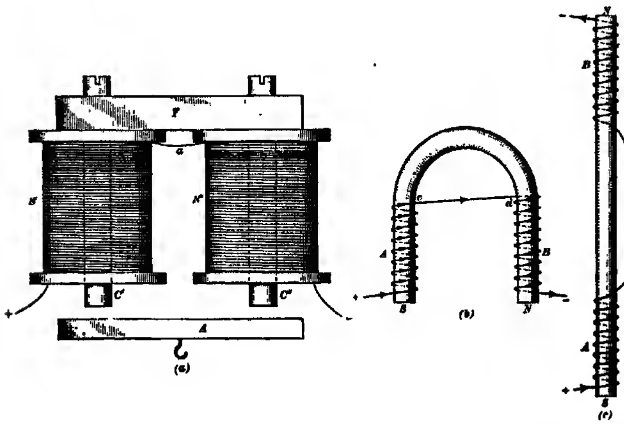


FIG. 35.

99. Electromagnets are made in many forms, one of which is shown at (*a*), Fig. 35. Here S' and S'' are spools or bobbins made up of a very great number of turns of fine, insulated copper wire. C' and C'' are soft iron cores, which pass through the spools and are held in place by the yoke Y. A is a soft iron armature, or keeper, to which is attached a hook, from which weights or loads may be hung. The wire is continuous, and after being wound around one spool, is continued around the other, the connection between the two being at a. The manner in which

the winding is done is shown at (*b*), which represents a bar bent into the form of a U. The wire is wound around *A*, passes over the top at *c*, then under *B*, at *d*, and around down to the end in the *opposite* direction. Observe that both coils are right-hand helixes, as they must be since the bar is a continuous one; if straightened out, it would look, with its winding, as shown at (*c*). According to the rule of Art. **97**, the end *A* is the south pole and the other end is the north pole. Had the winding been in the form of a left-hand helix, the poles would have been reversed. Note that the arrowheads on the two coils point toward one another; they indicate the direction of the current.

Contact of the armature with the poles of the magnet greatly increases the strength of the magnet, because practically all the lines of force pass through the armature and, consequently, through the poles. If, however, the armature is separated from

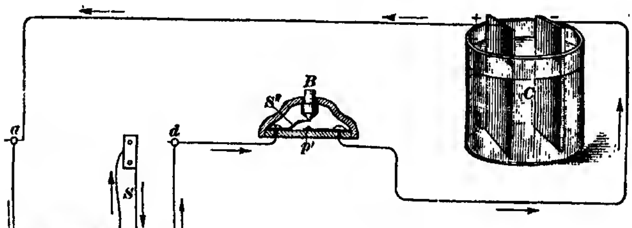
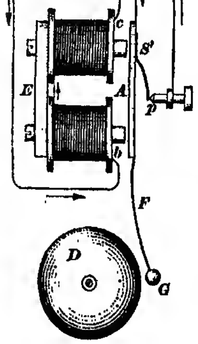



Fig. 36.

the poles by an *air gap*, many of the lines of force are diverted from the armature, and the number passing through the poles is smaller, which lessens considerably the strength of the magnet.

100. The Electric Bell.—One of the practical uses of the electromagnet is the electric bell. Referring to Fig. 36, *C* represents a cell, either wet or dry, but preferably a dry cell. *E* is an electromagnet, whose armature *A* is attached to a spring *S* that ordinarily keeps the armature from contact with the poles of the magnet. The armature carries a spring *S'*, which presses against the pivot *p*. The push button *B* is kept from contact with the pivot *p'* by the spring *S''*, thus leaving the circuit open. When

the button B is pushed down, the circuit is closed; the current flows from the positive electrode of the cell to the binding post a; enters the magnet at b and leaves at c; flows along the spring S and armature A to pivot p, to binding post d, and thence back to the negative electrode of the cell. But as soon as the current passes through the magnet coils, the poles of the magnet draw the armature A into contact with them; this takes the spring S' away from the pivot p and breaks the circuit. The coils then cease to be a magnet, and the spring S draws the armature away to its former position, spring S' comes into contact with p, the circuit is again closed, and the armature is again drawn to the poles. The armature carries a clapper F, the end G of which strikes the bell every time the armature is drawn to the poles. Therefore, the bell will ring as long as the push button B is held down. On releasing it, the circuit is open, and the bell will no longer ring.

101. Magnetic Leakage.—All the lines of force induced in an electromagnet do not follow a single path; some of them stray and take shorter paths, even though they have to pass through

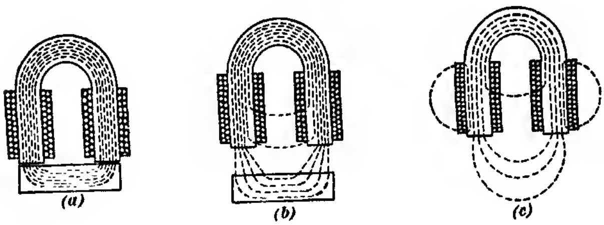


FIG. 37.

the air. If, however, the armature is in contact with the poles, practically all the lines of force are confined to the core and armature, as shown diagrammatically in Fig. 37 at (a). If the armature is at some distance from the poles, as shown at (b), some of the lines will fail to cross the air gap to the armature, as indicated. The total number of lines of force, the flux, will be less also. Assuming that the same magnet is used in both cases, the lifting or attractive power of the magnet in the second case is less than in the first case. If there is no armature, the result is shown diagrammatically at (c). Here there are more "stray" lines of force and a much smaller number of lines of force in the core; conse-

quently, the magnet in the third case is much weaker than in either of the two other cases.

Those lines of force that do not pass through the poles have no effect on the lifting or attractive power of the magnet; they constitute what is called the **magnetic leakage.** The magnetic leakage depends upon the magnetic substance composing the core of the magnet, the uniformity of the material (freedom from and distribution of impurities), whether or not there is an armature, whether or not it is in contact with the poles, and if not, upon the length of the air gap between the armature and the poles. The shorter the air gap the less is the magnetic leakage, and the greater the strength of the magnet. This confirms what was stated in Art. **99.**

ELECTRICAL MEASURING INSTRUMENTS

MEASURING CURRENT

102. Galvanometers.—Any instrument that measures electric currents by the effects produced by the electromagnetic action is called a **galvanometer.** The word means *galvanic* measurer, and

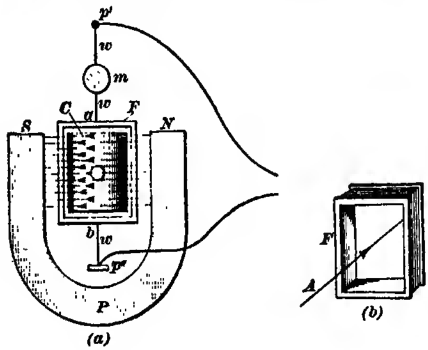


FIG. 38.

was named after Alvisio Galvani. There are many kinds and makes of galvanometers, but only the D'Arsonval galvanometer named after its inventor, will be described here. The instrument is constructed as follows:

Referring to Fig. 38, at (a) is shown a permanent magnet P, which stands in a vertical position. Between the poles of the magnet is suspended from a fine silver wire a light, hollow rectangular frame, shown in detail at (b); the thickness of the frame is quite small as compared with its height and breadth. The frame is wound with many turns of fine, insulated wire, thus making it a solenoid. The wire suspension w is pulled taut between the supports p' and p'', so as to support the solenoid and leave it free to turn. The solenoid is arranged relative to the poles of the magnet as shown at (a), with the axis A of the solenoid making an angle with the direction of the lines of force between the north and south poles of the magnet. Within the solenoid, is a soft-iron, cylindrical core C, which acts as an armature to attract lines of force and makes a strong, uniform magnetic field. The silver suspension wire is connected to one end of the coil at a and to the other end at b, and it is also connected to the circuit at p' and p''; thus when a current is flowing through the circuit, it passes through the coil of the solenoid. Since the lines of force through the solenoid make an angle with those passing between the poles of the magnet, the two sets of lines of force tend to coalesce, thus causing the solenoid to turn and twist the suspension wire w. When the current is shut off (by opening the circuit), the wire untwists, and the solenoid returns to its former position. Since the force required to twist the wire increases with the amount of twist, and since the force causing the twist increases with the strength of the current, it is evident that the angle turned through by the solenoid increases or decreases as the current increases or decreases. To measure the angle, a pointer or small mirror m is attached to the suspension wire. A beam of light thrown on the mirror from some point in a line perpendicular to the plane of the mirror will be reflected back to the point from which it came; but, as the solenoid turns, the mirror turns also, and the beam of light is reflected to some other point. Knowing the strength of the current at different times and marking the points to which the light reflects, a scale can be constructed that will measure the strength of an unknown current.

Instead of a mirror, a pointer may be attached to the wire, and a scale for measuring unknown currents may be constructed in a similar manner.

103. As previously stated, galvanometers are made in many forms. While any instrument used to measure currents by

means of its electromagnetic action is a galvanometer, the term is usually restricted to instruments used in laboratories and in precise measurements. As will presently be shown, a galvanometer can be so constructed that it will measure the e.m.f. of a current in volts, in which case, it is called a **voltmeter**; or, it can be so constructed that it will measure the strength of a current in amperes, in which case, it is called an *ampere meter* or **ammeter**, the latter being the name in commercial use.

104. The Weston Ammeter.—The Weston ammeter is a special form of the D'Arsonval galvanometer; its construction is shown in Fig. 39. A perspective view of the entire instrument is shown

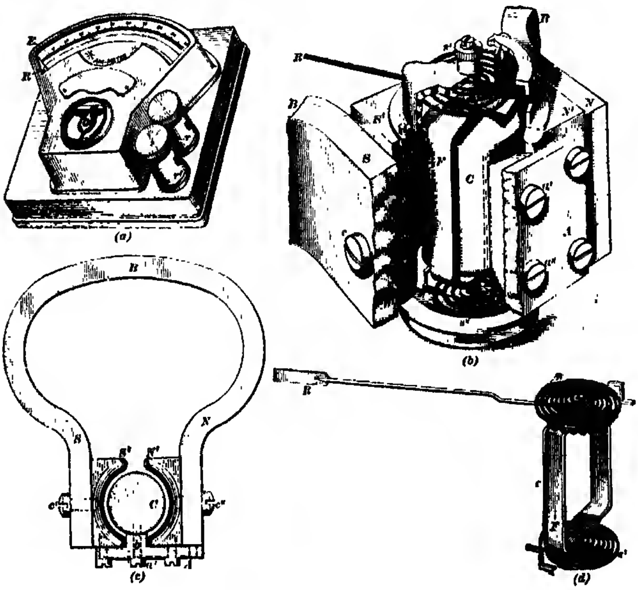


FIG. 39.

at (*a*); a perspective of the working parts, with a portion removed, is shown at (*b*); a plan view showing the magnet and soft-iron core is given in (*c*); and (*d*) is a perspective showing the solenoid, springs, and pointer. Referring to (*b*) and (*c*), *B* is a permanent magnet, which has attached to its poles *N* and *S* soft-iron pole pieces *N'* and *S'*, the inner surfaces of which are curved to a circular arc. A brass plate *A* is screwed to the outer ends

of the pole pieces and carries a lug b to which the round iron core C is attached by the screws a' and a''. The core C fits inside the solenoid F, shown at (d), in the manner illustrated in (b). The spiral springs s' and s'', at the top and bottom of the solenoid, tend to prevent the solenoid from turning. To the upper part of the solenoid is attached a light aluminum pointer R that moves over the graduated scale E, shown in (a). When the circuit is open, the springs bring the pointer to 0, the left-hand end of the scale which indicates zero; but when the circuit is closed, the coil (solenoid) turns, as explained in Art. **102**, carries tho pointer with it, against the resistance of the springs, and the reading of the scale shows the strength of the current in amperes. The action is exactly the same as in the D'Arsonval galvanometer. The current enters the instrument at the binding post r, which is marked + on the instrument, flows through the solenoid, and leaves the instrument at the binding post r'.

The distinguishing feature of any ammeter is its very low internal resistance; this is necessary, in order that the full strength of the current may pass through it. A Weston ammeter of the kind just described, and which will indicate up to 15 amperes, has an internal resistance of only .0022 ohm; hence, when indicating to full capacity, the drop in voltage between the two binding posts is, by Ohm's law ($E = IR$) only $15 \times .0022 = .033$ volt, and the loss in power is only ($P = I^2R$) $15^2 \times .0022 = .495$ watt, say half a watt. For lower values of I, the loss is much less; thus, for 5 amperes, the drop in voltage is only $5 \times .0022 = .011$ volts, and the loss in watts is $5^2 \times .0022 = .055$ watt. Note, however, that the entire current goes through the instrument.

105. Ammeter Connections.—All the current to be measured must flow through the ammeter; hence, the ammeter must be connected in *series* with the apparatus receiving the current to be measured, preferably *between* the apparatus and the source from which the current comes. Suppose, for example, that it were desired to measure the current that flows through the lamp L in (a), Fig. 40. If C is the conductor carrying the current from the source to the lamp, the ammeter must be placed on this wire, the binding post marked + being connected to that part coming from the source and the other binding post to that part leading to the lamp. All the current received by the lamp then passes through the ammeter.

If the connections at the binding posts were reversed, the pointer would tend to turn in the opposite direction, and the ammeter might either be damaged or destroyed.

Suppose that by mistake the ammeter were connected in parallel with the lamp, as shown at (b), Fig. 40. If the e.m.f. of the circuit were 110 volts and the internal resistance of the am-

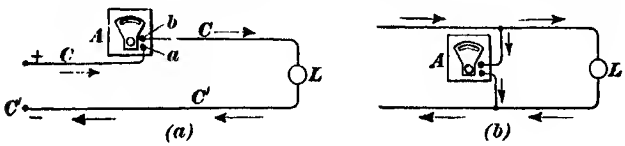


FIG. 40.

meter were .0022, the strength of the current passing through the instrument would be, by Ohm's law, $I = \frac{E}{R} = \frac{110}{.0022} = 50{,}000$ amperes, which would completely destroy the ammeter by melting the resistance.

106. The Weston Voltmeter.—The distinguishing feature of a voltmeter is the extremely high internal resistance, which is necessary in order to make the current flowing through it exceedingly small. The resistance of a voltmeter that will record up to 150 volts is 18,000 ohms. Therefore, by Ohm's law, when registering 110 volts, the strength of the current that flows through it is only $I = \frac{E}{R} = \frac{110}{18{,}000} = .00611+$ ampere, and the power it absorbs in watts is $P = \frac{E^2}{R} = \frac{110^2}{18{,}000} = .672+$ watt.

The Weston voltmeter, which is illustrated in Fig. 41, resembles very closely the Weston ammeter. The chief difference in external appearance is in the position of the binding posts, one being placed on either side of the magnet poles instead of both on one side, as in the ammeter. The solenoid is wound with finer wire, and there are many more turns. The current enters the instrument at the binding post r', which is marked $+$, and then passes to a high resistance R by the wire a; from R, it goes to the solenoid F by wire b, and leaves the solenoid and the instrument by wire c, which connects with the other binding post r''. The instrument frequently has a switch S, in the form of a push button, so that the instrument records only when the button is pushed, thus closing the circuit. The principal dif-

ference between the Weston voltmeter and the ammeter is the resistance R and the fact that the solenoid is wound with many more turns of finer wire; otherwise, the two instruments are practically alike in their mechanical construction.

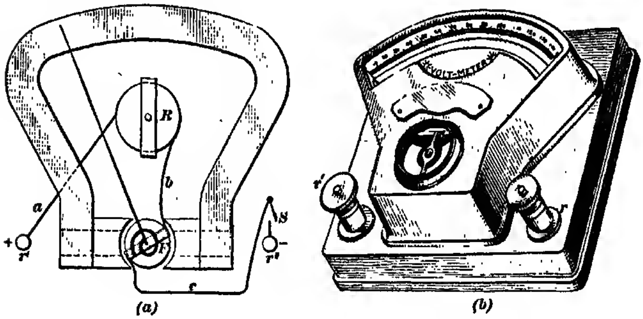


FIG. 41.

107. Voltmeter Connections.—A voltmeter must always be connected in parallel (multiple) with the apparatus whose e.m.f. it is desired to measure; the e.m.f. of the voltmeter will then be the same as that of the apparatus (Art. **37**). Thus, referring to Fig. 42, suppose it were desired to find the voltage of the lamp L. At a, a point near to where the current enters the lamp, a connection is made to the binding post marked +; at b, a point near to where the current leaves the lamp, a connection is made to the other binding post. The current thus divides at a and unites at b, and the *voltage* of the instrument is the same as that of the

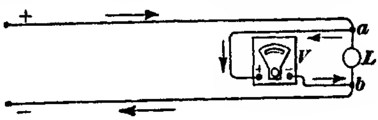


FIG. 42.

lamp. Observe that in the case of the ammeter, both connections are to the same (the +) wire; but, in the case of the voltmeter, the connections are made to both (the + and the −) wires of the circuit.

108. Watt Meters and Watt-Hour Meters.—Knowing the current in amperes and the e.m.f. in volts, the watts (electric power) can be found by multiplication, since $P = IE$, Art.

47. Insofar as the individual user of electric power is concerned, he does not wish to know the power for a particular time, but at any time; for this reason, instruments called **watt meters** are manufactured. The details of their construction and operation are somewhat complicated and will not be described here, further than to state that a watt meter does not have a permanent magnet, but two sets of coils (solenoids), one being connected to the circuit in series (like an ammeter) and the other being connected in multiple (like a voltmeter). One of the coils is free to turn, while the other is fixed and takes the place of a magnet. The effect is such that the pointer, as it moves over the scale, indicates watts instead of volts or amperes. For measuring the electrical energy supplied to houses, offices, etc., what are called **watt-hour meters** are employed. In these instruments a set of gears is caused to rotate; and since each revolution is proportional to a certain number of watt-hours, the number consumed in a given time is recorded in much the same manner as in a gas meter. As previously stated, the watt-hour indicates a certain number of units of work or energy, and is the basis on which the consumption of electricity is bought and paid for.

109. Detector Galvanometer.—When a galvanometer is so made that it merely detects the presence of a current and indicates its direction, but does not measure its value in amperes or

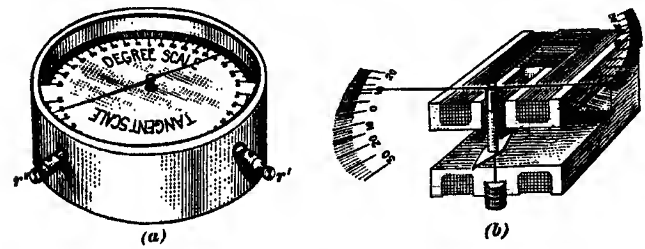


FIG. 43.

volts, it is called a **detector galvanometer**. One form of such an instrument is shown in Fig. 43. A perspective view is shown at (*a*), r' and r'' being the binding posts. At (*b*) is shown one-half of the solenoid, which consists of two coils, both wound in the same direction, and formed of a large number of turns of very fine, insulated wire. The middle part of the frame on which the coils are wound is cut out, to permit the insertion of a magnetic

needle and its staff. The staff carries a long aluminum pointer at its upper end, the pointer being placed at right angles to the axis of the needle. The dial has two scales, one being divided into degrees and the other into parts that correspond to the tangents of the angles indicated by the degrees. One of the halves of the pointer moves over the degree scale and the other over the tangent scale. When no current is flowing through the coils, the pointer rests over the zero mark in the middle of both scales; but, when a current is flowing, the needle is deflected to the right or left of the zero mark, according to its direction (see Art. 95), a certain amount, the value of which depends upon the strength of the current. The amount of the deflection from zero does not vary in direct proportion to the angle turned through when measured in degrees, but is in proportion to the tangent of the angle.

To use the instrument, turn it until the pointer rests over the two marks indicated by 0; the needle then points north and south. Now make connection with the circuit, and if a current is flowing the pointer will move. Assuming that the coils are wound into a right-hand helix, a movement to the left of 0 indicates that the current is flowing away from the observer, while a movement to the right indicates that the current is flowing toward the observer.

The instrument may be used to *compare* the strengths of two different currents. Thus, suppose that on being connected to one circuit, the needle deflects 21 degrees; this corresponds to .38 on the tangent scale. Suppose, further, that on being connected to another circuit, the needle deflects 36 degrees; this corresponds to .73 on the tangent scale. The strength of the current in the second circuit is then $\frac{.73}{.38} = 1.92$ times that in the first circuit. If the strength of the currents in the two circuits be denoted by I' and I'' and the readings on the tangent scale by T' and T'', respectively, $I' : I'' = T' : T''$, or $I'' = I' \left(\frac{T''}{T'}\right)$ Since the value of I' is not generally known, the above proportion is best expressed by writing it in the form $\frac{I''}{I'} = \frac{T''}{T'}$; and since T' and T'' are read directly on the scale, this will give the ratio of the currents in the two circuits.

In one make of this instrument, the coil is wound with No. 30

B. & S. wire of such length as to give a resistance of about 30 ohms; it is so sensitive that a current of .00001 ampere will deflect the needle about 1 degree. These instruments should be handled very carefully.

MEASURING RESISTANCE

110. Rheostats.—It is frequently desirable to increase or decrease the strength of the current flowing through a circuit or a shunt or to obtain a current of some particular strength. From Ohm's law, $I = \frac{E}{R}$, and if E, the e.m.f. of the circuit or shunt, remains the same, the value of I, the strength of the current in amperes, can be changed by changing the resistance R. Any device for changing R without opening the circuit is called a **rheostat,** which is derived from two Greek words—rheo (to flow) and statos (standing, stop); the word therefore literally means flow-stopper. An *adjustable* resistance or an apparatus for varying the resistance is a rheostat; it slows down the current by absorbing electrical energy, which heats the resistance and is thus prevented from doing useful work.

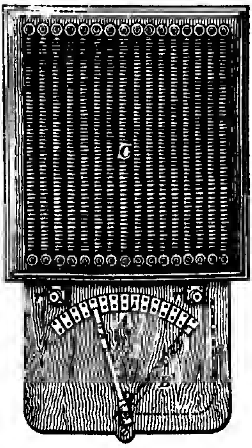


FIG. 44.

111. A form of **sliding contact rheostat,** to be placed against a wall, is shown in Fig. 44. There are 16 coils, the resistances of which are known, and each coil is connected to one of the contact pieces A, which are arranged in an arc of a circle. The first two (left-hand) coils are connected to the left-hand contact piece, the third coil is connected to the next contact piece, and so on, all the coils being connected in series. The current enters at the binding post r', passes through the wire a to the end b of the swinging arm B, then through the arm to the contact piece on which it rests, thence to the coil connected to the contact piece and all the other coils to the left of it; the first coil on the left connects to the other binding post r'', which is connected with the circuit.

With the arm in the position B′, the current has to pass through all the coils; but, as the arm is moved to the left, the coils are gradually cut out. When the arm is in the position B, only five of the contact pieces are in series, that is, the current passes through only 5 + 1 = 6 coils.

An adjustable resistance of a nature similar to the foregoing is frequently called a **resistance box**. Since the resistance of each coil is known, the strength of the current can be varied almost at will.

112. The Wheatstone Bridge.—A special form of rheostat that is used for measuring an unknown resistance is known as the **Wheatstone bridge**. To understand the principle of its operation, consider the diagram, Fig. 45. The current is supplied by a battery B; it flows from B to a, where it divides, a part going through the upper shunt U–X and the remainder through the lower shunt L–A. Both shunts are considered as being divided into two parts, the upper into U and X and the lower into L and A. U is called the **upper balance arm**, L the **lower balance arm**, A is called the **adjustable resistance**, and X is the **unknown resistance**, which is to be measured. The e.m.f. in both shunts is the same (see Art. **37**), the resistance in the arms U, L, and A is known, and the resistance X can be found by proportion, as will now be shown.

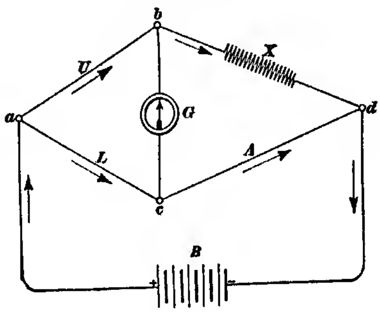


Fig. 45.

The fall of potential between the point a, where the current divides, and d, where it unites, is the same for both shunts, and for the same *proportionate distance* from a, the fall of potential is the same in both shunts. Denote the fall between a and b by U, between b and d by X, between a and c by L, and between c and d by A; then, when $U : X = L : A$, the fall of potential at b is equal to the fall of potential at c. Connecting a sensitive galvanometer at b and c, as shown, the current will divide at b whenever there is a difference of potential between b and c; but, if there is no such difference, no current will flow between b

and c. Therefore, when the current passes from the battery to a, there will usually be a movement of the galvanometer needle; but, by adding to or cutting out resistance in the adjustable arm A, the needle can be made to point to 0, in which case, the e.m.f. at b will be the same as at c, no current flows through the galvanometer, and $U : X = L : A$, from which $U \times A = L \times X$, or

$$X = A \times \frac{U}{L}$$

Thus, if the resistance $U = 100$ ohms, $L = 10$ ohms, and $A = 547$ ohms, the unknown resistance is $X = 547 \times \frac{100}{10} = 5470$ ohms. Again, if the resistance $U = 10$ ohms, $L = 100$ ohms, and $A = 206$ ohms, $X = 206 \times \frac{10}{100} = 20.6$ ohms.

113. In practice, the resistances are made up of standard coils of known resistance and wound non-inductively. If a wire (insulated) be folded in the middle and both parts wound together into a coil, the current induces lines of force that circulate around the two parts in opposite directions, thus preventing any magnetic action; in other words, the coils will not be solenoids, and they are then said to be wound **non-inductively.** The contact pieces, which form the arms, have a rectangular cross-section and are of sufficient area to offer very slight resistance. Connection is made by plugs p, as indicated in Fig. 46. One end of the coils is connected to one contact piece and the other to the next adjacent one, as indicated by the top view in Fig. 47. When a plug is *in*, the current flows through it instead of through the coil, the resistance of which is much higher; but, when a plug is *out*, the current must flow through the coil to complete the circuit through the arm. Hence, a resistance is thrown into the circuit only when a plug is *out*. Referring now to Fig. 47, the arms of the bridge are arranged somewhat in the form of a letter M. The wire from the battery connects at a, where the current divides, a part going to b, where connection is made with the galvanometer and the unknown resistance, and a part going to c, where connection is made with the galvanometer and the adjustable resistance. Evidently, ab corresponds to the upper arm, Fig. 45, ac corresponds to the lower arm, and *cefghid*

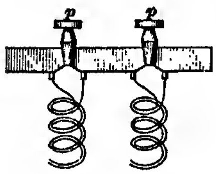


FIG. 46.

corresponds to the adjustable arm. For convenience, the upper and lower arms are also made adjustable, so as to allow for a wide variation in the unknown resistance, there being three resistances in each of 10, 100, and 1000 ohms respectively. The coils in the adjustable arm have resistances of 1, 2, 2, 5; 10, 20, 20, 50; 100, 200, 200, 500; and 1000, 2000, 2000, 5000, 10,000 ohms. The sum of these resistances is 21,110 ohms, and the plugs may be so set as to throw into the circuit from the adjustable arm any resistance expressed by an integer from 1 ohm to 21,110 ohms, and the range of the instrument is from .01 ohm to 2,111,000 ohms. Thus, if the 10 plug is out in the upper arm, the 1000 plug in the lower arm,

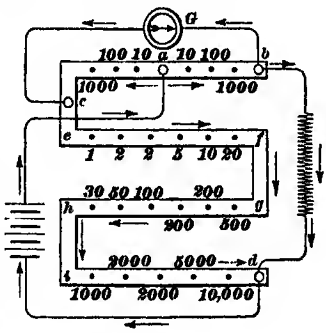


FIG. 47.

and the 1 plug in the adjustable arm, $X = A\left(\frac{U}{L}\right) = 1 \times \frac{10}{1000}$ $= .01$ ohm; or, if the 1000 plug is out in the upper arm, the 10 plug in the lower arm, and all the plugs are out of the adjustable arm, $X = 21{,}110 \times \frac{1000}{10} = 2{,}111{,}000$ ohms.

114. Measuring Resistance with an Ammeter and Voltmeter. The resistance of any part of a circuit may be determined with a voltmeter and ammeter and an application of Ohm's law. Thus, let the diagram, Fig. 48, represent a circuit, S being the source of the current (battery or dynamo), and suppose it is desired to measure the resistance between a and b, the resistance consisting of one or more lamps, a motor, a rheostat, or anything else that offers resistance. Connect the ammeter in series with the resistance,

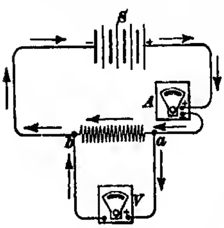


FIG. 48.

so that all the current that goes through the resistance goes through the ammeter; then connect the volt-meter in parallel with the resistance, the current being divided at a and united at b. Be sure that the current enters both instruments at the binding

posts marked +. Read both instruments at as nearly the same instant as possible. Suppose that the voltmeter reads 216 volts and the ammeter reads 4.8 amperes; then, by Ohm's law, $R = \frac{E}{I} = \frac{216}{4.8} = 45$ ohms, the resistance between a and b.

Example 1.—Referring to Fig. 47, suppose the 10-ohm plug between a and b, the 100-ohm plug between a and c, and the following plugs 1, 5, 50, 200, 500, 1000, and 2000, between c and d are out; what is the value of the unknown resistance?

Solution.—The total resistance represented by the plugs that are out in the adjustable arm is $2000 + 1000 + 500 + 200 + 50 + 5 + 1 = 3756$; hence, by Art. **112**, $X = 3756 \times \frac{10}{100} = 375.6$ ohms. *Ans.*

Example 2.—Suppose a voltmeter to be connected in parallel with the upper and lower carbons of an arc lamp and that an ammeter is connected in series with the same lamp. If, when the lamp is burning, the voltmeter reads 44 volts and the ammeter reads 9.9 amperes, what is the resistance of the lamp when hot?

Solution.—The e.m.f. of the current flowing through the carbons is 44 volts, and the strength of the current is 9.9 amperes; hence, by Ohm's law, $R = \frac{E}{I} = \frac{44}{9.9} = 4.44+$ ohms. *Ans.*

The result just obtained in the last example is called the **resistance hot** of the lamp. The resistance when the carbons are cold is considerably higher, because the resistance of carbon decreases as the temperature increases.

115. Ohmmeters.—What are called **ohmmeters** are instruments made on much the same principle as voltmeters. The scales, however, read ohms instead of volts. The connections for an ohmmeter are the same as for a voltmeter, that is, the instrument is connected in parallel with the resistance to be measured.

EXAMPLES

1. Show by a sketch how you would make an electro-magnet, indicating direction of current and the north and south poles.

2. Why does a solenoid behave like a magnet?

3. What is the purpose of a voltmeter? Of an ammeter? How is each connected in a circuit? What would happen to a voltmeter if it were connected like an ammeter?

4. How much work is required to run a grinder six days of 24 hours each if driven by a motor that takes an average of 460 amperes at 550 volts? Express the answer in kilowatts-hours and horsepower-hours.

Ans. 36,432 k.w.h.; 48,836 + h.p.h.

ELECTROMAGNETIC INDUCTION

116. Inducing a Current.—Let M, Fig. 49, be a magnet (either a permanent magnet or an electromagnet), and let AB be a conductor, the ends of which are connected to a sensitive galvanometer G, thus making a complete circuit consisting of the conductor AB, the wires Br' and $r''A$, and the galvanometer. Then suppose that the conductor be moved suddenly downward past and near one of the poles of the magnet, in this case, the north pole. The galvanometer needle will be seen to deflect, say to the right, thus indicating that a current has passed through the circuit. If the conductor be moved suddenly upward, the needle will deflect again, but in the opposite direction, showing that a current has passed through the circuit in a direction opposite to that which flowed through it when the conductor passed downward. When an electric current is created in this manner, it is said to be created by *induction*, and it is called an **induced current**, the entire process being called **electromagnetic induction.**

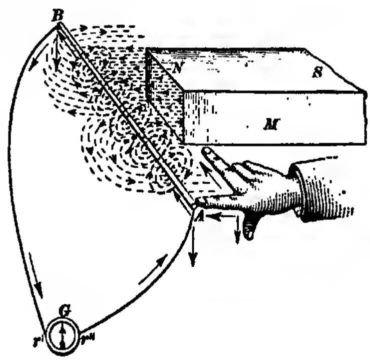


FIG. 49.

The reason for the flow of current is because the conductor as it passes through the magnetic lines of force issuing from the pole of the magnet has set up around it magnetic whorls, which correspond in every respect with those set up around a conductor when a current is flowing through it. If the circuit is complete, any conductor having magnetic whorls around it must have a current flowing through it. When the whorls are caused by lines of force coming from a magnet or electromagnet, as in the present case, the current is an induced current.

117. To determine the direction of the current, suppose the lines of force to be capable of being stretched and bent around. As the conductor moves down, the *under* side presses against the lines of force, stretching them and tending to bend them

around the conductor in the direction of the hands of a clock hence, the lines of force in the whorls will have a clockwise direction around the conductor, assuming that the observer be looking from A toward B, and the current will flow from A to B. (See Art. **92.**) When the conductor is moving upward, the *top* side presses against the lines of force, bending them around the conductor in a direction opposite to the hands of a clock, the lines of force in the whorls have a counterclockwise direction around the conductor, assuming the observer to be looking from A toward B, and the current flows from B to A. It may be considered that the lines of force in the whorls are the same as those of the magnet that were touched by the conductor, that they were stretched until they broke in two places, the ends uniting to make the lines of force in the whorls.

Another way to determine the direction of the current is to use Fleming's rule, usually called the *rule of the right hand.* Assume a bar magnet grasped at the north end, with the right hand, as shown in Fig. 50, with the index finger pointing in the direction of the lines of force, the thumb at right angles to the index finger, and the middle finger pointing downward at right angles to the thumb and index finger. If a conductor C, parallel or approximately parallel to the middle finger, be moved across the magnet in front of its pole and in the direction of the thumb, the direction of the current will be downward from A to B, in the direction pointed by the middle finger. If the direction of movement of the conductor be opposite to that here indicated, turn the hand and magnet over, so the palm will be upward and the thumb will point in the direction the conductor is moving; the middle finger will then point upward, showing that the current is moving from B to A. Hence, the rule:

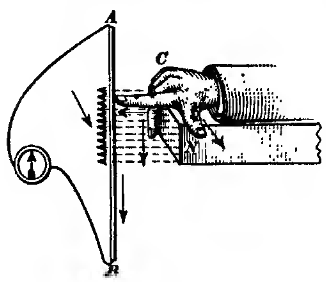


FIG. 50.

Rule.—*To determine the direction of the induced current, place the thumb and index and middle fingers of the right hand at right angles to one another; hold the hand so the index finger will point in the direction of the lines of force, the thumb in the direction of*

motion of the conductor, and the direction pointed to by the middle finger will be the direction of the current.

As an example, note the position of the fingers in Fig. 49. Applying the first method to Fig. 50, assume the observer is at *B;* the lines of force are bent around the conductor counter-clockwise; hence, the current is flowing toward the observer, that is, from *A* to *B*. The observer is supposed to be looking along the conductor from *B* toward *A*.

118. The creation of magnetic whorls by the breaking of the lines of force is called **cutting lines of force;** the more whorls there are about the conductor, the stronger will be the current passing through it; therefore, the greater the number of lines of force that are cut in a given time the stronger will be the current. If the circuit is not closed, as in the case of a wire moving across a magnetic field, no current can flow, but an electrical stress will be produced in the conductor; in other words, an e.m.f. will be generated. Consequently, the voltage, or e.m.f., also depends upon the number of lines force cut per unit of time, say per second.

119. Referring to Fig. 51, let *abcd* be a U-shaped frame made of some conducting material and supported between the poles of a magnet in such a manner that it will not cut lines of force; let *C* be a conductor resting on this framework, and cutting lines of force as it moves from *b* to *a* or from *a* to *b*. So long as *C* is stationary, no current will be generated, since it does not cut lines of force; but, when *C* moves, say, in the direction of the arrow toward *a*, it cuts lines of force, and according to the rule of the right hand, the current will flow from *f* toward *e*, thence to *b*, *c*, and *f*, again. That part of the conductor that cuts lines of force and is represented by *ef* is called the **internal circuit,** and the part *cbcf* of the frame through which the current flows is called the **external circuit.**

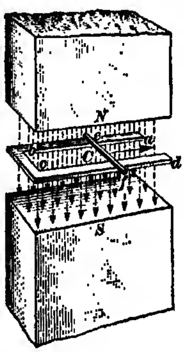


FIG. 51.

The number of lines of force cut by any movement of the conductor depends upon the flux density (see Art. **89**), the length of the internal circuit, the velocity of the conductor through the

field, and the direction in which the conductor cuts the lines of force. Thus, suppose that $be = cf = 6$ inches, and that the points e, b, c, and f all lie in a plane that is parallel to the planes of the pole faces. Suppose further that the lines of force are parallel straight lines. A movement of the conductor from b to e will then cut a greater number of lines of force than for any other position of the frame, because, the distance bc remaining the same, if the frame be tipped so bc lies lower than cf, the rectangle $ebcf$ will enclose a smaller number of lines of force than before; and the same will be true if cf be lower than be. Therefore, the conductor will cut the greatest number of lines of force for any particular distance traveled when it cuts them at right angles. Evidently, also, the longer the internal circuit the greater will be the number of lines of force cut; the greater the flux density the greater the number of lines of force cut; and the greater the velocity of the conductor the greater the distance it will travel in a unit of time, and the greater will be the number of lines of force cut in a unit of time.

120. It will be evident from the foregoing that an induced current is the result of motion; it does not matter whether the conductor is moved or whether the magnet is moved, so long as lines of force are cut. If either move so that lines of force are not cut, no whorls will be generated and no current or e.m.f. will result. For example, suppose the conductor be moved in the direction of its axis, that is perpendicular to the plane of the paper; no lines of force are cut and there will be no induced current or induced e.m.f. The same result will be obtained if the magnet be moved in the same direction, the conductor being stationary.

121. Inducing Current in Closed Coil.—Let $ABCD$, Fig 52, be a continuous conductor forming a rectangular-shaped coil; if passed between the poles of a magnet in the direction of the arrow so one side can cut lines of force, magnetic whorls will be set up around the conductor, and a current will flow in the direction of the arrows, Fig. 52. Suppose, however, that the length $CB = DA$ is small compared with the width of the pole faces, the result being that BA and CD are in the magnetic field at the same time. The whorls around BA are produced by the pressure of the *outside* of BA against the lines of force and are clockwise; the whorls around CD are produced by the pressure of the *inside*

of CD against the lines of force and are clockwise also. Consequently, the current in CD will be from D toward C, and since the current in BA is from A toward B, the two currents oppose each other. Therefore, if the flux density is uniform across the pole faces, making the flux density of the entire field uniform, there will be no current in the coil as long as the sides BA and CD are both in the field. If the field is not uniform, then that side which is in the part of the field of greater density will have the greater e.m.f., there will be a difference of potential, and the current will flow from that side having the greater e.m.f. to that side having the smaller e.m.f.

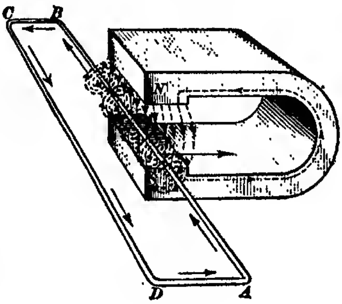


FIG. 52.

122. Suppose a circular coil to be placed in a uniform magnetic field, as indicated in Fig. 53, and suppose the coil to turn on the diameter ab, the half E moving down and the other half moving up, ab being horizontal. When the coil is in position (a), with the plane of its face parallel to the lines of force, none of them pass through the coil, and the slightest movement of the coil cuts them at right angles, thus generating an e.m.f. As the coil revolves, see (b), lines of force pass through the coil, and for the same amount of movement of the coil, the number of lines of force cut decreases; hence, the e.m.f. also decreases. The number of lines of force cut for a given arc moved through by the coil, and consequently, the e.m.f. generated, continue to decrease until the coil reaches the position (c), where the greatest number of lines of force pass through the coil and none are being cut for a slight movement of the coil. As the coil continues to turn, the number of lines of force being cut, and, consequently, the e.m.f. generated increase until they become a maximum when the coil again

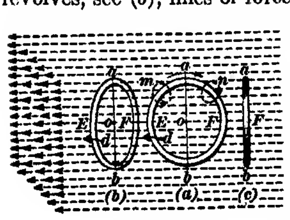


FIG. 53.

reaches the position indicated in (*a*), with its plane parallel to the lines of force, and cutting them at right angles. Since the number of lines of force cut is continually changing, the e.m.f. is also continually changing, and varies from 0, in position (*c*) to a maximum in position (*a*). Since there is a continual change in the e.m.f., a current will flow through the coil, but the strength of the current will not be uniform. If the coil turn about an axis passing through its center *o* and perpendicular to the face of the coil, there will be no change in the number of lines of force passing through the coil, no lines of force will be cut, no e.m.f. will be generated, and there will be no current.

123. Mechanical Work Necessary to Induce a Current.—Every electric current possesses energy, and this energy must be supplied from some outside source in some way. Every induced current produced by electromagnetic induction is the result of motion; and since motion implies the action of a force through a distance, it follows that work is done in producing the motion, it being stored up as electric energy. To do work, a force must act, and it is desired to ascertain the nature of the force in this case.

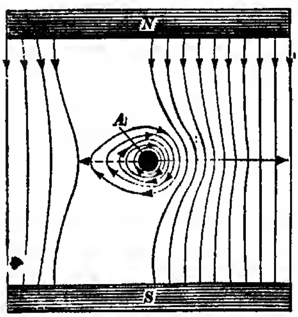


FIG. 54.

Referring to Fig. 54, *A* is a cross-section of a conductor moving through a magnetic field at right angles to the lines of force. As the conductor moves, it bends the lines of force, and it continues to bend them until they finally break and become whorls around the conductor. This action in forming the whorls can be produced only by the action of a force; in other words, the lines of force object to being broken and having their shape changed, and they react on the conductor with a force directly opposite to that which moves it. The full arrow indicates the force that moves the conductor and the dotted arrow indicates the force that opposes the motion—the resisting force. This force multiplied by the distance traveled by the conductor is the work done by the conductor and equals the energy of the current.

From the foregoing, it will be evident that to produce an elec-

tric current by induction, in the manner heretofore described, mechanical work must be done. In the case of a dynamo, the mechanical work is performed by the engine, waterwheel, or other prime mover that turns the armature or field, whichever revolves.

124. As another illustration, consider a circular closed coil, Fig. 55, and let a magnet M be moved into the coil, as shown. The coil will cut the lines of force coming out of the north pole of the magnet, magnetic whorls will be set up around the coil, moving counterclockwise, and induce an e.m.f. A current will flow in the direction indicated, because as the magnet passes through the coil, the flux density decreases, being greatest at the

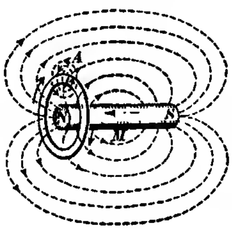


FIG. 55.

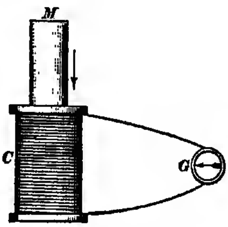


FIG. 56.

end, thus changing the e.m.f. and producing a movement of the current. Now referring to Fig. 32, it will be noticed that when a piece of soft iron is brought to the coil, the current must flow in the opposite direction around the coil in order that the iron may become a temporary magnet, with its poles the same as in Fig. 55. The direction of the current is therefore opposed to the magnetism, and tends to stop the movement of the magnet into the coil.

If the movement of the magnet be stopped, the current will also stop, since no lines of force are being cut. If the magnet be pulled out of the coil, the current will reverse in direction, it will act with the magnetism, and a force must be exerted to pull the magnet out. These effects are more strikingly evident when a solenoid is used instead of a coil. Thus, referring to Fig. 56, let C be a solenoid, the ends of the wires being connected to the galvanometer G. On inserting the magnet M, the galvanometer needle will deflect, say to the right; when the magnet stops moving, the needle moves back to 0; and when the magnet is

pulled out of the solenoid, the needle deflects in the other direction, to the left. The action is exactly the same as with the coil and magnet in Fig. 55, the coils that form the solenoid cutting the lines of force. If the magnet be reversed, with the other end entering the solenoid, the deflections of the needle will also be reversed.

125. Referring again to Fig. 55, consider the sides of the coil that would be touched by flat plates laid against them, as the poles of a magnet. When the magnet is entering the coil as shown in the figure, the result of the induced current is to make the right-hand side a north pole and the left-hand side a south pole; but, when the magnet is pulled out of the coil, these poles are reversed. Since like poles repel each other and unlike poles attract, a repelling force tends to resist the magnet when entering the coil, and an attractive force tends to resist it when leaving the coil. The same effect is produced in the solenoid, Fig. 56.

The whole matter is summed up in what is called Lenz's law. *The direction of an induced current is such that its magnetic field opposes the motion of the magnet that produces the field.*

126. Self-Induction.—What is called **self-induction** occurs whenever there is a sudden change in the current flowing through a circuit. For instance, suppose a steady current is flowing through a coil, the current being derived from a battery, dynamo, or other outside source; this current sets up a magnetic field about the coil and causes lines of force to pass through it. If the circuit be suddenly opened by, say, throwing a switch, the result is practically the same as withdrawing the magnet in Fig. 55, and a momentary current is induced in the circuit. The same effect, though not so marked, will be observed if a resistance is suddenly cut out of the circuit. Or, if the current be suddenly increased, as by throwing the switch so as to close the circuit, an induced current will result, in the same manner as when a magnet is suddenly inserted into a coil. These induced currents are said to be caused by *self-induction;* they oppose the original current when the current is increased, and they add to the strength of the original current when it is decreased. It therefore requires a small amount of time for a current to increase to its maximum or decrease to its minimum value on account of this self-induction, though the time is very short—only a very small fraction of a second.

127. Mutual Induction.—When two coils, one carrying a current, are so situated relatively to each other that the magnetic field of the coil carrying the current encloses the other coil, a current will be induced in the coil not carrying a current by what is called **mutual induction.** The coil carrying the current is called the **primary coil,** and the other coil is called the **secondary coil.** The secondary coil is usually placed so as to enclose the primary coil within it, but it may be placed outside of the primary, both having the same size, as in Fig. 57, where p and s are the primary and secondary coils, respectively. The current, in this case, is supplied by the battery B, and it flows through the coil p (which is a right-hand helix) as indicated by the arrows, that is, from the battery, through a, through the coil, and then through b. By the rule of Art. **97,** the end of the helix marked S is the south pole, and the right-hand end of the soft-iron core that passes through both coils is a north pole. The lines of force therefore have the direction indicated by the arrowheads, and some of them spread out so as to enter the other coil. The flux density is, of course, greatest at the poles S and N. Now if the current be suddenly "made" (turned on) by closing the switch k (pressing the key), the effect will be the same as though the secondary coil s had been suddenly moved along the core nearer the south pole S of the primary coil, since it then moves into a *denser* field, and the number of lines of force passing through the secondary coil is suddenly increased also, thus inducing an e.m.f. in the secondary coil. If the external circuit cd of the secondary be closed, a current will flow through it. It is important to observe that the current flows only during the short period following the "make" or "break" of the circuit and ceases to flow as soon as the stability of the circuit is established.

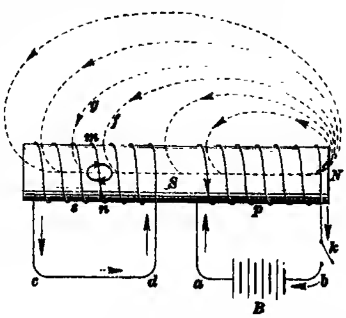


Fig. 57.

To determine the direction of the current, consider the turn mn of the coil and the line of force f. As mn moves to the right, it presses against f and bends it so that the direction of f around mn

will be clockwise; hence, by the rule of Art. **92,** the current is flowing away from the observer, or from *n* toward *m*, as indicated by the arrowheads. The direction may also be determined by the right-hand rule, the hand being held over the coil, the index finger pointing down (the direction of *f*), the thumb pointing to the right toward *N* (the direction of movement of the secondary coil), and the middle finger pointing away from the observer (the direction of the current). The arrows indicate the direction of the current in the external circuit and coil of the secondary.

Note that the direction of the current in the secondary is *opposite* to that in the primary. But, if the circuit be suddenly opened, by releasing the key *k*, the number of lines of force passing through the secondary will be suddenly decreased; the effect will be the same as though the secondary coil had been suddenly moved along the core to the left; *mn* will then press against the line of force *f*, which will wind around it counterclockwise, the direction of the current in the secondary will be reversed, and will be the *same* as that in the primary. Therefore, when the e.m.f. is *increasing*, the current in the secondary tends to move in a direction *opposite* to that in the primary; but when the e.m.f. is *decreasing*, the current in the secondary tends to move in the *same* direction as that in the primary.

128. From the foregoing, it is seen that an induced current may be generated in three ways: by electromagnetic induction, by self induction, and by mutual induction. When generated by electromagnetic induction, it is the result of motion; when generated by self induction or mutual induction, it is the result of a sudden change, usually by making or breaking the circuit, but will occur whenever there is a sudden change in the number of lines of force passing through the circuit.

It is likewise clear that: (*a*) whenever an electrical conductor cuts magnetic lines of force, an e.m.f. (electromotive force) is induced in it; (*b*) this e.m.f. is greater as the number of lines of force cut per second is greater; (*c*) the direction of the magnetic field produced by the induced current always *opposes the motion that produces the field,* and the e.m.f. produced is electrical energy derived from mechanical work.

DIRECT CURRENT DYNAMOS

129. Principle of the Dynamo.—Referring to Fig. 58, *M* represents a magnet, between the poles of which, a rectangular coil turns. The ends of the coil are attached to two metal rings whose centers lie in the axis of the coil and the axis of the shaft *T*, to which the rings are keyed. As the shaft *T* turns, the rings and coil turn with it. Pressing against the rings are two metal strips *B′* and *B″* each having attached to the other end a wire conductor leading to the galvanometer *G*. The rings are called **collecting rings, collectors,** or **slip rings,** and the metal strips *B′* and *B″*,

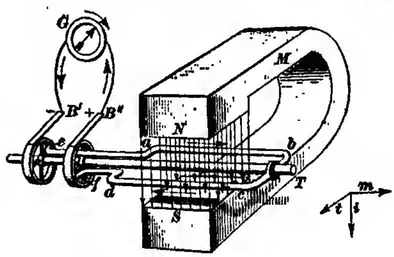


Fig. 58.

are called **brushes.** In the figure, ring *e* is connected to the farther wire *ab* of the rotating coil and ring *f* is connected to the nearer wire *dc*. Suppose the coil is rotating so that when viewed from the end *T* of the shaft, it turns counterclockwise; that is, when the coil is viewed from the side, looking toward the poles *N* and *S* of the magnet, the farther wire *ab* is coming upward toward the observer and the nearer wire *dc* is moving downward and backward.

When the coil is in the position shown (Fig. 58) with the end *bc* horizontal, it is not cutting any lines of force at that instant; but as *ab* moves up and *cd* moves down, they cut lines of force, the number cut increasing until the greatest (maximum) number are being cut when *bc* is vertical, in the position indicated diagrammatically in Fig. 59 (*a*); here *ab* and *cd* are cutting the lines of force at right angles. As the coil continues to turn, the number of lines of force cut decreases and becomes 0, a minimum, when *bc* again becomes horizontal, as indicated at (*b*), at which point the coil has made one-half a revolution. Continuing the rotation, *cd* moves upward and *ab* moves downward, and the number

of lines of force cut increase and becomes a maximum when cb becomes vertical, as indicated at (c); here the coil has made three-fourths of a revolution. During the next quarter turn, the number of lines of force cut decreases to zero, a minimum, the coil completes the revolution, and it returns to its original position as indicated in Fig. 58.

130. To determine the direction of the current induced by cutting the lines of force, note that during the first half-revolution, *ab*, Fig. 59(*a*), tends to make the lines of force wind counterclockwise about it (when viewed from the end *T*); hence, the

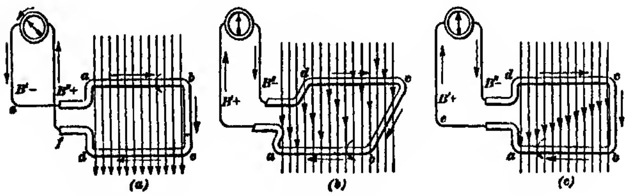


FIG. 59.

current is flowing from *a* toward *b*. At the same time, *dc* tends to wind the lines of force about so they will whirl clockwise; hence, the current is flowing from *c* toward *d*. Since both currents are flowing in the same direction around the coil, the total current is the sum of the currents in the wires *ab* and *cd*; it leaves the brush B'', which is therefore +, flows through the external circuit, through the galvanometer, and back to brush B', which is therefore −. . During the second half-revolution, the current continues to flow in the same direction around the coil, as indicated by the arrows in Fig. 59(*c*), but it flows in the *opposite* direction in the wires *ab* and *cd*; the brush B' becomes + and B'' −, and the current goes through the galvanometer in the opposite direction. Hence, for one half-revolution, the current flows through the galvanometer in one direction and for the other half-revolution in the other direction; in other words, this arrangement generates an alternating current (see Art. **14**).

131. Graphical Illustration.—The current is proportional to the e.m.f., and the e.m.f. is proportional to the number of lines of force cut. When the current is going in one direction, say the first half-revolution, call the e.m.f. positive (+), and when going in the other direction, call it negative (−). Draw a hori-

zontal line *ae*, Fig. 60(*a*), of any convenient length, and let it represent to some scale the *time* of one revolution of the coil; the middle point *c* will represent the time of one half-revolution; that is, *ac* = first half and *ce* = second half. Dividing *ac* and *ce* into halves at 4 and 9, *a*4 represents the time for the first quarter revolution, 4*c* for the second quarter, *c*9 for the third quarter, and 9*e* for the fourth quarter. Divide these quarters into any convenient number of equal parts, say 4; then *a* 1 = 1 2 = 2 3 = etc. = $\frac{1}{16}$th of the time for one revolution.

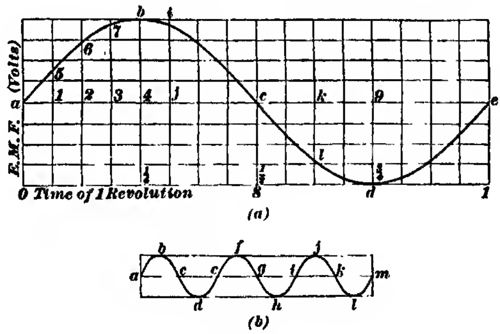


Fig. 60.

Through each of these divisions draw vertical lines (they will be at right angles to *ae*), and on them lay off the values of the e.m.f. at those points; when the e.m.f. is positive, lay it off *above* the line *ae*, but when it is negative, lay it off *below ae*. Thus, when the coil has made $\frac{5}{16}$th of a turn, the time in seconds will be represented by *aj*, and the e.m.f. in volts by +*ji*; and when the coil has made $\frac{10}{16}$th = $\frac{5}{8}$th of a turn, the time in seconds will be represented by *ak*, and the e.m.f. in volts by −*kl*. At the beginning *a*, the middle *c*, and the end *e*, the e.m.f. is 0; at the quarter points 4 and 9, the e.m.f. is a maximum. The smooth curve drawn through the various points above and below *ae* shows at a glance the variation in e.m.f. as the coil turns. Thus, at *a*, the e.m.f. is 0; it increases until the point *b* is reached, then decreases to 0 at *c*, becomes negative and increases numerically until *d* is reached, then decreases numerically, becoming 0 at *e*. During the next revolution, the curve is repeated identically, as shown at (*b*), Fig. 60, which represents (to a smaller scale three revolutions.

132. Changing Alternating to Direct Current.—Fig. 61 shows the coil in the same position as in Fig. 58, but there is only one collecting ring instead of two, as before, and this ring is split into two equal segments having a small air gap between. When the plane of the coil is horizontal, as shown at (*a*), the coil is not cutting lines of force and the e.m.f. is 0; the brushes touch both segments across the air gap, as indicated at (*b*). With the coil turning in the direction of the arrows, the slightest movement brings segment *ef* in contact with brush *B''* only, and segment *gh* in contact with brush *B'* only, and this condition is maintained

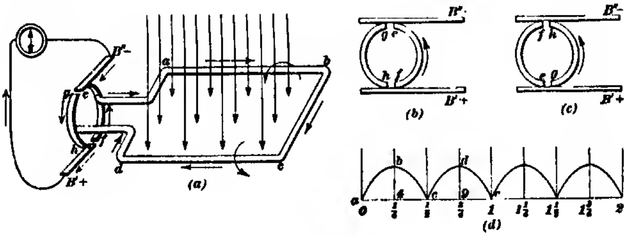


FIG. 61.

for a half-revolution, when the brushes again momentarily touch both segments, as indicated at (*c*), the e.m.f. again being 0. During this half-revolution (the first half-revolution), the current flows through the coil in the direction of the arrows and reaches the brush *B'* by means of the segment *gh*; it returns to the coil by the segment *ef*. At the beginning of the second half-revolution, the direction of the current in *ab* and *cd* reverses, and at the same time, the segments to which these wires connect reverse their contacts with the brushes, *ef* being in contact with *B'* and *gh* with *B''*; hence, the direction of the current flowing out of the brushes is the same as before, and, consequently, the direction of the current through the external circuit is unchanged. In other words, the current has been changed from an alternating to a direct or continuous current in the *external* circuit.

133. The variation in e.m.f. may be represented graphically as before, as shown at (*d*), Fig. 61, which shows the curve for two revolutions, there are no values to be laid off below *ae*, because the e.m.f. is never negative in this case. The current increases from 0 to 4*b* in the first quarter, decreases to 0 at the half, increases to 9*d* at three quarters, and decreases to 0 again at *e*, the

end of the revolution. The curve is repeated identically during the next and subsequent revolutions, assuming that the speed of turning does not change. A current of this kind is direct and continuous, but is not constant, varying up and down; such a current is called a **pulsating current.**

As will be seen, the e.m.f. fluctuates greatly, varying between 0 and its maximum value for every half-revolution. This fluctuation may be greatly decreased and the average value of the e.m.f. greatly increased by having two coils at right angles to each other and by dividing the collecting ring into four segments instead of two, as shown in Fig. 62. Here coil A' is vertical and

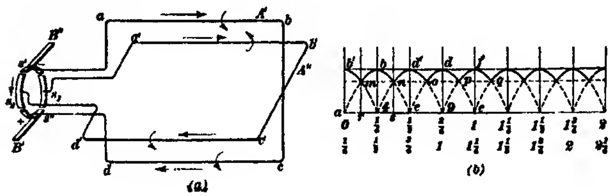


FIG. 62.

A'' is horizontal, and both are turning in the same direction as before, the direction of the current being indicated by the arrows. A' is cutting the maximum number of lines of force and A'' is not cutting any, when in the position shown. The wire *ab* is connected to the upper segment s' and the wire *cd* is connected to segment s'' diametrically opposite. At this instant, the brushes B' and B'' bear directly on the middle lines of the segments, and the maximum current is going through the external circuit. As the coils turn, the e.m.f. begins to decrease until it reaches the split, which is one-eighth of a turn from the position shown. Here the brushes leave the segments s' and s'' and bear on the segments s_1 and s_2, which are connected to the other coil A'', the e.m.f. of the current in which is the same at this point as that of the current in A'; and since A'' is approaching a vertical position and cutting a greater number of lines of force, the e.m.f. immediately begins to increase, reaching its maximum value when coil A'' becomes vertical, which it will do in another one-eighth of a turn. The e.m.f. then begins to decrease again for the next one-eighth of a turn, and then to increase, as segments s'' and s' come into contact with the brushes.

The variation in e.m.f. is graphically shown in (*b*), Fig. 62. The curve *abcde* is for the coil A'' alone for one revolution, and the curve $b'4d'9f$ is for coil A' alone during the same revolution. At the start, ab' is the e.m.f. (in volts) in coil A', while in A'', the e.m.f. is 0. The e.m.f. then declines in A' and rises in A'' until they become equal at r, where ar = one-eighth revolution, the value at this point being rm. Here A' is cut out of the circuit and the e.m.f. in A'' takes its place, the latter increasing to its maximum value $4b$ at one-quarter revolution. While this was occurring, the e.m.f in A' has fallen to 0 at c. During the next eighth of a turn, the e.m.f. falls in A'' and rises in A' until they become equal at s, the value at this point being $sn = rm$. This rising and falling is constantly repeated, the minimum value being $rm = sn =$ etc., or about three-fourths of the maximum value ab'.

The fluctuation in e.m.f. is very much less than before; the average e.m.f. for one revolution is considerably greater, as will be seen by comparing (*b*), Fig. 62, with (*d*), Fig. 61. If the curve has been accurately drawn, the average distance between *ae* and the curve (= average e.m.f.) is equal to (see Fig. 62) the area $ab'mbnd'odpfe$ divided by *ae*, and this is evidently much greater than (see Fig. 61) the area *abcde* divided by *ae*. By further increasing the number of coils (and also the segments), the fluctuations become so small that the e.m.f. is practically constant.

It is to be noted that the number of segments is twice the number of the coils, since each coil has two ends and each end must have its own segment; further, the ends of the coils must be so connected to the segments that the segment connected to one end of any coil must be diametrically opposite to that connected to the other end of the same coil.

134. Parts of a Dynamo.—The apparatus just described is a **dynamo**; its coils are caused to rotate by some mechanical means, and as they cut lines of force issuing from the poles of the magnet, they generate an e.m.f., which when the circuit is closed, causes a current to flow through it. In toys, the magnet may be a permanent magnet; but in practice, where the current is used to drive motors, light lamps, etc., the magnet is an electromagnet. The revolving coil is called the **armature,** the region through which the magnetic lines of force travel is called the **field,** and the segments that make up the ring touched by the

brushes is called the **commutator,** because it *commutes* (transforms) an alternating current into a direct current.

135. Increasing the e.m.f.—The e.m.f. generated by a dynamo such as just described would be very low, but may be increased in a number of ways: First, by wrapping the coils about a drum of some magnetic material, as soft iron, cast iron, mild steel; this attracts the lines of force, offers them a better and easier path than the air; and greatly increases the density of the magnetic flux. Second, curving the pole faces to an arc of a circle, thus lessening the average length of the air gap between the armature and the pole faces. Third, winding the poles of the field with many turns of wire; this increases the strength of the field, since the greater the number of turns of wire the greater will be the magnetic flux. Fourth, increasing the speed of rotation of the armature; the faster the armature turns the greater the number of lines of force the coils will cut per second.

136. Bipolar and Multipolar Dynamos.—A dynamo having but two poles is called a **bipolar dynamo** (*bi* means two); if it has

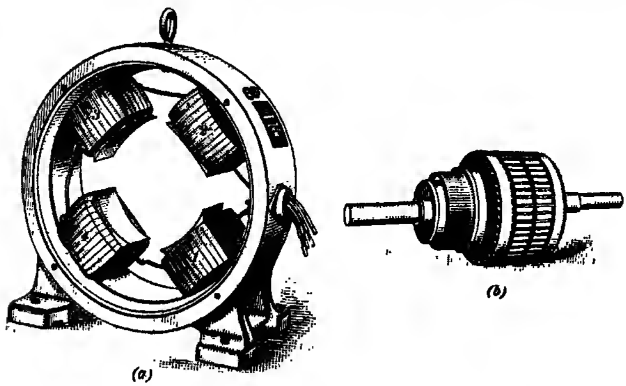


FIG. 63.

more than two poles, it is called a **multipolar dynamo.** All dynamos must have an even number of poles, since for every north pole there must be a corresponding south pole. Most dynamos in use have 4, 6 ,or 8 poles, and some have many more. A multipolar dynamo must have more than two brushes; in fact, it must have as many brushes as it has magnetic circuits through the field, and it will usually have as many circuits as it has poles.

A 4-pole dynamo field frame with its pole pieces is shown in Fig. 63 (*a*), the armature being shown at (*b*). If pole 1 is a north pole, pole 2 is a south pole, pole 3 a north pole, and pole 4 is a south pole; that is, poles of opposite polarity are adjacent to one another. The winding must be in opposite directions around each pole, and must be carried from one pole to the next, so that the poles are in series with one another, as shown in the figure. This last is shown more clearly in the diagrammatic view of the field frame in Fig. 65, which will be referred to more fully later. Referring to (*b*), Fig. 63, the coils and core of the armature are clearly seen. The coils must be carefully insulated from one another and from the core; otherwise, the current will be short-circuited through the core and very little will go to the external circuit. In practice, a coil may consist of more than one turn, but is regarded as being a simple coil of one turn.

137. Magnet Circuit in Dynamo.—Fig. 64 represents diagrammatically a bipolar dynamo, the armature being a hollow cylinder. The winding on both poles begins at the end nearest the armature, and since the same wire is used, it is necessary to wind the poles in opposite directions. According to the rule previously given (Art. **97**), the right-hand pole is the north pole and the left-hand pole is the south pole, and the lines of force have the directions indicated by the arrowheads. As the lines of force enter the south pole, they divide, the upper half going through the upper part of the frame and the lower half through the lower part of the frame, uniting again as they enter the north pole. Consequently, two magnetic circuits are formed, as indicated, and two brushes are required on the commutator.

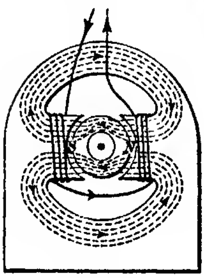


FIG. 64.

Fig. 65 represents diagrammatically a 4-pole dynamo. As indicated by the winding and the direction of the current in the wire (shown by the arrowheads), 1 and 3 are north poles and 2 and 4 are south poles. The lines of force pass from one north pole to an adjacent south pole in the manner indicated and back to the pole from which it came. There are thus four magnetic circuits, and there must be four brushes on the commutator. The brushes are so arranged that the two positive brushes are

diametrically opposite each other and are connected in parallel so that their combined current flows to the external circuit. The two negative brushes are arranged and connected in a similar manner, the line (diameter) joining their points of contact on the commutator being at right angles to the line joining the points of contact of the positive brushes.

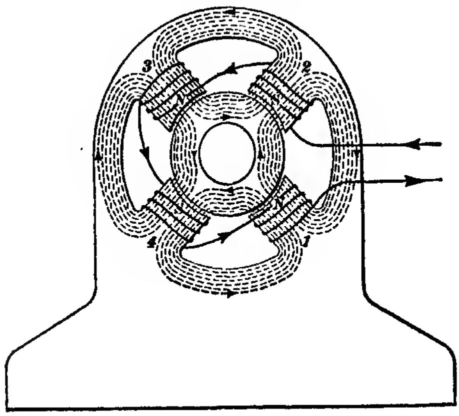


FIG. 65.

138. The coils are wound on the armature core in such manner that the conductors (wires) lie on the surface parallel to the axis. The greater the number of coils, the greater the number of lines of force cut, and the greater the e.m.f. generated. Suppose there are 50 coils in the armature of a certain dynamo; there will then be $50 \times 2 = 100$ segments in the commutator. If the outside diameter of the commutator is 12 in. the space between any two segments is .03 in. This width is so small that it is impracticable to use an air gap between the segments; hence mica strips are generally used. The segments must also be insulated from the shaft, to prevent short circuiting, and vulcanite or micanite strips are generally used for this purpose.

139. Value of Induced e.m.f.—When a conductor cuts lines of force at right angles, an e.m.f. is induced in the conductor, and its value is proportional to the rate of cutting; the rate of cutting is equal to the total number of lines cut divided by the time in seconds taken to cut them. *When the rate of cutting is* 100,000,000 $= 10^8$ *lines of force per second, an e.m.f. of* 1 *volt is generated.*

Suppose the projected area of the pole faces of a bipolar dynamo is 150 sq. in., the flux density (average) is 30,000 lines of force per square inch, the number of coils in the armature is 50, and the revolutions per minute of the armature is 2400. If the armature is wound so that the wire passes over and through it, as in winding a ring, the wires passing inside the core do not cut any lines of force, and there will be 50 wires on the outside that do the cutting. Each wire will cut the lines of force twice—once when passing the north pole and once when passing the south pole; hence, in one revolution the number of lines cut will be $150 \times 30{,}000 \times 50 \times 2 = 450{,}000{,}000$; the number of revolutions per second is $2400 \div 60 = 40$; hence, the lines of force cut per second is $450{,}000{,}000 \times 40 = 18{,}000{,}000{,}000 = 18 \times 10^9$; and the e.m.f. generated is $18 \times 10^9 \div 10^8 = \frac{18{,}000{,}000{,}000}{100{,}000{,}000} = 180$ volts.

140. Strength of Current.—According to Ohm's law, $I = \frac{E}{R}$, in which, for a dynamo, R is the resistance of the circuit through which the current passes. This may be divided into the internal circuit and the external circuit. The internal circuit is the total length of the conductors between the brushes and on the armature, and the external circuit is the total length of the conductors forming the circuit from brush to brush. If the resistance of the circuit, both internal and external, in the case referred to in the last article is 19.3 ohms, the strength of the current is $\frac{180}{19.3} = 9.326+$ amperes. The **internal circuit** may also be defined as the length of the circuit from the negative brush (where the current enters the dynamo) to the positive brush (where the current leaves the dynamo), and the **external circuit** as the length of the circuit from the positive brush to the negative brush.

141. Strength of Field.—If a stronger magnetic field is desired, that is, if it is desired to increase the flux density at the poles, it may be obtained in two ways: by increasing the strength of the current flowing through the field windings; by keeping the current at the same strength and increasing the number of turns of wire in the solenoids formed by the coils wound around the pole-pieces. For example, if a current of 1 ampere be sent through a coil of one turn, it will induce a certain magnetizing effect. If the strength of the current be doubled, the magnetizing effect will be doubled. If the length of the wire be doubled and wound

into two equal coils, the current being constant, the magnetizing effect will be doubled also. In other words, the magnetizing effect is directly proportional to the product of the current in amperes and the number of turns in the coil, that is, it is proportional to the number of **ampere-turns.** For example, suppose a current of 8 amperes is sent through a coil having 150 turns; the number of ampere-turns is $8 \times 150 = 1200$ ampere-turns. If another coil have the same diameter and 400 turns, a current of 3 amperes will give it $3 \times 400 = 1200$ ampere-turns, and since both coils have the same number of ampere-turns, they will exert the same magnetizing effect.

A little consideration will show the correctness of the preceding statement. Thus, a coil of one turn will make a solenoid of certain strength that is equivalent to a magnet of the same strength; two turns will be equivalent to two magnets of equal strengths or one magnet of twice the strength; n turns will be equivalent to n magnets of equal strength, and since the turns are all connected in series and the current is the same in all, the result is a solenoid equivalent in strength to n magnets. If the strength of the current varies, the strength of the n magnets will vary in proportion, and since n is the number of turns (magnets), the strength of the solenoid is proportional to $I \times n$, that is, to the number of ampere-turns, I being the strength of the current.

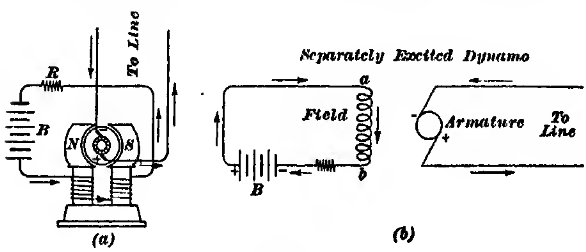


Fig. 66.

142. Sources of Field Current.—Direct-current dynamos are classified according to the source from which the field coils receive their current,—that is, according to the field excitation,—as *separately excited* and *self excited.* When separately excited, the field current comes from some outside source, as a battery of primary cells, a storage battery, a special dynamo, etc.

The connections for a separately excited dynamo are shown in

Fig. 66, in which (*a*) is a conventional representation of a bipolar dynamo and (*b*) is a diagram of the connections. In this case, the source of current for the field coils, called the *field* in the diagrams, is a storage battery, but might be a small dynamo. In the diagram, the loops between *a* and *b*, marked field, indicate that the coils of the field are in series. It will be observed that the current flowing through the field coils has no connection whatever with the current flowing through the brushes and external circuit; and the stronger the current through the field the greater the e.m.f. generated by the armature, since the flux density will then be greater and the armature coils will cut a greater number of lines of force. A field rheostat or resistance box *R* serves to vary the strength of the current to the field to allow for fluctuations in the e.m.f. of the line due to variations in the load.

143. Self-excited Dynamos.—In self-excited dynamos, the field current is supplied by the armature of the dynamo itself; they are divided into three classes: shunt-wound dynamos, series-wound dynamos, and compound-wound dynamos. They all work on the principle of what is called residual volts, which will now be explained.

After a dynamo has once been operated, it does not lose *all* its magnetism when standing still, even if there is no current flowing through the field coils. Consequently, when the armature is started revolving, its conductors cut a certain number of lines of force, thus generating a weak e.m.f., which varies in value from 2 to 10 volts; these are called **residual volts.** The manner of building-up the field will be described presently.

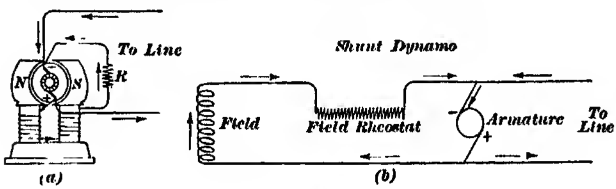


Fig. 67.

144. Shunt Dynamo.—The connections for a shunt-wound dynamo are shown in (*a*) and (*b*), Fig. 67. The field coils are in parallel with the main line circuit, the current dividing at the positive brush and uniting at the negative brush. The shunt

consists of the field coil winding, which is made up of a large number of coils of very fine wire, thus making a high resistance, so that only a small part of the current goes to the field coils, the greater part going to the line.

When the dynamo is started, the conductors cut the lines of force due to the residual magnetism, thus creating the residual volts and causing a small current to flow through the shunt, that is, through the field coils, thus increasing slightly the strength of the magnetic field. The revolution of the armature in this increased field raises the generated voltage somewhat, which, in turn, sends a larger current through the field coils, thus increasing the magnetic field; the e.m.f., and the field current; this is called the process of **building up the field.** The building up of the field continues until the fields have enough magnetism to produce the maximum voltage, the value of which depends upon the speed of rotation of the armature and the number of ampere-turns of the fields. This process usually takes from 10 to 30 seconds.

After the voltage has become constant, it can be controlled or regulated to any definite value within the limits of the machine by means of the field rheostat *R*, see Fig. 67, which must be adjustable. If the voltage is too high, it can be lowered by increasing the resistance through the rheostat; this lowers the field current, which weakens the magnetic field, lessens the number of lines of force, and decreases the voltage produced in the armature conductors. Decreasing the resistance through the rheostat will produce the opposite effect. The voltage may also be raised by increasing the revolutions per minute (r.p.m.) of the armature, and it may be lowered by decreasing the r.p.m. of the armature.

145. Series Dynamo.—The connections for a series-wound dynamo are shown in (*a*) and (*b*), Fig. 68. A small number of turns of large wire are used for field coils, and they connect in series with the main circuit. The current leaves the positive brush, passes through the coils, out to line, and back to the armature through the negative brush. Any change in the resistance of the external circuit produces a change in the e.m.f. and current, the armature rotating at constant speed, because a change in current alters the field magnetism and thus changes the induced e.m.f. in the armature conductors. In practice, the voltage of a series dynamo is altered by suitable means to

suit circuit conditions, with the result that it becomes a constant-current dynamo, and is therefore well adapted to arc lighting circuits. It is obvious that a series dynamo will not self-excite when the external circuit is open, since no additional e.m.f. can then be generated; neither will it build up, if the external resistance is very high. Series dynamos are not much used.

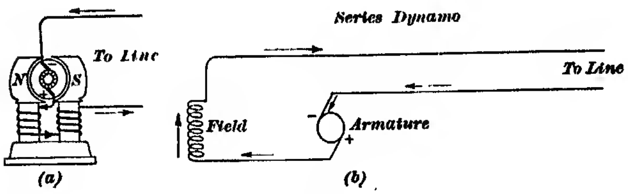


FIG. 68.

146. Compound Dynamos.—The connections for a compound-wound dynamo are shown in (*a*) and (*b*), Fig. 69. The field coils are wound first like a shunt-wound dynamo, with field rheostat *R*, fine wire being used for the coil; on top of this, another winding of coarse, heavy wire is wound, as in a series dynamo. The current divides at the positive brush, the greater part going to the mains through the series winding.

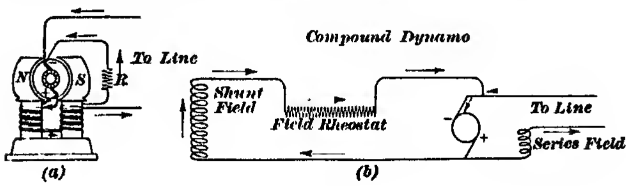


FIG. 69.

The object of a compound-wound dynamo is to secure a constant potential. In the case of a shunt-wound dynamo, as soon as any current flows into the line, the e.m.f. falls a little; and it continues to fall, as more and more current is used, unless some means are taken to prevent it. For every condition of load current, the voltage will be of a certain value, but it will always be less than the voltage at no load. To counteract this fall of potential as the load increases, is the object of the compound-wound dynamo. The magnetizing effect of the series coil is added to that of the shunt coil, both being wound on the same

pole-pieces; the greater the amount of current taken by the mains the greater the amount flowing through the series field coils, and the greater the magnetizing effect, with a consequent increase in the e.m.f. to counterbalance the drop in the shunt coils. Thus the voltage is kept constant, no matter how much current (within the limits of the machine) is taken by the line. Most of the flux in the field is generated by the shunt coils, the series coils having just enough ampere-turns to increase this magnetism sufficiently to make up for the tendency of a shunt-wound dynamo to lower slightly in voltage as the load increases.

147. Commutating Poles.—In between the main pole-pieces, many of the modern machines have smaller pole-pieces, called commutating poles. These poles are intended to generate e.m.f. in the armature, their object being to keep the brushes from sparking badly, which is likely to occur in shunt- or compound-wound dynamos when the load is heavy or they are running at high speed. The coils on the commutating poles are always in series with the armature, and the strength of the current in them depends upon the strength of the current in the armature conductors.

DIRECT CURRENT MOTORS

148. Movement of Current-Carrying Conductor in Magnetic Field.—If a conductor be placed in a magnetic field, as shown at (*a*), Fig. 70, and circuit is open, the conductor will remain where-

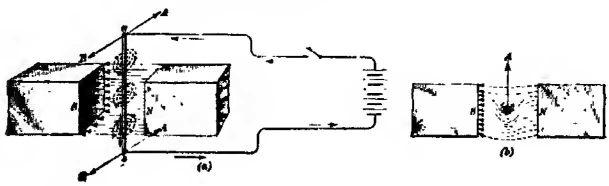


FIG. 70.

ever it is placed and will have no tendency to move. If, however, the circuit be closed and a current be sent through it, as indicated by the arrows, flowing through the conductor from *a* toward *b*, the conductor will move to the right, across the lines of force, in the direction of the arrows *A*. The reason for this

movement is simple. When the current flows through the conductor, it sets up magnetic whorls around it, as indicated by the dotted circles, and when looking along the conductor from *a* toward *b*, these whorls will be clockwise. Those parts of the whorls in front of the conductor, looking in the direction of the arrows *A*, have the same general direction as the lines of force, and they make the field in front of the conductor denser; those parts of the whorls back of the conductor have a general direction opposite to the lines of force, with the result that the magnetic field is rendered less dense. Since the conductor tends to move from a point of greater density to one of less density, if free to move, it will, in this case, move in the direction of the arrows *A*, as indicated at (*b*), which is a view looking down the conductor from *a* to *b*.

If the direction of the current in the conductor be reversed, the direction of whorls will also be reversed, and the conductor will move in the direction of the arrows *B*; the same will be true if the direction of the current in the field coils of the magnet be reversed, since this will reverse the magnetic poles. But if both currents be reversed, there will be no change in the movement of the conductor, which will then move in the direction of the arrows *A*.

The force which acts to move the conductor will be directly proportional to the strength of the current in the conductor, the strength of the magnet remaining the same. If the strength of the current be doubled, the force will be doubled; and if the strength be halved, the force will also be halved.

149. Direction of Movement of Conductor.—To determine the direction of movement of the conductor when carrying a current, use the *left* hand in the same manner that the right hand was used in finding the direction of an induced current. Place the thumb, index finger, and middle finger of the left hand at right angles to one another; point the index finger in the direction of the lines of force, the middle finger in the direction of the current, and the thumb will point in the direction that the conductor moves. Tho manner of using the left hand to find the direction of the conductor in Fig. 70 is clearly shown in Fig. 71. Here the left hand holds the north end of the magnet, with the index finger pointing in the direction of the lines of force, the middle finger points down (the direction of the current in the conductor), and the thumb indicates the direction in which the conductor will

move. Note that the direction of the conductor is opposite to that which it must have in order to induce a current in the conductor from *a* toward *b*; this is why the left hand must be used to determine the direction of movement of a conductor.

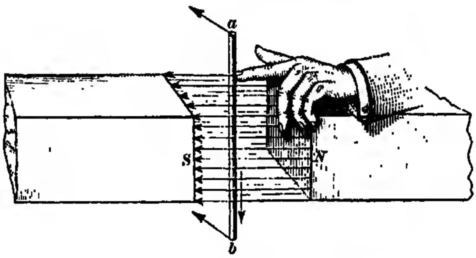


FIG. 71.

150. Why a Motor Rotates.—A motor differs from a dynamo only in certain mechanical details, it does not differ in principle. Any dynamo can be used as a motor if an electric current be sent through the brushes, the armature, and the field coils. The wire wound around the core of the armature parallel to the axis, forms a series of conductors, all parallel to one another, and the current sets up magnetic whorls around each conductor, thus producing the same effect as described in connection with Fig. 70. These whorls being in a magnetic field are acted on by the lines of force passing between the pole pieces, and the conductors are forced to move, that is to rotate; and the armature, to which the core is attached, rotates also. The direction of rotation can always be determined by placing the fingers of the left hand as instructed in Art. **149.**

151. The Commutator.—It is just as necessary to have a commutator on a motor, when the current driving the motor is direct, as on a dynamo. In the case of the dynamo, when a conductor passes one pole, the current has a certain direction, and when it passes the next pole, the current has the opposite direction, and without the commutator, the current in the line would be alternating. The current in the *armature* is *always* alternating; hence, if the armature is to be made to rotate by the action of an electric current, the current must be supplied in such a way that when any conductor passes a pole, the current in it must be opposite in direction to that which it will have when the

the conductor passes the next pole; and this result is obtained by means of the commutator, which reverses the current at just the right instant.

152. The voltage of direct-current dynamos and motors may run as high as 600 volts, and in special cases, as high as 1500 volts, but they are not adapted to much higher voltages. It is frequently desirable to generate current at much higher voltages than this, particularly for long distance transmission of power, and when such is the case, alternating-current machines are used. If the motor be driven by an alternating current, it will, in many cases, have no commutator.

Since the motor is used to drive other machines, the electric energy supplied to the motor is converted into mechanical energy.

153. Motor Connections.—As in the case of dynamos, motors may be shunt-wound, series-wound, or compound-wound, the connections being the same as in the corresponding type of dynamo. In a shunt motor, the armature and field are connected in parallel, the current dividing, with a part of it going to field and the greater part to the armature. In a series motor, the field and armature are connected in series, the entire current going through both. The compound motor is a combination of both of these.

154. Speed of a Shunt Motor.—If a shunt motor be connected to a circuit having a constant e.m.f., current will flow through the field coils and create lines of force in the field; a current will also flow through the armature conductors, and the armature will rotate. As the armature rotates, the conductors cut lines of force and induce an e.m.f. that acts in the opposite direction, which is known as the **back** or **counter electromotive force** (*counter e.m.f.*); this fact might be inferred from the last sentence of Art **149.** The counter e.m.f. is opposite in direction to the e.m.f supplied by the current; hence, the voltage that is forcing a current at any instant through the revolving motor armature is not the voltage of the line, but the e.m.f. of the line minus the counter e.m.f. in the armature. To apply Ohm's law to this case, let I = current through armature, E = e.m.f. of line − counter e.m.f. of armature (in volts), and R = resistance of armature (in ohms); then,

$$I = \frac{E}{R}$$

The faster the motor runs, the greater is the counter e.m.f.; consequently, when the motor is speeded up to a point where the counter e.m.f. is nearly equal to the supplied e.m.f., the current in the armature will greatly decrease, and the motor will not tend to speed up further. The shunt motor then runs at a constant speed, which is slightly less than the speed it would have when running as a dynamo and generating the same line e.m.f. that is being supplied to it as a motor. If the line e.m.f. be kept constant and the field of a shunt motor be weakened, by connecting a resistance (rheostat) in series with the field (thus lessening the field current), the motor will run faster; because, if the field is weakened, the motor must run faster in order to generate the same counter e.m.f., the counter e.m.f. being proportional to the strength of the field and the speed of rotation of the armature.

If when a shunt motor is standing still it is connected to the line, the current that flows through the armature winding will be limited only by the resistance of the winding, as there can then be no counter e.m.f. The current will be excessive, and, in the case of larger sizes, injurious to the motor; for this reason, a starting box (see Art. **157**) is used. As soon as the motor speeds up, the counter e.m.f. prevents a large current from flowing.

155. Speed of a Series Motor.—As has been explained, due to the counter e.m.f., the current through the armature of a shunt motor becomes less and less, as it speeds up and until a point is reached where its speed remains constant. This occurs when the counter e.m.f. balances the e.m.f. of the line, which may be called the applied e.m.f.

As a series motor speeds up, the current through the armature becomes less and less, because of the counter e.m.f.; but, since the armature and field are connected in series, the current in the field also becomes less, and the strength of the field (the flux density) becomes less. Since the counter e.m.f. depends upon the strength of the field, it will become less when the field is weakened. Therefore, if a series motor without load be connected to the line, it will tend to speed up more and more, but it can never reach the point where the counter e.m.f. will balance the applied e.m.f. Hence, when a series motor is used, it must be connected to heavy machinery, to keep it from excessively speeding up or "running away."

156. D. C. Motor Speed Independent of Dynamo Speed.—If a d.c. (direct-current) motor have no load or if it have a defi-

nite load and its field and armature circuits are kept constant, the speed of the motor will depend only upon the e.m.f. of the supply circuit, that is, upon the applied e.m.f. If the applied e.m.f. be increased, the motor speed will be increased, and vice versa. The speed of a d. c. dynamo supplying power does not affect the motor speed as long as the e.m.f. of the current supplied to the motor remains constant.

157. Starting Box.—When starting a motor of any size, large or small, it is not safe to throw the full line voltage across it at once. The armature has a very low resistance; hence, the full line voltage would force a large current through it and burn it up. It is therefore necessary to put a box containing an adjustable resistance, called a **starting rheostat** or **starting box** in series with the armature, in order to cut down this current. This rheostat is always arranged so that, as the motor gets up speed, the resistance can be gradually cut out; and when it is entirely cut out, the full line voltage is across the motor. The counter e.m.f. then takes care of the line current.

A d. c. motor is started with a strong field, but a d. c. dynamo is started with a weak field.

158. Speed Control of Series Motor.—Since in a series motor all the current that flows through the armature flows through the field, the speed of rotation is controlled by a separate resistance, in series with the field. Increasing the resistance, with the current flowing, reduces the voltage at the terminals of the motor, and thus reduces its speed; decreasing the resistance increases the speed. The current first passes through the adjustable rheostat, next through the field, and then through the armature.

Series motors are used where a widely varying load is to be taken care of and where a strong starting effort without drawing excessive current is needed; they are therefore used on street cars and for every kind of traction work, on electric cranes and elevators, electric automobiles, etc.

159. Speed Control of Shunt Motor.—The shunt-wound motor has two excellent points: (*a*) nearly constant speed at all loads; (*b*) possibility of controlling the speed by field resistance and armature resistance, as explained in Art. **153.**

To decrease the speed of a shunt-wound motor, resistance may be inserted in the armature circuit; this cuts down the power of the motor, which makes it an expensive method of control.

To increase the speed, simply insert a resistance in series with the field coils. This decreases the flux density and increases the current in the conductors of the armature; the difference between the applied e.m.f. and the counter e.m.f. becomes greater, and the speed increases. In fact, if all the current is cut out from the field of a shunt motor, by inserting too much resistance or by opening the field circuit, the speed of the armature becomes so great that it flies apart and wrecks the machine.

Motors are now made in which the field may be weakened to a great extent and large changes in speed made possible by means of an adjustable rheostat in series with the field; such a motor is called an **adjustable speed motor,** and it is made with commutating poles, such as were mentioned in Art. **147.**

By reason of its tendency to run at constant speed, the shunt-wound motor is used on pulp grinders, pumps, beaters, machine tools, printing presses, and other machinery, such as the constant speed line of a paper machine, where a steady speed of rotation is necessary. Some machinery needs to be run at different speeds at different times, but steadily at the speed required at any particular time; for such cases, the adjustable speed motor is most suitable, the best that can be used.

EFFICIENCY AND CAPACITY OF MOTORS

160. Motor Losses.—It was stated in Art. **51** that no machine could give out as much work or power as it received; consequently, all the energy supplied to a motor cannot be utilized in doing useful work. The efficiency of a motor is always less than 1, or 100 per cent. The difference between the input and the output is called the **motor losses,** and these are usually grouped under three headings:

(*a*) The **copper** or I^2R, **losses** are those required to furnish the power used to force the current through the copper wire in the armature and field windings. These losses may be calculated if the resistance of the armature and field coils and the strength of the current flowing in each circuit are known.

(*b*) The **friction and windage losses** represent the power required to overcome the friction of the bearings and brushes and by the rotation of the armature in the air.

(*c*) The **core, or iron, losses** are due to the alternations in the flux passing through the iron magnetic circuit in the core.

It is not the individual losses just specified that are considered, so much as the sum of the three, which is called the **total losses.** The latter may be found if the input and output of the motor are known, and it is to determine these that motors are tested.

161. Efficiency of Motors.—The input and output can be determined with the aid of a watt meter. By measuring the power required to run the motor at no load and at different loads, the efficiency of the motor under different conditions can be found by using formula (1), Art. **51,** which may be written $e = \frac{\text{output}}{\text{input}}$. The losses evidently equal input − output.

All losses result in heating the motor; and if a machine is compelled to deliver more power than its rated capacity, that is, if it is overloaded, it heats more and more, and may become seriously damaged. A machine underloaded is also less efficient than when running at its rated capacity, which is the load for which the machine was designed. A small motor is less efficient than a large one of the same type. For motors running at their proper speed and under their proper load, the efficiency varies between .65 and about .93, that is from 65% to 93%, according to size.

Dynamo losses are of the same nature as motor losses, and the efficiency of dynamos is about the same as for motors of similar size and type.

162. Relation of Capacity to Load.—It is worth while here to note a point that sometimes is not fully understood concerning electrical apparatus in general. The output of a dynamo, motor, or transformer at any particular time is the load that is required of it at that time—no more and no less. The input is the load plus the losses. A 100-K.W. dynamo may, at some particular time, have only one 40-watt lamp on its circuit; the dynamo is not then generating $100 \times 1000 = 100{,}000$ watts, but 40 watts (the load) + all the losses necessary to operate the dynamo and carry the current to the lamp. These losses will be a large part of the losses incurred when running under full load and the efficiency under such a condition of operation will be correspondingly low.

The capacity of a motor is determined by the temperature of the wires, commutation, insulation, etc. The rated, or full load, capacity is the load that the motor (or dynamo) was de-

signed to carry to secure the greatest efficiency. The only relation between the load and the rated capacity of a motor (or any machine) is that it is not economical to run it either much underloaded or much overloaded; the best way is to run it as nearly as possible at its rated load.

163. Electrolytic Processes.—Besides the conversion of electrical energy into heat energy (as in a lamp) or mechanical energy (as in a motor), electrical energy may be changed to chemical energy and do chemical work. This has been mentioned in connection with storage batteries. If a direct current of proper voltage be passed into a cell (see Fig. 1) through a plate of a metal, as silver, and a solution containing the same metal, an article of another metal may be suspended in the solution to act as the other pole and silver, for example, will be deposited on the article. This is **electro-plating.**

The pulp and paper industry is more interested in **electrolysis** (see *Chemistry*, also *Bleaching of Pulp*), the breaking up of a chemical compound under the influence of an electric current. Thus, if a direct current is passed into a solution of common salt, the *chlorine* will be liberated and pass off as a gas to be used in bleaching pulp, while the *sodium*, which is simultaneously set free, will combine with water to form caustic soda. This product may be used for cooking rags or wood, or sold for its chemical value.

ALTERNATING CURRENT GENERATORS

164. Simple Single-Phase Alternator.—The machine that transforms mechanical energy into electrical energy and delivers it as a direct current is called a dynamo; but when it is delivered as an alternating current, the machine is called an **alternating-current generator** or, more simply, an **alternator.** In this section, a direct-current generator will be called a dynamo, as heretofore, and an alternating-current generator will be called an alternator. It is frequently desirable to abbreviate the terms direct-current and alternating current to d.c. and a.c., respectively. Some writers on electrodynamics use the word "dynamo" to include both the generator and the motor, what is here called a dynamo being called a d. c. generator. The definition of dynamo as above given is preferred by the writer, and is one used in this Section.

A simple single-phase alternator has already been described in Art. **129**, and illustrated diagrammatically in Fig. 58. The variation in e.m.f. is shown by the diagrams in Fig. 60. Instead of a commutator, the one-piece collector rings, commonly called *slip rings*, and denoted by *e* and *f* in Fig. 58, are used to take off the current. Such a machine is called a **single-phase alternator,** because only one curve can be drawn to show the variation in the e.m.f. In other types, two and even three such curves may be drawn, in which case, the machines are called **two-phase** and **three-phase alternators,** respectively. The single-phase alternator supplies current to the external circuit from a single, continuous armature winding.

165. Alternator Field Excitation.—Since the magnetic flux must always have the same direction, the current in the field coils must be a direct current, the same as with a dynamo. But the alternator armature supplies only alternating current; hence, the field-exciting current is supplied from a separate source, such as a small dynamo, which may be driven from the same engine or other mechanical source of power that drives the alternator; or it may be driven from a separate source. Both methods are used in practice.

166. Armature Windings of an Alternator.—The armature of an alternator is wound in a very different manner from the armature of a dynamo. This is necessary in order to secure a powerful alternating current. A brief explanation of this difference in winding will now be given.

Referring to (*a*), Fig. 72, suppose a flat coil be moved across the pole pieces of a U-shaped magnet, the movement being from left to right. When the coil is in position 1, no lines of force are being cut and no e.m.f. is being generated; this corresponds to position 0 on the line *ag* in (*b*). A slight movement to the right causes the edge *a* of the coil to cut some lines of force and create whorls in a clockwise direction around the conductors of the coil, and the number of lines cut increases as each layer of the coil passes the edge *m* of the magnet. A further movement of the coil causes the edge *b* to cut lines of force going into the south pole of the magnet; and since the whorls around the conductors, which are created by this action, are counter clockwise, the induced e.m.f. has the same direction around the coil as that induced when the edge *a* cuts lines of force, and the total e.m.f.

is the sum of the two. This increase in e.m.f. is represented graphically by the line *ab* in (*b*). The e.m.f. now continues fairly uniform until the edge *a* of the coil reaches the edge *n* of the magnet, as indicated by the line, *bc* in (*b*); but from this point on,

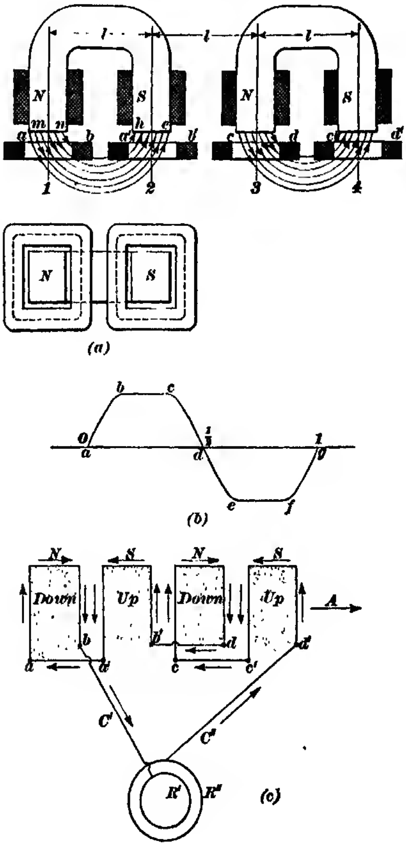


FIG. 72.

the e.m.f. decreases, as indicated by the line *cd* in (*b*), until, when the coil has reached position 2, it is no longer cutting lines of force, and the e.m.f. is 0. Edges *a* and *b* now occupy positions *a′* and *b′*. A further movement to the right, and *a′* begins to cut

lines of force; but, since these are flowing in the opposite direction, the direction of the e.m.f. is reversed. If a second magnet be placed alongside of the first, so that the distance l between any two consecutive poles is the same, the line *defg* in (*b*) represents graphically the variation in e.m.f. while the coil is passing from position 2 to position 3. The curve *defg* is symmetrical to *abcd* with reference to the point *d* as the center of symmetry; it is located below the line *ag* to indicate that the e.m.f. is opposite in direction. Any further movement of the coil to the right will result in a repetition of the curve *abcdefg*, beginning at *g*.

It will be noted that when the coil is in position 3, it occupies the same relative position as regards the north pole that it occupied in position 1; in other words, the curve in (*b*) represents a complete cycle of changes in the e.m.f. Any further movement to the right from position 3 will start another cycle, and if more magnets are placed alongside the two shown, a new cycle will be started every time the center line of the coil passes the center line of a north pole of a magnet.

167. Referring again to (*a*), Fig. 72, suppose that the four coils under the two magnets be connected in series. The manner of doing this is illustrated diagrammatically in (*c*), where the partial rectangles represent one turn of each coil and the dots represent a cross-section of the lines of force, the whole corresponding to a bottom view of the coils in (*a*). Note that the corresponding (inner) edges a, a', c, and c' are connected, and that the outer edges are similarly connected, except the first and last, which are connected to the external circuit through the slip rings R' and R''. The direction of the current is indicated by the arrows, when the coils are moving in the direction of the arrow A. The e.m.f. in the conductors C' and C'' will evidently be that due to the sum of the e.m.f.'s in the four coils. If more magnets are placed in line with the two shown and at the same distances apart, the e.m.f. curve will resemble that shown at (*b*).

It may here be remarked that the outline of the e.m.f. curve may have various shapes, depending upon the construction of the alternator, but one half-cycle will always be symmetrical to the other half-cycle with respect to the point where the curve crosses the base line *ag*, Fig. 72(*b*).

168. If the magnets be arranged in a circle and the coils be placed in slots cut in the circumference of a wheel, as shown in

Fig. 73, the same conditions will obtain as in Fig. 72 when the wheel revolves and the magnets are stationary. The wheel *W* with the coils make up the armature, and the magnets and stationary frame constitute the field. The wires of the coils are heavily insulated, the coils are insulated from the iron core in the manner indicated by the black sectioned parts *i*, and are kept in place by the wooden plugs *p*. The coils are connected in

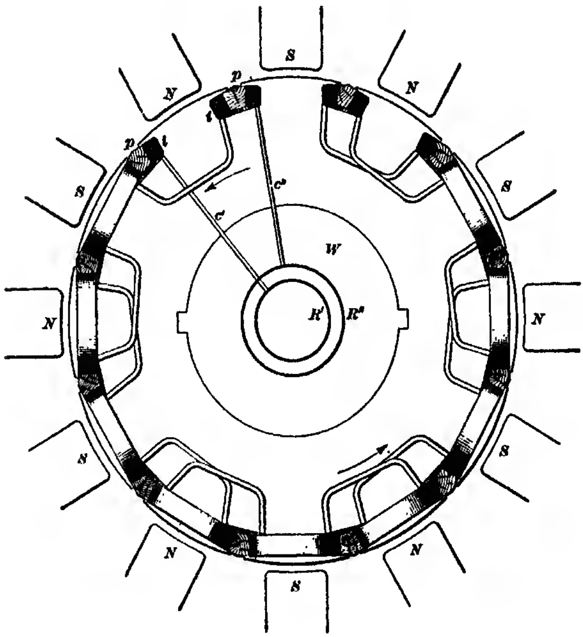


Fig. 73.

series and the terminals are connected to the slip rings R' and R'' in the manner shown. The armature is supposed to be revolving counterclockwise, as indicated by the arrows. It will be noted that this alternator has 12 poles and 12 coils.

169. Alternations and Frequency.—Every time the centerline of a coil passes the centerline of a pole piece, there is a change in the direction of the e.m.f., and the e.m.f. curve passes from one side of the base line to the other; this is called an **alternation,**

and there are evidently two alternations for every cycle (see Art. 14). Consequently, the coil must pass two poles, one north and one south, to complete a cycle; and for every revolution, there will be as many alternations as there are poles, and the number of cycles will be half as many as the number of poles, or $\frac{p}{2}$, where p is the number of poles. The number of cycles per second is called the **frequency**; and if F is the frequency, and n is the number of revolutions per second made by the armature,

$$F = \frac{p}{2} \times n = \frac{pn}{2} \qquad (1)$$

If N is the number of revolutions per minute, $n = \frac{N}{60}$, and

$$F = \frac{p}{2} \times \frac{N}{60} = \frac{pN}{120} \qquad (2)$$

If the frequency and number of poles are known or have been decided on, the number of revolutions per minute can be found by solving formula (2) for N, obtaining

$$N = \frac{120F}{p} \qquad (3)$$

For example, how many revolutions per minute must the armature of the alternator of Fig. 73 make to give a frequency of 60 cycles per second? Here the number of poles is 12, and $F = 60$; substituting in formula (3), $N = \frac{120 \times 60}{12} = 600$ r.p.m.

The frequency most used is 60 cycles per second, though 25 cycles per second is much used; frequencies of 30, 50, 125 and 133 cycles per second are also employed. The number of poles used varies from 4 to 80, the larger machines usually having the greater number of poles.

170. Revolving Field Alternators.—An inspection of Fig. 73 will make it clear that insofar as frequency, cutting lines of force, and generating e.m.f. are concerned, it makes no difference whether the armature is revolved or the field is revolved; hence, the coils and poles might be interchanged as regards position, and the field revolved. When this done, as is frequently the case, the stationary part carrying the coils in which the e.m.f. is induced is called the **stator**, and the revolving part, which carries the magnets, is called the **rotor**.

171. Two- and Three-Phase Alternators.—Referring to Fig. 74, the full line represents the e.m.f. curve for a single-phase alternator, such as has just been described. Instead of letting the base line *ac* represent the *time* for completing one cycle, suppose it to represent one circle, or 360°; then the point *b*, midway between *a* and *c*, will be 180°, and *ab* represents a semicircle; the point *a′*, midway between *a* and *b*, will be 90°, and *aa′* represents a quadrant. Every cycle may be represented by a circle of 360°. In the case of a bipolar machine a coil in going through one cycle goes through a circle of 360°, from the north pole, say,

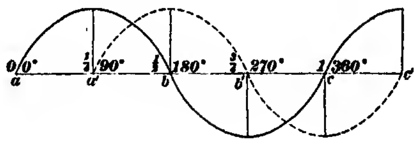


FIG. 74.

to the south pole and back to the north pole again. In the case of an 8 pole machine as in Fig. 75, the coil goes from one north pole passing a south pole and to the next north pole in completing one cycle. Now for convenience in figuring, the arc from the center line of one pole to that of the next like pole is considered to be divided into 360 equal spaces and to eliminate confusion these spaces are called **electrical degrees.** In a bipolar machine it is readily seen that an electrical degree is equal to a circular degree. In a four-pole machine an electrical degree is only ½ a circular degree, in a six-pole machine ⅓, and so on. Hereafter difference of phase will be expressed in electrical degrees.

Now suppose that the coils on the armature of an alternator were so connected that they formed two sets, the coils of each set being connected in series, and each set supplying its own circuit in the line. This may be done by placing the second set midway between the first set. The result of this is that the cycle for the e.m.f. curve (shown dotted) of the second set, Fig. 74, starts at *a′* instead of *a*, the e.m.f. being 0 when the e.m.f. of the first set is a maximum; 90° farther, and the e.m.f. of the second set will be a maximum, while for the first set it is 0; etc. In other words, there are two different phases, and the second set (or, rather, the e.m.f. of that set) is said to be 90° out of phase with

the first set; the phase of the second set being 90° *behind* the first set, since its cycle begins 90° *after* the first cycle.

172. One form of connection for a two phase alternator is shown in Fig. 75. There are 8 poles and two sets of coils, 4 in each set. There are also 4 slip rings. Note that the coils *B* are so placed that their center lines are 90° ahead of the center-lines of the coils *A*. Coils *A* are connected in series, and their

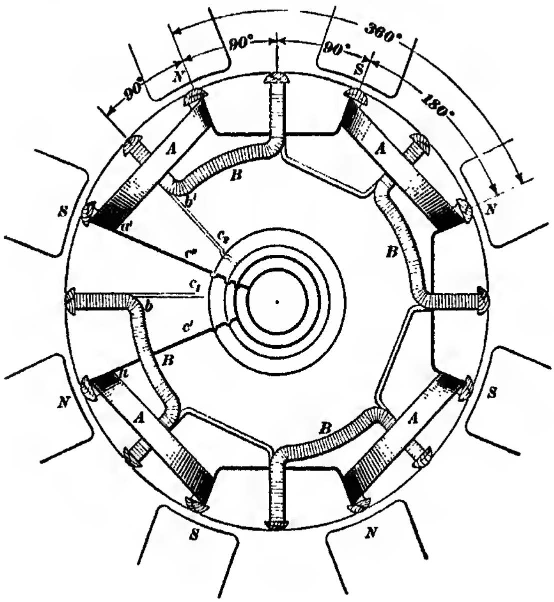


Fig. 75.

terminals are connected to the two inner slip rings by the conductors c' and c''. The coils *B* are also connected in series, and their terminals are connected to the two outer slip rings by the conductors c_1 and c_2. In reality, the slip rings are all of the same diameter, different sizes of circles being used here to separate the rings. It should be noted also that each pair of slip rings serves its own circuit.

173. In the three-phase alternator, the phases are 120° apart, the second being 120° behind the first, and the third being 120° behind the second and 240° behind the first. The e.m.f. curve

for this arrangement is shown in Fig. 76. Here 3 cycles are shown for curve A, the first phase; curve B represents the second phase and starts 120° after A; curve C represents the third phase and starts 240° after A, the distances $ab = bc = cd$ representing 360°. The number of slip rings may be 6, 4, or 3. When 6 slip rings are used, each phase supplies its own external circuit. When 4 slip rings are used, three wires lead to the external circuits, and all have a common return wire. When 3 slip rings are used, one end of each phase (set of coils) is joined to a common connection, and the other ends are joined to the three rings;

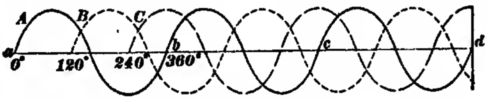


FIG. 76.

or, the phases are connected to form a closed circuit, the slip rings being attached to the points where the phases join. Both methods of connecting when 3 slip rings are used give the same output in watts, but the former method gives a higher voltage, while the latter method gives a higher current.

The three-phase alternator is the type of polyphase generator most used in practice, **polyphase** meaning more than one phase, and the three-slip-ring type is the one almost universally used in three-phase alternators.

ALTERNATING CURRENT MOTORS

174. Alternating current motors may be divided into two general classes: *synchronous motors* and *induction motors.*

The word **synchronous** means at the same time, that is, simultaneous, and has reference in this connection to the fact that the motor must run in phase with the current that is supplied to it; in other words, the alternations of both must be equal and simultaneous.

175. Synchronous Motors.—Synchronous motors may be operated on either a single-phase or a polyphase system, but the field must be excited by a separate direct-current from some source. If a single-phase motor be connected to a single-phase alternator, the motor will not start up and run, because the cur-

rent in its armature is rapidly alternating in direction; this tends to make the armature turn first one way and then the other, and it does not get started from rest. If, however, some means are employed for starting and bringing the armature up to speed, then the motor will operate, provided the armature is supplied with the proper alternating-current voltage of correct frequency, and its field is supplied with the proper direct-current voltage. A polyphase motor, however, will start and run up to synchronism, when the armature is supplied with alternating current of the proper voltage, frequency, and number of phases and its field is supplied with the proper direct current.

176. Disadvantages of Synchronous Motor.—The principal disadvantage of the synchronous motor is that its speed is not adjustable, since it depends upon the generator frequency and not upon any variation in the e.m.f., as in the case of d.c. motors. In other words, the motor must run in synchronism with its supply current, which means that the motor speed must be that which it would have if running as an alternator and supplying current of the same frequency. Hence, variations in the speed of the motor depend upon variations in the speed of the alternator that supplies the current. Another disadvantage is that a synchronous motor must be supplied with both alternating current and direct current, the first for the armature and the second for the field; in many cases, this is not only inconvenient but also expensive. The starting process is more complicated than with other forms of motors, whether run with alternating or direct current. The synchronous motor, however, is very valuable in cases where a constant speed motor can or must be used.

177. Speed and Direction of Rotation.—The speed of a synchronous motor may be found by the formula

$$N = \frac{120F}{p}$$

in which N = revolutions per minute of armature, F = frequency, and p = number of poles. Thus, if the frequency is 30 cycles per second and the number of poles is 8, $N = \frac{120 \times 30}{8}$ = 450 r.p.m. Observe that the formula is the same as formula (3), Art. **165**. Consequently, if a motor have the same number of poles as the alternator that supplies its current, the speed of both will be the same.

A synchronous motor will run in either direction; but the direction in which it actually turns will be that in which it revolves when started by its auxiliary motor. If a synchronous motor be started by simply allowing current to flow through its armature, the direction in which it revolves will depend upon the way in which the terminals of the armature coils are connected to the external circuit.

178. Induction Motors.—The induction motor works on the principle of mutual induction, which was explained in Art. **127.** Such motors are generally operated on two-phase or three-phase circuits, but may be operated on single-phase circuits.

Induction motors always consist of a stationary part, called the **stator,** and a rotating part, called the **rotor.** The current is usually led to the stator, the coils on which form the field, and which are also called the **primary**; the rotor consists of the iron core and conductors, which are also called the **secondary.** The names primary and secondary have the same meaning as in Art. **127.** Either the primary or the secondary may revolve, but they may always be distinguished by the fact that the primary is the part that receives the current. The usual arrangement is for the secondary to rotate, and this will be assumed to be the case in what follows.

179. It will help in understanding the action of an induction motor by considering what will happen to a *direct-current* motor under the following conditions. Suppose the motor to have, say, four poles, and that the field has been excited and a current sent through the brushes in the usual manner; the armature will rotate, and the greater the load on the motor the greater must be the current strength through the brushes. It will be noted that in this case the armature is caused to rotate by the reaction between the magnetism of the field and the whorls set up by the current through the conductors of the armature and received through the brushes. Now suppose that the brushes be disconnected and that the entire outside surface of the commutator be enclosed by a copper ring, with which it forms close contact; this connects together the ends of all the armature conductors, short-circuiting them, and making each coil a *closed* circuit. If the armature be fixed, so it cannot rotate, and the field be revolved about it, the lines of force from the field pole-pieces will be cut by the conductors on the armature, and an e.m.f. will

be set up in the armature coils; and since the coils are all short-circuited (connected in parallel with the copper ring on the armature), a heavy current will be set up in them, which current will react on the field, producing a heavy drag on the armature. If the armature were not fixed, this drag would cause it to turn, following the field. The armature, however, cannot turn at the same rate as the field, for if it did, the armature coils and field coils would both be turning at the same number of revolutions per minute, and no lines of force would be cut; hence, the armature must always turn slower than the field. The difference in the relative speeds of the armature and field is called the **slip**, which is usually expressed as a per cent. Thus, suppose the speed of the field is 480 r.p.m., speed of armature is 460 r.p.m.; then, $480 - 460 = 20$, and $\frac{20}{480} = .04\frac{1}{6} = 4\frac{1}{6}\%$, the slip.

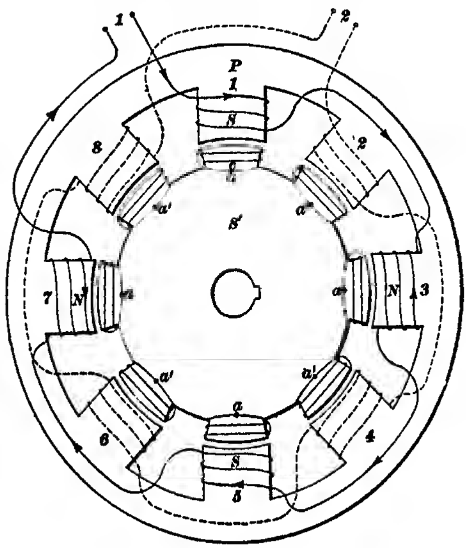


Fig. 77.

180. Much the same effect can be produced by a stationary field and a two-phase or three-phase alternating current. For instance, suppose an arrangement similar to that in Fig. 77, in which P is a stationary field having 8 pole-pieces, wound so as to make two phases of 4 poles each. Poles 1, 3, 5, and 7 belong

to one phase and 2, 4, 6, and 8 to the other phase, the second being 90° out of phase with the first. If, now, an alternating two-phase current, one phase being 90° ahead of the other, be sent through the field coils, the e.m.f. in the coils will rise and fall in exactly the same manner as it was generated in the alternator. Thus, it will rise in the coils of poles 1, 3, 5, and 7 until it reaches its maximum value; then as it falls in the coils of the odd numbered poles, it rises in the coils of the even-numbered poles, the result being practically the same as though a direct current were sent through the field coils and the fields were revolved. The effect is much the same as in the case of a moving electric sign, where a light ahead goes on as the one behind goes out, producing an appearance of movement, as of a snake; there is no movement, but there appears to be one. Opposite the poles of the field are the poles of the armature, the ends of the coils being joined at a and a', thus making each coil a closed circuit. The rise and fall of the e.m.f. in the field coils (the primary) induces a current in the armature coils (the secondary) in exactly the same manner as described in Art. **127**, and as the magnetism goes from one set of poles to the other, it drags the armature (rotor) after it with a turning force, called the **torque**, that is proportional to the slip; the greater the percentage of slip the greater the torque. The resistance of the conductors of the rotor is so small that the current is very heavy, with the result that the percentage of slip is quite small, varying from about 1% at no load to about 5% or 6% at full load on the motor, and being greater for small motors than for large ones.

181. The rise and fall of the e.m.f. in the field coils, in the above case, may be represented graphically in the same manner as in Fig. 74. While the field is stationary, the effect is the same as though the field revolved, as in Art. **179**; hence, this apparent movement is called the **revolving field.** The speed of rotation of the revolving field is the same as that of an alternator of the same number of poles that produces a like current. Therefore, it is customary to consider this revolving field as producing lines of force that are cut by the rotor conductors, thus generating an e.m.f. in them. In reality, however, the current in the rotor conductors is caused by mutual induction.

182. Induction motors are not usually made in accordance with the arrangement described in connection with Fig. 77.

Instead of pole-pieces projecting out from the inner surface of the stator, slots are cut in the stator, and the coils are placed in the slots, as shown at (*a*), Fig. 78. Further, each pole is usually

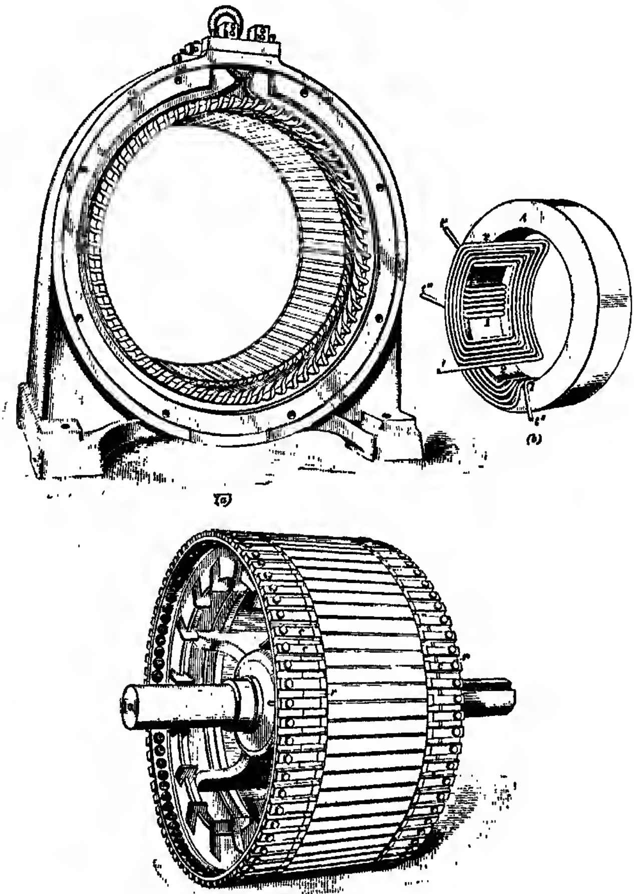


Fig. 78.

subdivided into 2, 3, or 4 parts. For example, if the poles of a 6-pole, 3-phase motor were subdivided into three equal parts, there would be $3 \times 6 = 18$ pole spaces, and the number of slots

would be $3 \times 18 = 54$. One method of connecting the windings is shown in (*b*). Although the poles here appear to overlap one another, they really do not, on account of the difference in phase.

The rotor is shown at (*c*). The conductors *c* are usually heavy copper bars of rectangular cross-section; placed in slots in the rotor core. The ends of the bars are fastened to the copper rings *r*, thus short-circuiting the conductors and connecting them all in parallel. On account of its resemblance to a squirrel cage, a motor having this kind of a rotor is called a **squirrel-cage motor.**

The rotor of an induction motor may be a wound rotor, especially where it is desirable to obtain small variations in speed or where a high starting torque is desired combined with a low starting current. The windings must be well insulated. When bars are used in the rotor of the squirrel-cage motor, they should be very well insulated or very well grounded. The practice is not to insulate, but ground them well.

183. On account of the comparatively large size of the copper conductors of the rotor, the resistance is very small, thus making the current very heavy. This current is confined to the conductors, moving in a short, closed circuit, and produces powerful magnetic effects, with the consequence that a small percentage of slip will result in a strong torque, thus enabling the motor to pick up a heavy load. It is to be noted that there are no slip rings and no commutator, and that the motor if polyphase is self starting. A single phase induction motor is not self starting.

The induction motor is essentially a constant speed motor, and is not adapted to use where variable speeds are desired. In some forms of these motors, variations in speed may be secured by inserting a variable secondary resistance, but the variations in such cases should be moderate.

One of the many advantages of the induction motor is that it does not require any direct-current supply; it is operated entirely on alternating current. This greatly simplifies its use in mills, and it makes possible the use of small individual motors in places where it would not be possible otherwise, on account of complications and expense.

The power factor (Art. **192**) of the current taken by an induction motor is low when operated at less than full load and increases as the capacity of the motor is approached. It is lower for small than for large motors of the same speed, and is lower for slow speed than for high speed motors of the same capacity.

Polyphase alternators and motors have the great advantage that they are well adapted to use on high voltage circuits; this is largely due to the absence of a commutator. Alternating-current motors are made that will operate on voltages as high as 13,200 volts, though the more usual voltages are 220, 440, 550, 2200, and 6600.

184. Single-Phase Motors.—Without going into details, it may be stated that a direct-current series motor will run if used on one phase of an alternating-current circuit; but if intended to run on an alternating-current circuit, the construction is somewhat different, to gain in efficiency and in operation. The series a.c. motor is used for the same purpose as the d.c. series motor—railways, cranes, hoists, etc.

In small sizes, series motors are often made so as to operate about equally well on either a.c. or d.c. circuits, and are then called universal motors; such motors are used for driving small fans, electric drills, etc.

While the single-phase synchronous motor is not self-starting, the alternating-current single-phase series motor is self-starting. Special forms of induction motors are manufactured that are self-starting and operate on single-phase circuits, with constant speed characteristics that are similar to the polyphase induction motor and the direct-current shunt motor; but these motors are used only where it is not easy to obtain polyphase current, and their construction usually involves commutators or somewhat complicated mechanism in the rotor, which often makes them undesirable for general use.

TRANSFORMERS

185. Object of Transformer.—It has already been shown that when power is transmitted at a low voltage, the voltage drop and power loss are considerable, unless the wires (conductors) are very large and, hence, expensive. (See Art. 70.) Direct current dynamos are usually not made for voltages higher than 600 volts, but alternators can be built for voltages as high as 13,000 volts, and even this is not high enough for transmitting large amounts of power long distances. For this reason, a device is employed to raise the voltage for transmission and to lower it for use. Devices of this kind are called **transformers**. When a

transformer is used to increase the voltage it is called a **step-up transformer** and when used to decrease the voltage it is called a **step-down transformer.**

With a step-up transformer, the voltage may be increased to as high as 220,000 volts, but the more usual voltages for power transmission are 22,000, 66,000, or 110,000 volts. At the point where the current is to be used, these are stepped down to 110, 220, 550, etc., volts. Similarly, transformers are often used to step down from, say, 550 volts (which is suitable for the motors in a mill) to 110 volts (which is suitable for the lamps), thus using the same source of power for both motors and lights.

It is to be noted that (assuming there are no losses) the transformer does not alter the power of the current. Thus, by formula (2), Art. **47**, $P = IE$; hence, if E is increased, I is decreased in the same proportion. For instance, if the dynamo generates 20 amperes at 1000 volts, and the transformer steps this up to 20,000 volts, the voltage has been increased $20{,}000 \div 1000 = 20$ times, the strength of the current has been reduced 20 times and is $20 \div 20 = 1$ ampere, and the power in both cases is $P = IE = 20 \times 1000 = 20{,}000 \times 1 = 20{,}000$ watts = 20 k.w.

186. Principle of the Transformer.—The transformer acts on the principle of mutual induction, explained in Art. **127.** Referring to Fig. 79, let A be the primary coil and B the secondary coil, C being an iron core. If an alternating current be sent through the primary, the increase and decrease in its e.m.f. will induce a current in the secondary; and if the secondary be made of the same size of wire of the same number of turns as the primary, the voltage will be the same in both, as may be ascertained by connecting a voltmeter to the terminals of the secondary; and the current in both will also be the same. Now assume that there is no loss of power in the transformation; then, if the number of turns in the secondary be doubled, the resistance will be doubled, and by Ohm's law, the current will be halved. But, since the power does not change, if the current is halved, E must be doubled, in order to keep the power con-

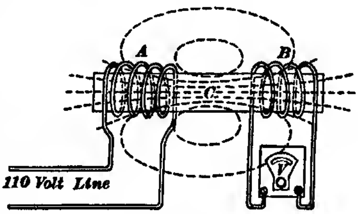


Fig. 79.

stant. Consequently, the ratio of the voltages in the primary and secondary must equal the ratio of the number of turns in them. For example, if the primary have 200 turns, the secondary 1200 turns, and the voltage of the current supplying the primary be 110, the voltage of the secondary will be 660 volts, because $\frac{200}{1200} = \frac{110}{E}$, from which $E = 660$ volts = voltage in secondary.

Evidently, a transformer cannot be used on a direct-current circuit, since there would then be no variation in the e.m.f. of the primary and no current could then be induced in the secondary.

187. Diagram of Step-down Transformer.—Fig. 80 represents in diagrammatic form a transformer that steps a current of 2200

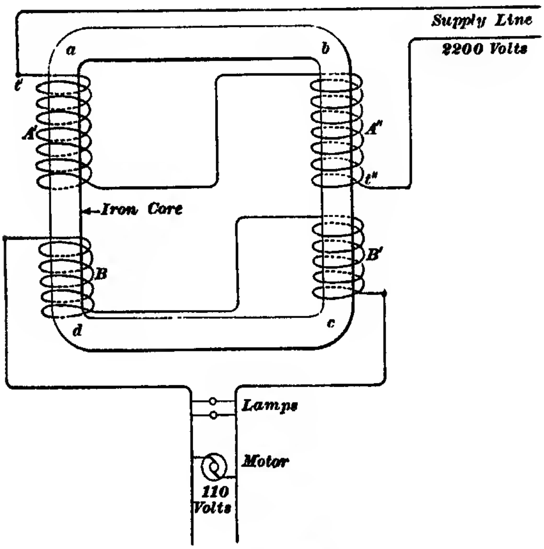


FIG. 80.

volts down to 110 volts, for use on a lamp circuit. The number of turns in the primary coil A will be 20 times the number in the secondary B, because $2200 \div 110 = 20$. The iron core *abcd* is made in one piece, as shown. The current from the alternator enters the primary at terminal t' and leaves at terminal t''. The external circuit is connected to the two terminals of the

secondary in the manner shown. It will be observed that the circuit taking the current is not electrically connected to the supply circuit, the current in the external circuit supplying the lamps and motor being an induced current.

188. A perspective view of a transformer of the shell type is shown in Fig. 81. The iron core C is made up of a large number of sheet-iron punchings. The primary coil is made in two parts, connected in series, as indicated by the wire s; and the secondary coil, also made in two parts, is placed between the parts P' and P'' of the primary. The terminals of the primary are indicated by t' and t''. The terminals of the secondary S' are a and b, and of the secondary S'' are c and d; consequently, the two secondary coils may be connected in series or multiple, as desired.

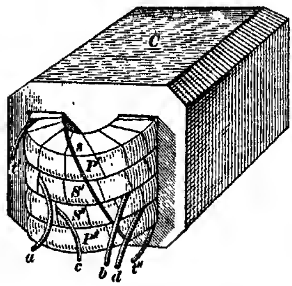


FIG. 81.

189. Efficiency of Transformers.—The transformer is one of the simplest and cheapest of electrical devices, chiefly because there are no moving parts. As a consequence, there are no losses due to friction, but there are other losses, comparatively small, which cause the efficiency to be less than 100 per cent., as in every case of transformation of energy, though no useful work is done. Small transformers have a lower efficiency than large ones; the efficiency of a small transformer may be as high as .95, or 95 per cent., and of a large one, as high as 99 per cent.

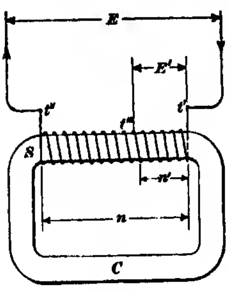


FIG. 82.

When the transformer is well made and the coils are properly insulated from each other, it requires practically no attention; simply an inspection once or twice a year.

190. Autotransformers.—What is called an autotransformer uses only a single coil; it is shown diagrammatically in Fig. 82 and operates on the following principle: The coil S encloses

an iron core, which forms a complete magnetic (iron) circuit. The current from the alternator enters the coil at the terminal t' and leaves at t''. One wire of the secondary is connected at t' and the other at some point t''' between the two terminals of the coil. If the difference in voltage between t' and t'' be E and between t' and t''' be E', and if n = the number of turns in the coil and n' = the number of turns between t' and t''', then $E' : E = n' : n$, from which,

$$E' = \frac{E \times n'}{n}$$

For example, if $E = 600$ volts, $n = 240$ turns, and $n' = 60$ turns, $E' = \frac{600 \times 60}{240} = 150$ volts = voltage in the secondary circuit. The voltage in the secondary is here ¼th that in the primary; hence, the current in the secondary is 4 times that in the primary.

The principle use of the autotransformer is for starting alternating current motors, where a low voltage is required to start and while running up to speed; it is for this reason that they are also called **starting compensators.** The chief advantage of autotransformers is their cheapness and small size. They are not adapted to use on power and lighting circuits, because the secondary and primary have direct electrical connection; and since the voltage of the primary is very much higher in such cases than the voltage of the secondary, it would be dangerous to have the secondary circuit in connection with the primary.

POWER FACTOR

191. Real and Apparent Power of Alternating Current.—As has been shown in Art. **131** and Fig. 60, the e.m.f. of an alternating current starts at 0, increases to a maximum in one direction, decreases to 0, then increases to a maximum in the other (opposite) direction, and falls to 0 again, this sequence being repeated over and over again. It has also been shown that the strength of the current in a closed circuit depends upon the applied e.m.f., the greater the e.m.f. the greater the current, other conditions remaining the same. Consequently, as the voltage increases in value in one direction, the current also increases in the same direction; when the voltage decreases, the

current decreases, and when the voltage (e.m.f.) is reversed and increases in the opposite direction, the current likewise reverses and increases in that direction. Therefore, if the resistance is constant, the current is proportional to the e.m.f., and the rise and fall of the current may be indicated by means of a curve that will be exactly similar in outline to the curve of e.m.f. in Fig. 60. If the only opposition to the flow of current in the circuit is the resistance, the e.m.f. and the current will both reach the maximum value at the same instant, and they will both decrease and become 0 at the same instant. In such case, the e.m.f. and the current will be running along together, so to speak, and are then said to be **in phase**.

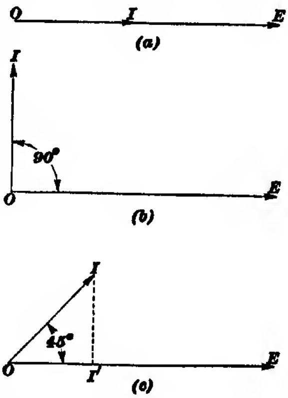


Fig. 83.

This condition may be indicated graphically as in Fig. 83 (*a*), where the horizontal line OE represents the number of volts to any convenient scale. For example, if the scale is 100 volts = 1 inch, then if OE is to represent 220 volts, its length must be $220 \div 100 = 2.2$ in. The current may be laid off in the same manner, and for the same instant, on the same line. For example, if the current is 16 amperes and a scale of 16 amp. = 1 in. be selected, lay off $OI = 16 \div 16 = 1$ in. At this instant then, OE represents the e.m.f. (220 volts) and OI represents the current (16 amperes).

If there is any *inductance* in the circuit and an e.m.f. is applied to it, the current will not immediately reach its full value of $I = \frac{E}{R}$; an appreciable time elapses before this value is attained. Hence, when an alternating-current e.m.f. is applied to a circuit containing inductance, the current will not reach its maximum value until *after* the e.m.f. has done so; the current is then said to **lag** behind the e.m.f. The time required for the e.m.f. to complete the curve (Fig. 60) from 0 to the maximum in one direction and back to 0, then to the maximum in the other direction and back to 0 is considered as 360°; from 0 to a maximum in either direction is $\frac{1}{4} \times 360° = 90°$. Consequently, if the

current does not reach its maximum until the voltage has decreased to zero, the current *lags* 90°, and no power can be transmitted. This effect is shown graphically in Fig. 83(*b*). Here OE is a horizontal line representing the e.m.f. and OI is a line at right angles to it representing the current. If the current reaches a maximum when the e.m.f. is half way between its maximum and 0, the line OI, which represents the current, will then make an angle of 45° with OE, as shown in Fig. 83(*c*), and the current is said to lag 45°. In such case, the effective power called the **real power,** will not be that due to the current OI, but that represented by the *projection* of OI on OE, represented in the figure by OI'. The power resulting from the current represented by OI is called the **apparent power.** The ratio $\frac{OI'}{OI} = \frac{\textit{real power}}{\textit{apparent power}} = f$ is called the **power factor.** In (*a*), Fig. 83, $OI' = OI$, and the power factor is $f = \frac{OI}{OI} = 1 = 100\ \%$; in (*b*), $OI' = 0$, and $f = \frac{0}{OI} = 0$; in (*c*), $OI' = \frac{1}{2}\sqrt{2} \times OI = .707 + \times OI$, and $f = \frac{.707 + \times OI}{OI} = .707+ = 70.7\ \%$.

192. The Power Factor.—In alternating currents, that part of the total current which is in phase with (in the same direction as) the e.m.f. is the only part that will perform useful work in conjunction with the voltage; this is the reason why the component parallel to the e.m.f. is called the real power. In (*a*), Fig. 83, it is evident that 100% of the current is in phase with the voltage (e.m.f.); hence, in this case, the useful current is 100 % of the total current, and the real power equals the apparent power. In (*b*), it is evident that there can be no part of the current in phase with the voltage, the useful current is 0% of the total current, and the real power is 0. For any lag between 0° and 90°, let θ = angle of lag; then, by trigonometry, OI' in Fig. 83(*c*) is determined by the equation $OI' = OI \times \cos\theta$, and the power factor is equal to

$$f = \frac{OI \times \cos\theta}{OI} = \cos\theta$$

If the angle of lag is 30°, $f = \cos 30° = \frac{1}{2}\sqrt{3} = .866+$; if the angle of lag is 45°, $f = \frac{1}{2}\sqrt{2} = .707+$; and if the angle of lag = 60°, $f = \cos 60° = \frac{1}{2} = .5$. In Fig. 83(*b*), $\theta = 90°$, cos

$90° = 0$, and $f = 0$; in Fig. 83(*a*), $\theta = 0°$, $\cos 0° = 1$ and $f = 1$. In the foregoing expressions, *cos* is the abbreviation for cosine.

The power factor is usually expressed as a percentage, in which case, a power factor of, say, 65% means that the angle of lag is one whose cosine is .65, which corresponds to an angle of about 49° 28′.

Equation (2), Art. **47**, $P = EI$, holds only for direct currents or for alternating currents whose power factors are 100 per cent. The real power (useful power) of an alternating current circuit is expressed by

$$P = EI\cos\theta = fEI \qquad (1)$$

This formula is true only for single-phase circuits. In a two-phase circuit, the real power is not equal to twice the power of either phase, but $\sqrt{2} = 1.414+$ times the real power of one phase; that is,

$$P = \sqrt{2}EI\cos\theta = 1.414fEI \qquad (2)$$

For a three-phase circuit, the real power is expressed by

$$P = \sqrt{3}EI\cos\theta = 1.732fEI \qquad (3)$$

Example.—What is the real power of, the useful power delivered by, a single-phase generator, if the e.m.f. is 2200 volts, the current is 300 amperes, and the power factor is (*a*) 80%, (*b*) 60%, and (*c*) 100%?

Solution.—Applying formula (1),

(*a*) $P = .80 \times 2200 \times 300 = 528000$ watts $= 528$ k.w. *Ans.*
(*b*) $P = .60 \times 2200 \times 300 = 396000$ watts $= 396$ k.w. *Ans.*
(*c*) $P = 1.00 \times 2200 \times 300 = 660000$ watts $= 660$ k.w. *Ans.*

Thus, it is seen that with a power factor of 100%, the real power delivered by the generator is 660 k.w., but when the power factor is 60%, the real power delivered by the generator is only 396 k.w.

BUYING ELECTRIC POWER

193. The Time Element.—When buying electrical energy, it is just as necessary to specify a definite quantity as when buying coal, gasoline, or any other commodity. If, for example, one should attempt to buy a kilowatt of power at a central station, it would be impossible to tell the cost until the length of time that it was to be used had been stated. Power is a relative term; hence, the time element is always considered in the sale of electric power, and the kilowatt-hour (k.w.h.) is generally the unit

employed in making out the charges, except where large amounts are used.

194. Cost per Kilowatt-Hour.—The common price for electric energy used for lighting and other household purposes is 10 cents per k.w.h. It may be higher for some steam-power plants and less for some water-power plants. When used in considerable quantities for power purposes, the charge may be 1 or 2 cents per k.w.h. The wide difference in the two prices is due to the fact that lights are used, as a rule, for only a few hours during the day; but the electric company must nevertheless keep its apparatus ready and running at all times to supply the current when wanted. Thus a lighting load on an electric light plant generally means considerable idle apparatus for the greater part of the day. This, of course, is expensive, and hence, electricity for house lighting must be charged for at a higher rate than when used continuously between definite hours for power purposes.

195. Cost of Lighting.—Suppose a room in a mill requires 960 c.p. (candlepower); if 60 carbon lamps of 16 c.p. each be used, each lamp will require about $3\frac{1}{8}$ watts per candlepower, or $3.125 \times 16 = 50$ watts for each lamp, and the total watts for 960 candlepower will be $50 \times 60 = 3000$ watts. If the lamps are burned 10 hours each day and the price charged for the current is 2 cents per k.w.h., the cost per day will be $\frac{3000}{1000} \times 10 \times 2 = 60$ cents. For a year of 365 days, the cost will be $365 \times 0.60 = \$219.00$.

If 12 80-candlepower tungsten lamps are used, each lamp will take about 1 watt per c.p., and the number of watts required will be 960, or .96 k.w. At 2 cents per k.w.h. and 10 hours per day, the cost per day will be $.96 \times 10 \times 2 = 19.2$ cents; the cost per year will be $365 \times 19.2 = 7008$ cents $= \$70.08$ and the saving over the carbon lamps will be $\$219.00 - \$7008 = \$148.92$.

196. Cost of Operating a Motor.—Suppose that it is found that a certain pulp grinder uses 500 horsepower; what will be the cost of operating it by an electric motor? Since 746 watts $= 1$ horsepower, 500 horsepower $= 746 \times 500 = 373{,}000$ watts $= 373$ k.w. If the motor has an efficiency of 90%, the input to the motor must be $373 \div .90 = 415$ k.w., nearly. If the power charge is 1 cent per k.w.h. and the motor is operated 24 hours per day, the cost per day will be $415 \times 24 \times 1 = 9960$ cents $= \$99.60$ per day.

197. Block Power.—These methods of calculating the cost of electric power are employed when rather small amounts of power are used. Whenever large amounts of power are required, to be used throughout the year, it is customary to sell the power in *blocks*, as it is termed, the unit being the **horsepower-year.** For instance, suppose a mill requires an available maximum of 2000 horsepower; it might arrange with an electric company to pay, say, \$25.00 per h.p. per year; the bill would then be \$50,000 per year, since 2000 × 25 = \$50,000. The mill would have to pay this amount, whether the 2000 h.p. were used all the time or whether, as is usually the case, the 2000 h.p. were used only a part of the time or even not at all. Suppose that the average amount of power used all the time throughout the year were 1000 h.p.; this would cost \$50,000, as per contract. But at 1 cent per k.w.h., assuming the mill to run 300 days per year and 24 hours per day, the cost would be 1000 × .746 × 24 × 300 × .01 = \$53,712, or \$3,712 more.

If the average horsepower used throughout the year were 1500, the cost at 1 cent per k.w.h. would be 1500 × .746 × 24 × 300 × .01 = \$80,568, and the saving would then be \$30,568.

198. The Fixed-and-Meter Rate.—Another method of buying power is by what is called a **fixed-and-meter-rate** contract. The fixed rate is, say, \$9 per h.p.-yr., and is designed to cover the investment charges on the power held in reserve for the customer; this fixed rate is applied either to the maximum demand or to the connected load (combined rating of motors, lamps, etc.); the former is generally better for the customer, since most plants have more motor capacity than they actually require. If, therefore, the maximum demand in the example of the last article were, say, 2000 h.p., the fixed charge would be \$9 × 2000 = \$18,000 per year. The meter rate is designed to cover the generating expenses of the power company; for hydro-electric power, this is about 0.7 cents per k.w.h. This may again be modified by using a sliding scale, the rate decreasing as consumption of power increases. If the average load is 1000 h.p., the power used would be at the rate of 1000 × .746 × 24 × 300 = 5,371,200 k.w.h. per year, or 5371200 ÷ 12 = 447,600 k.w.h. per month. The meter rate might be, say, 1 cent per k.w.h. for the first 100,000 k.w.h. per month, 0.6 cent for the second 100,000 k.w.h., and 0.3 cent for any amount above 200,000 k.w.h. per

month. Under these conditions, the cost of power in the above case would be

Fixed charge, $18,000 ÷ 12		=	$1500.00 per month
100,000	k.w.h. at 1 cent	=	1000.00 per month
100,000	k.w.h. at 0.6 cent	=	600.00 per month
247,600	k.w.h. at 0.3 cent	=	742.80 per month
447,600	Total	=	$3842.80 per month

The total cost under these conditions is $3,842.80 per month, or $46,113.60 per year, a saving of 53,712.00 – 46,113.60 = $7,598.40 per year over the flat rate of 1 cent, but 50,000 – 46,113.60 = $3886.40 less per year than under the block power rate, for the conditions mentioned.

If an average of 1500 horsepower were used throughout the year, the average meter rate per month would be 671,400 k.w.h. Then,

Fixed charge			$1500.00 per month
100,000	k.w.h. at 1 cent	=	1000.00 per month
100,000	k.w.h. at 0.6 cent	=	600.00 per month
471,400	k.w.h. at 0.3 cent	=	1414.20 per month
671,400	Total	=	$4514.20 per month

The total cost per month would then be $4514.20, or $54,170.40 per year, which is $4170.40 more than under the block power rate.

If the average horsepower used throughout the year is about 1240, the cost by either method will be the same; if less than this, the fixed-and-meter rate will give a lower total cost, but if greater than this, the block-power rate will give a lower total cost.

EXAMPLES

1. A 4-pole dynamo armature has 100 conductor segments (50 coils) and makes 2000 r.p.m., the cross section of each pole piece is 120 sq. in. and the average flux density 25,000 lines of force per sq. in. What e.m.f. will be generated? *Ans.* 200 volts.

2. Referring to example 1, what will be the strength of the current in amperes if the resistance of the circuit (internal and external) is 120 ohms? *Ans.* 1⅔ amps.

3. What is the speed of a 6-pole synchronous motor on a 60-cycle circuit? *Ans.* 1200 r.p.m.

4. An alternator generates current at 550 volts, which is to be transmitted at 22,000 volts and used on a motor designed for 440 volts. What must be the ratio of turns in primary and secondary coils in (*a*) the step-up transformer? (*b*) in the step-down transformer?

Ans. (*a*) 1:40. (*b*) 1:50.

5. Allowing for Sundays and holidays, a mill requiring 2000 h.p. on full load but taking an average day load of 1500 h.p. is operated 300 days per year. In addition, 30 h.p. is used all the time (365 days) for lights, fans, etc., and 150 h.p. is required in the finishing room, which runs only 8 hours a day. What will be the yearly rate at (*a*) 1½ cents per k.w.h. for power actually used, and (*b*) at block power rates, 2000 h.p., at $40 per h.p. per year? (*c*) How much is saved per year by the cheapest method of buying?

Ans. (*a*) $127,821.13. (*b*) $80,000. (*c*) $47,821.13.

50. The chemical name for water is hydrogen oxide, the molecular formula for which is H_2O. But oxygen forms another and very different compound with hydrogen, called ***hydrogen peroxide***, having a molecular formula of H_2O_2, which is a clear, sirupy liquid, having a specific gravity of 1.458, that of water being 1; it is usually obtained as a solution in water. Hydrogen peroxide (sometimes called hydrogen dioxide) is a very unstable compound, a very slight heating being sufficient to release the extra atom of oxygen, causing the peroxide to reduce to water and oxygen The atoms of oxygen thus freed are in a great hurry to combine with other atoms to form molecules, and are said to be in a **nascent** condition or state, the word nascent here meaning *just set free, just born.* When atoms are in a nascent state, they are far more active in seeking combination than at any other time. Consequently, the desire of the oxygen atoms in the nascent state to unite with other atoms is so powerful that it makes hydrogen peroxide a strong bleaching agent and an extremely good germicide. The union of oxygen with another element or compound is called **oxidation**, and bleaching is a process of oxidation, as will be explained later; combustion, the rusting of iron, etc. are all processes of oxidation.

51. Ozone.—Oxygen usually forms molecules consisting of two atoms, the molecular formula of which would be O_2; but there is a form in which the molecule contains three atoms, the molecular formula of which is O_3; oxygen in this state is called **ozone.** Ozone in its natural state is a gas of pale blue color, and has a characteristic odor; like hydrogen peroxide, it is very unstable, the extra atom being easily released, leaving ordinary oxygen and the nascent oxygen, which is, as in the case of hydrogen peroxide, a strong oxidizing agent, thus making ozone a strong bleaching agent, a disinfectant, and germicide.

52. Oxidation and Reduction.—Oxidation has already been defined; it signifies the union of oxygen with some other element or compound, the object being to increase the amount of oxygen in it or to form a compound containing oxygen. In the case of combustion of wood and coal, the oxygen is obtained from the air; but in other chemical reactions, the oxygen is usually obtained from another compound containing it, in which case, the compound losing its oxygen is said to be **reduced.** The principal metals rarely occur in the free state; they are usually

combined with oxygen, and are chemically known as **oxides.** When these oxides are found in considerable quantities, they are called **ores.** An ore, however, is not necessarily an oxide, an ore being any compound containing a metal, which occurs in considerable quantity, as, for instance, galena, which is lead sulphide, a compound of lead and sulphur.

Carbon takes oxygen from metallic ores containing it, oxygen having a greater liking for the carbon than for the metal. By heating the mixed carbon and ore in a place from which air is excluded, the oxygen of the ore combines with the carbon, the carbon is oxidized, and the ore is reduced, leaving the metal free. This process is called **reduction.** Oxidation and reduction are exactly opposite processes; in almost every case, the reduction of one compound is brought about by the oxidation of another. When the object of the process is to oxidize an element or compound, the reduction part is not considered, and the process is called *oxidation;* but if the object is to get rid of the oxygen in a compound, then the process is called *reduction.* The materials used to effect the oxidation or reduction are called **oxidizing agents** and **reducing agents,** respectively.

53. Nitrogen.—Nitrogen, the other chief constituent of the air, is a strange acting element. Though it forms nearly 80 per cent (by volume) of the air, it is only with great difficulty that it can be obtained from the air in a form available for use in industry and for food. Yet no plant or animal can live without nitrogen.

A certain family of plants known as legumes, of which beans and peas are the most common varieties, have the power of taking nitrogen directly from the air and storing it in the seed, plant, and roots; but other plants must get it from the soil. The manure of animals and decaying animal and vegetable matter give up nitrogen as a compound with hydrogen, which is known as **ammonia.** Up to quite recent times, these and the nitrate beds of Chili were the chief sources of available nitrogen. In the last few years, by means of the electric current, nitrogen compounds have been produced directly from the air, with the result that great plants have been erected to manufacture nitrogen compounds from the free nitrogen of the atmosphere.

54. As nitrogen compounds are essential to the making of explosives the manufacture of these compounds received a great impetus during the war; they are also extremely important in

agriculture, being the most expensive ingredient in fertilizers. Nitrogen compounds are now obtained in large quantities by means of calcium carbide, a compound of calcium and carbon, CaC_2, which has a great attraction for nitrogen at a certain stage of its manufacture and absorbs a large amount of that element when cooling. The resulting compound is called *calcium cyanamide*, CaN_2C.

55. Nitrocellulose.—As previously mentioned, there are five oxides of nitrogen; several of these form the basis of explosives used in war and industry. When properly prevailed upon, the nitrogen atom can climb into the molecule of a well-behaved substance, like paper, which is chiefly cellulose, and taking with it a little more oxygen, the result is nitrocellulose—a smokeless gunpowder. A better quality is obtained when cotton is used instead of wood cellulose. Cellulose so treated is said to be *nitrated*. See Art. **221.**

Nitrated cellulose is made by immersing cotton or other cellulose in a mixture of nitric and sulphuric acids; it does not lose its fibrous appearance, and the acid may be washed out with water. When dry, the nitrocellulose may be dissolved in a mixture of alcohol and ether. When this solution is poured out on a flat surface and the alcohol-ether evaporated, a colorless film called collodion is produced, which is insoluble in water and is used by the photographer as a medium for his sensitized solutions on plates and films.

A form of artificial silk is prepared by forcing the alcohol-ether solution of nitrocellulose through fine orifices into either water or warm air, thus removing the volatile solvent and producing fine threads, which are denitrated, so as to be non-explosive and less combustible.

Nitrogen is the basis of two very important compounds that are exact opposites from a chemical standpoint; these are nitric acid and ammonia, and they are typical of two great classes of compounds—acids and bases—to which attention must now be given.

CHEMICAL EQUATIONS

56. In order to show with more or less accuracy just what happens as the result of a chemical reaction, what are called chemical equations are employed. For example, in Experiment

2, it was shown that the union of zinc with sulphuric acid produced hydrogen. The chemical equation expressing this reaction is written as follows:

$$H_2SO_4 + Zn = ZnSO_4 + H_2$$

That part on the left of the sign of equality represents the conditions before the reaction, and the part on the right represents the conditions after the reaction. The equation shows, in this case, that the zinc unites with the sulphuric acid and the result of the reaction is free hydrogen and a compound called zinc sulphate, $ZnSO_4$. It will be noted that the zinc and the hydrogen have exchanged places, the zinc being free before the reaction and the hydrogen being free after the reaction.

In every chemical equation, no matter how simple or how complicated it may be, *there must be the same number of atoms on one side of the sign of equality as are on the other side; and there must be the same number of atoms in each element concerned in the reaction on both sides.* This follows at once from the theory that an atom cannot be divided or destroyed. In the above equation, there are 8 atoms on one side and 8 atoms on the other side; there are also 2 atoms of hydrogen, 1 of sulphur, 4 of oxygen, and 1 of zinc on one side and the same number respectively on the other. This fact affords a sure way of checking the correctness of the equation, mistakes often being made, principally through carelessness, in writing the right-hand side of the equation. Since the number of atoms on the two sides is equal, it follows that the sum of the atomic weights of the elements on one side must equal a similar sum on the other side. Thus, in the above equation, the sum of the atomic weights on the left side is $2 \times 1 + 32 + 4 \times 16 + 65.4 = 163.4$, and this equals $65.4 + 32 + 4 \times 16 + 2 \times 1$. Ordinarily, this would be expressed thus:

$$\underset{98}{H_2SO_4} + \underset{65.4}{Zn} = \underset{161.4}{ZnSO_4} + \underset{2}{H_2}$$

As another simple example of a reaction, consider the following equation, which shows the reaction between calcium oxide (lime) and sulphuric acid

$$\underset{56}{CaO} + \underset{98}{H_2SO_4} = \underset{136}{CaSO_4} + \underset{18}{H_2O}$$

Here the calcium and hydrogen trade places, the result of the reaction being water and a compound called calcium sulphate. Also, $56 + 98 = 136 + 18 = 154$.

The reaction that occurs when calcium phosphate, $Ca_3(PO_4)_2$, is heated in contact with silica, SiO_2, is shown by the equation

$$\underset{310}{Ca_3(PO_4)_2} + \underset{180.9}{3SiO_2} = \underset{348.9}{3CaSiO_3} + \underset{142}{P_2O_5}$$

The result of the reaction is calcium silicate and phosphoric anhydride, P_2O_5. Notice that $310 + 180.9 = 490.9 = 348.9 + 142$; also, on the left side of the equation, there are 3 atoms of calcium, $2 \times 1 = 2$ atoms of phosphorus, $2 \times 4 + 3 \times 2 = 14$ atoms of oxygen, and 3 atoms of silicon, and it will be found that there are the same number of atoms of the same kinds on the right side. The total number of atoms on either side is $3 + (1 + 4)2 + 3(1 + 2) = 22 = 3(1 + 1 + 3) + 2 + 5$.

QUESTIONS

(1) Using the atomic weight given in the table at the end of this Part, find the percentage of each element in a molecule of calcium bisulphite, $CaH_2(SO_3)_2$.

Ans. $\begin{cases} \text{Ca}, 19.81\% \\ \text{H}, \ 1.00\% \\ \text{S}, \ 31.72\% \\ \text{O}, \ 47.47\% \end{cases}$

(2) What is (*a*) the authority for a chemical law? (*b*) How does a law differ from a hypothesis?

(3) (*a*) What law states that 40 pounds of calcium will combine with 16 pounds of oxygen? (*b*) How can this be proved?

(4) If the valence of oxygen is 2 and two atoms of oxygen combine with one atom of sulphur to form sulphur dioxide SO_2, what is the valence of sulphur in this compound? What is the valence of sulphur in H_2S? in SO_3?

ACIDS, BASES, AND SALTS

57 Metals and Non-metals.—In accordance with their physical and chemical properties, the elements are classified as **metals** and **non-metals.** In some cases, the distinction is not very sharp, because in accordance with some of its properties, an element would ordinarily be called, say, a metal, while by reason of certain other properties, it would be called a non-metal. Arsenic and antimony, for example, are elements of this kind; they are closely related chemically, and will here be regarded as metals. Although mercury is a liquid at ordinary temperatures, it is nevertheless, always considered to be a metal.

Elements that are generally regarded as non-metals are: boron,

bromine, carbon, *chlorine*, *fluorine*, *hydrogen*, iodine, *nitrogen*, *oxygen*, phosphorus, selenium, silicon, sulphur, and tellurium; to this class also belong the five rare gases of the atmosphere—*argon*, *neon*, *helium*, *krypton*, and *xenon*. Those elements (including the last five) whose names are printed in Italics are gases at ordinary temperatures. All the other elements may be considered to be metals.

58. Acids.—The word **acid** is derived from the Latin and means *sour;* in fact, nearly all acids have a decidedly sour taste, which is one of their distinguishing characteristics. The sour taste of vinegar and lemon juice, for example, is due to the presence of acetic acid and citric acid, respectively.

Another characteristic feature of an acid is that if a strip of blue litmus paper be dipped into an acid or, in some cases, its solution, its color is changed from blue to red; for this reason, litmus paper is called an **indicator.** If dipped into concentrated (undilute) sulphuric acid, it will turn black, because, as previously stated, sulphuric acid has a great liking for water; it therefore takes the water out of the paper and turns its color black. But if the acid be diluted with water, it turns the blue litmus red.

59. Bases.—A **base** is the chemical opposite of an acid; those bases that are soluble in water are called **alkalies,** and solutions of alkalis have a soapy taste. Ordinary washing soda, commonly called sal-soda, when dissolved in water, will be found to have a soapy taste. The soapy taste is a distinguishing characteristic of alkaline solutions.

A second characteristic of soluble bases is that after a strip of blue litmus paper has been turned red by an acid, it may be turned back to its original blue color by dipping it into an alkaline solution.

Any solution that turns blue litmus red is either an acid or is partly acid; and any solution that turns red litmus blue is either a base or is partly basic.

Some bases are powerful and corrosive, with very decided chemical properties, while others are not so definitely marked. **Soda** $NaOH$, (also called *sodium hydroxide*, *sodium hydrate*, but commonly *caustic soda*), **potash,** KOH, (also called *potassinm hydroxide*, *potassium hydrate*, but commonly *caustic potash*), and ammonium hydroxide, NH_4OH, (also called ammonium hydrate) are well-known and powerful bases, as is also ordinary household

ammonia, which is a solution of ammonia gas in water. Lime, CaO, is a strong base, but is not readily soluble in water, and so does not readily exhibit its basic properties; magnesia, MgO, is a similar base. These two compounds belong to the class known as the **alkaline earths.**

All bases are either oxides or hydrates of a metal, a **hydrate** being a compound that contains the combination OH (called *hydroxyl*). The alkaline hydrates as $NaOH$, KOH, etc. are frequently called hydroxides (see last paragraph).

60. Chemical Definition of Acid.—Here are three acids:

Chemical Name		Common Name
Hydrochloric acid	HCl	Muriatic acid
Nitric acid	HNO_3	Aqua fortis
Sulphuric acid	H_2SO_4	Oil of vitriol

Note first that all these acids contain hydrogen; a great many other compounds contain hydrogen, but there is something about the hydrogen of acids that is quite distinctive, as will presently appear.

In experiment 2, zinc was treated with sulphuric acid, and hydrogen was given off as a free gas; the same effect is produced when zinc is treated with either of the other two acids just mentioned, the reactions in all three cases being indicated in the following equations:

$$\begin{array}{cccc} H_2SO_4 & + Zn = & ZnSO_4 & + H_2 \\ 98 & 65.4 & 161.4 & 2 \\ 2HCl & + Zn = & ZnCl_2 & + H_2 \\ 73 & 65.4 & 136.4 & 2 \\ 2HNO_3 & + Zn = & Zn(NO_3)_2 & + H_2 \\ 126 & 65.4 & 189.4 & 2 \end{array}$$

It will be remembered that hydrogen has only one bond (its valence is always 1); zinc ordinarily has two bonds (its valence is 2). Now when a dyad and a monad come together chemically, each bond of the dyad must take hold of a monad or it must hold the same amount as two monad atoms. In the first of the above equations, there are two atoms of hydrogen, and they hold the combination SO_4 (called a **radical**); consequently, one atom of zinc will hold the same radical. In the second equation, there is only one atom of hydrogen in the acid molecule, and it holds the radical Cl; but zinc holds and must have twice as much as the one hydrogen atom, and consequently,

there must be two molecules of the acid and the zinc must have two of the radicals. The same is true of the third equation, the radical in this case being NO_3.

Referring again to the first equation, if an unlimited amount of sulphuric acid (mixed with some water) were available for combination with 65.4 g. of zinc, the reaction would use only 98 grams of the acid, and only 161.4 grams of zinc sulphate would be formed (see Art. **42**); and if 98 g. of the acid and an unlimited amount of zinc were available, the reaction would use only 65.4 g. of zinc, and the amount of zinc sulphate formed would be 161.4 g.

Referring again to the second equation, if an unlimited amount of hydrochloric acid were available and only 65.4 g. of zinc, only 73 g. of the acid would be used, and only 136.4 g. of zinc chloride would be formed; or, if an unlimited amount of zinc were available and only 73 g. of the acid, only 65.4 g. of the zinc would be used, and only 136.4 g. of zinc chloride would be formed.

61. It will now be seen that although a molecule cannot be weighed, the results of a chemical reaction can be predicted and desired results obtained by using weights of material in proportion to their molecular weights; whence the term **gram molecule,** which is the number of grams of a compound or element equal to the molecular weight of a molecule of that compound or element. Thus, a gram molecule of sulphuric acid is 98 g. of sulphuric acid, a gram molecule of zinc chloride is 136.4 g. of zinc chloride, etc. In the case of an element, a gram molecule is the number of grams equal to the atomic weight of the element; thus, a gram molecule of zinc is 65.4 grams of zinc.

What is called the **gram molecular volume** is the volume occupied by a gram molecule of any gas under standard conditions (0°C. and 760 mm.); it is always 22.4 liters. Consequently, if the weight in grams of 22.4 liters of any gas (under standard conditions) be found, the result will be the molecular weight of the gas. The constant 22.4 is determined as follows: a molecule of oxygen contains 2 atoms; hence, a gram molecule of oxygen weighs 32 grams. Since 1 liter of oxygen weighs 1.429 grams under standard conditions, 32 grams will occupy $32 \div 1.429 = 22.39+$, say 22.4 liters = 22,400 c.c. Hence, by Avogadro's law, 1 gram molecule of any gas under standard conditions will always occupy a volume of 22.4 l. = 22,400 c.c.

Equation two of the last article shows that it is necessary to use 2 gram molecules of hydrochloric acid to combine with 1 gram molecule of zinc (or this proportion must be used) if all the acid and zinc is used up in the reaction. If only 1 gram molecule of the acid be available to 1 gram molecule of zinc, only half the zinc will be used and only half as much zinc chloride will be formed. Plumbers add zinc to hydrochloric acid until all action ceases, that is, until no more hydrogen gas bubbles up; they then say that the acid is "killed," which is a good term to use, because the acid has then lost the characteristics of an acid. The term used by chemists to express the same idea is **neutralize,** that is, they would say that the acid has been neutralized. This term will be explained more fully later.

62. The most important part of an acid is its hydrogen, an element that every acid must contain. Moreover, this hydrogen must be so attached to its radical (radical comes from a Latin word meaning *root*) that it can be replaced by a metal during a chemical reaction. The metal itself may be an element, as in the equations of Art. **60,** or it may be the metal element in a base, as in NaOH, in which Na is the metal and OH is the radical of the base. An acid may therefore be defined as *a chemical compound containing hydrogen, provided the hydrogen is capable of being replaced by a metal, or by a group of elements acting like a metal, during a chemical reaction.* For convenience, it is customary to refer to the two parts into which an acid or base is divided as **ions.** In the case of an acid, the hydrogen is one ion and the radical is the other ion; in the case of a base, the metal is one ion and the radical is the other ion. One ion is considered to be *electropositive* and the other *electronegative,* hydrogen and the metal ions being positive and the radicals negative. This matter is mentioned because it has to do with why things dissolve and why electric currents make bleach out of salt; it will be discussed more fully later.

63. Anhydrides.—When sulphuric acid is heated to boiling, a dense, white, choking fume is given off; this is sulphuric anhydride, SO_3, which means sulphuric acid without water. In the word **anhydride,** *an* means *without* and *hydride* refers to water. The chemical reaction may be expressed by the equation

$$H_2SO_4 - H_2O = SO_3$$

In fact the formula for sulphuric acid might be written $SO_3 \cdot H_2O$

or $SO_3(H_2O)$; in either case, removing the water leaves the anhydride.

In this connection, it may be remarked that a very important acid, carbonic acid, has never been isolated; its formula would be H_2CO_3, but every attempt to produce it results only in its anhydride CO_2, the hypothetical reaction being expressed by the equation

$$H_2CO_3 - H_2O = CO_2$$

What is ordinarily called carbonic acid is carbonic anhydride, which is also frequently and correctly called carbon dioxide. Many acids form anhydrides, but a few cannot. Hydrochloric acid, for instance, cannot form an anhydride, because it has only one hydrogen atom and no oxygen.

64. Alkalis.—Consider the three following compounds:

Ammonium hydroxide, NH_4OH, commonly called ammonia.

Sodium hydroxide, $NaOH$, commonly called caustic soda or soda lye.

Potassium hydroxide, KOH, commonly called caustic potash or potash lye.

These are all soluble bases, have a soapy taste, and are called alkalis. Their peculiar properties are involved in the common possession of the radical OH, which is called **hydroxyl,** and the three compounds belong to what is known as the **hydroxyl group;** the hydroxyl radical is as much a characteristic of this group as the hydrogen ion is of the acid group.

It will be recalled that hydrogen has only one bond while oxygen has two; hence, in a combination of 1 atom of hydrogen and 1 atom of oxygen, one of the oxygen bonds of the hydroxyl will hold the hydrogen and the other will be free to take hold of anything within reach. As a consequence, the chemical activity of the hydroxyl radical does not lie in the hydrogen, but in the oxygen. This is fair arithmetic, but rough chemistry; still, it explains in a general way why the hydroxyl is so active. To sum up, the alkalis have activities incidental to the hydroxyl; the acids have activities incidental to the hydrogen.

When acids and alkalis meet, there is more or less excitement—in some cases, a regular riot—and much energy is released in the form of heat; for instance, when sulphuric acid is added to a strong soda solution. Again, when sulphuric acid (dilute) is brought into contact with the element sodium, the reaction is so violent that an explosion takes place.

65. Sodium; Symbol, Na; Atomic Weight, 23.—Sodium is a metal—a base; it is very soft, like wax, and its density is less than that of water. Air affects it rapidly, the oxygen uniting with it to form an oxide; consequently, it can be kept only when immersed in a liquid that does not contain oxygen, such as petroleum or kerosene. When sodium is brought into contact with water, there is an immediate reaction,

$$Na + H_2O = NaOH + H$$

which results in the formation of sodium hydroxide and free hydrogen. The action is so violent and the heat so intense (especially if the water is hot) that the hydrogen is ignited as fast as produced; the sodium is melted, and it floats like a ball of orange fire on the surface of the water. This makes a striking experiment; but only a small piece of sodium, about the size of a little pea, should be used and great care should be exercised, since, otherwise, troublesome burns may be caused or particles may sputter into the face and eyes. (Note the similarity in results of the sodium on water and an acid on a metal, as zinc; in both cases, hydrogen is given off). After the sodium has disappeared, the water will have a soapy taste, containing sodium hydroxide in solution, and will turn red litmus paper blue.

The chief source of the sodium compounds is common salt, or sodium chloride, NaCl, which is found in many parts of the world as rock salt, and is present in the ocean in unlimited quantities. Some of the sodium compounds are of great importance in paper making, and will be discussed later in connection with their application.

66. Potassium; Symbol, K; Atomic Weight, 39.1.—Potassium is a metal that resembles sodium very closely in its physical and chemical properties. It combines with water in a manner similar to sodium, but the flame has a lilac instead of an orange color.

In some chemical reactions in industrial work, sodium or potassium compounds may be used interchangeably; the sodium compounds are preferred, however, because they are cheaper. Advantage was taken of this fact during the late war, when sodium compounds were frequently substituted for the potassium ones formerly used. The chief source of potassium compounds is from the immense deposits of potash found in Germany, practically all the potash in commercial use coming from this supply. A mineral called *feldspar*, which is also quite common, contains

potassium compounds; but as the mineral is not soluble in water, the potash can be obtained only by expensive, indirect means. The potash minerals obtained from the German deposits are soluble in water; hence, they may be dissolved and may then be purified by crystallization. Potash is one of the three principal ingredients of fertilizers, the other two being a nitrogen compound, and phosphoric acid.

67. Soaps.—**Soaps** are made by causing an alkali, as soda or potash, to combine with fats, which are really acids (though of a more complex character than any so far discussed), and have hydrogen atoms that behave in the same way as the hydrogen ions of the simple acids. It may here be stated that *sodium* produces the so-called "hard soaps," while *potassium* produces the "soft soaps."

Palmitic acid is found in palm oil, and is a vegetable fatty acid; it forms sodium or potassium palmitate soap—a soap that floats on water. The basis of castile soap is olive oil. The animal fats contain stearic and oleic acids, and the greater part of the ordinary soap of commerce is produced from the non-edible fats of the slaughter houses. See Art. **215.**

Rosin consists chiefly of abietic acid (Art. **42**); consequently, when rosin and soda are heated together, a rosin soap is formed. This soap is soluble in water, and is the basis of rosin size. The old brown size used in earlier rosin sizing was completely neutralized rosin. In addition to the rosin soap, the sizes now in use contain a large amount of uncombined rosin acid, which is held in suspension by the soap. This white size milk, in which free (uncombined) rosin is held in suspension, is a good example of a class of mixtures called emulsions. When properly made, the particles of rosin are so very small that the liquid is practically a solution. An **emulsion** is a suspension (non-settling mixture) of very small liquid or solid particles in a liquid which does not dissolve them. By prolonged and violent shaking, for example, oil and vinegar may be so mixed that they will not afterwards separate. The result is not a true mixture, but an emulsion. Soaps greatly assist in the formation of emulsions.

Lime, which is found in greater or less amounts in all mill-supply waters, forms lime soaps with rosin; as these soaps are insoluble, some of the rosin is lost (insofar as sizing is concerned) when it comes into contact with "hard" waters. The sizing of paper is fully explained in a subsequent Section.

68. Ammonium; Symbol, NH_4; Molecular Weight, 18.042.—This is the queer member of the alkali family; it is not an element, but a pair of elements—an ill-assorted pair. It will be recalled that nitrogen is inert until compelled to combine by indirect means. In **ammonium,** four atoms of hydrogen are attached to one atom of nitrogen, but this strange partnership is found only in company with other elements. If any attempt be made to isolate ammonium as NH_4, the result is sure to be a gas having the molecular formula NH_3, called **ammonia.** The other hydrogen atom is set free, and these free atoms unite to form molecules of hydrogen. Yet the combination NH_4 can be bandied about considerably, so long as other groups of elements, or even single elements, are available, acting in this respect just like a metal. For example, take ammonium hydroxide and add to it dilute nitric acid or hydrochloric acid; the reactions are shown by the following equations:

$(NH_4)OH$	+	HNO_3	=	$(NH_4)NO_3$	+	$HOH(=H_2O)$
Ammonium hydroxide		nitric acid		ammonium nitrate		water
$(NH_4)OH$	+	HCl	=	$(NH_4)Cl$	+	HOH
Ammonium hydroxide		hydrochloric acid		ammonium chloride		water

Compare these with:

$NaOH$	+	HNO_3	=	$NaNO_3$	+	HOH
Sodium hydroxide		nitric acid		sodium nitrate		water
$NaOH$	+	HCl	=	$NaCl$	+	HOH
Sodium hydroxide		hydrochloric acid		sodium chloride		water

Observe that the group NH_4 replaces the hydrogen of the acid in the same manner that the sodium element replaces it. In fact, the chemical action of ammonium is so much like that of a metal that it is sometimes called a **hypothetical metal.** A reason will now be seen for writing the molecular formula for ammonium hydroxide as $(NH_4)OH$ instead of NH_5O or H_5NO, because when a chemical combination takes place, the groups NH_4 and OH separate from each other and combine as groups and not as separate atoms.

69. Until recently, the chief source of ammonium compounds was from decaying animal matter containing nitrogen or from coal as an incident to the production of coal gas. If the top of a

manure pile or compost heap be removed, fumes will be seen coming off and a strong scent of ammonia will be observed. At the present time, ammonia is produced commercially from the nitrogen of the air by action of the electric current and, also, by the cyanamide process previously referred to. In either case, other compounds are produced first, and the ammonia is obtained from them by various processes.

70. When compounds are built up from their elements or from other compounds, the process is called **synthesis**, and the compound is said to be produced **synthetically** or by a **synthetic** process; but when a compound is produced by tearing apart another compound, the process is called **analysis**, and the compound is said to be produced **analytically** or by an **analytic process**. Consequently, the production of ammonia, nitric acid, etc., from the air is a synthetic process, while the production of hydrogen from water or by the combination of a dilute acid and zinc is an analytic process.

71. Salts.—When an acid and a base combine chemically, the result is a **salt** and water; this is indicated in the following equation:

$$\underset{\text{acid}}{HNO_3} + \underset{\text{base}}{NaOH} = \underset{\text{salt}}{NaNO_3} + \underset{\text{water}}{H_2O}$$

When hydrochloric acid acts upon soda (sodium carbonate), the result is expressed by the equation

$$Na_2CO_3 + 2HCl = 2NaCl + CO_2 + H_2O$$

Here the sodium carbonate acts like a base, the sodium replacing the hydrogen of the acid. In this case, there are three products of the combination, which are: common salt, $NaCl$, carbon dioxide or carbonic anhydride, CO_2, and water, H_2O. The action of zinc on dilute sulphuric acid gives

$$H_2SO_4 + Zn = ZnSO_4 + H_2$$

and the zinc sulphate thus formed is a salt, the zinc acting as a base. In general, it may be stated that *a salt is a compound derived from an acid through the replacement of the whole or a part of the hydrogen of the acid by a metal or by a group of elements that act as a metal.* When the word salt is used without any qualification, sodium chloride (common salt) is always meant.

72. It may be mentioned that the term **sal** is sometimes used instead of salt. Thus, *sal soda* means salt of soda, a common

name for washing soda (sodium carbonate); ***sal ammoniac*** means salt of ammonia, and is a common name for ammonium chloride $(NH_4)Cl$; *sal volatile* means volatile salt, and is a common name for ammonium carbonate, the basis of smelling salts.

73. When an acid, like sulphuric acid, H_2SO_4, for example, has two or more hydrogen atoms, it is possible to have two or more different salts, according as a part or all of the hydrogen is replaced by the metal. When all the hydrogen is replaced with a metal, the acid is said to be neutralized, and the salt is called a **normal** or **neutral salt**; thus, the salts formed in the equations of the last article are all normal or neutral salts. When only a part of the hydrogen is replaced, a salt is formed that retains acid properties; these salts are termed **acid salts,** their solutions will turn blue litmus red, and their names contain the prefix ***bi***. A well known example is ***baking soda,*** properly called ***bicarbonate of soda*** or ***sodium bicarbonate.*** The molecular formula for washing soda (sodium carbonate) is Na_2CO_3, while the formula for baking soda (sodium bicarbonate) is $NaHCO_3$. Note that in the former, both atoms of hydrogen in carbonic acid, H_2CO_3, have been replaced by the sodium, but in the latter, only one atom of hydrogen has been replaced; note also that both molecules contain the same number of atoms and the same radical CO_3. Baking soda is also frequently called ***saleratus.*** Baking powder is a mixture of two acid salts—bicarbonate of soda and bitartrate of potash. Another acid salt, one that is familiar to sulphite mill men, is calcium bisulphite, $Ca(HSO_3)_2$.

Phosphoric acid, H_3PO_4, has three hydrogen atoms, and it forms three salts with monad metals, as follows when sodium is the metal used:

Na_3PO_4	Trisodium phosphate, or normal sodium phosphate
Na_2HPO_4	Disodium phosphate, or monohydrogen-sodium phosphate
NaH_2PO_4	Monosodium phosphate, or dihydrogen-sodium phosphate

Note that in the first salt, all the hydrogen has been replaced by the sodium; in the second, two atoms of hydrogen have been replaced; and in the third, only one atom of hydrogen has been replaced by the sodium. The prefix ***di*** has the same meaning as ***bi***.

Sometimes the hydrogen is replaced by two different metals; thus, calling ammonium a metal, the compound called sodium-ammonium-hydrogen phosphate has the molecular formula $Na(NH_4)HPO_4$, ammonium-magnesium orthophosphate has the formula $Mg(NH_4)PO_4$, and sodium-potassium carbonate has the formula $NaKCO_3$. The last formula is justified because the radical CO_3 requires either a dyad or two monad atoms, and potassium and sodium are both monads. Salts of this kind are called **double salts.**

74. The Alums.—One series of double salts, called the **alums,** is quite important in dyeing and was formerly used for sizing; they are sulphates of aluminum with sodium, potassium, or ammonium, as follows: potassium alum (common alum), otherwise called aluminum-potassium sulphate, has the formula $Al_2(SO_4)_3 \cdot K_2SO_4 \cdot 24H_2O$; sodium alum, otherwise called aluminum-sodium sulphate, has the formula $Al_2(SO_4)_3 \cdot Na_2SO_4 \cdot 24H_2O$; and ammonium alum, otherwise called aluminum-ammonium sulphate, has the formula $Al_2(SO_4)_3 \cdot (NH_4)_2SO_4 \cdot 24H_2O$. There are other alums, but these are the most important. Papermaker's alum, aluminum sulphate, is not really an alum, in the proper sense, though in use, its properties are similar, and it is cheaper.

Aluminum is a triad (it has three bonds); hence, one atom of aluminum cannot combine with one atom of oxygen, which is a dyad and has only two bonds. It is therefore necessary to take two atoms of aluminum in most cases (thus giving 6 bonds), which require 3 atoms of oxygen (giving $3 \times 2 = 6$ bonds) or 6 atoms of hydrogen. An atom of aluminum, therefore, cannot replace the hydrogen in a molecule of sulphuric acid, H_2SO_4, which would be one hydrogen atom short, or with two molecules, which would be one atom too many; consequently, it is necessary to use two atoms of aluminum and three molecules of sulphuric acid to effect the combination, as shown by the equation

$$3H_2SO_4 + 2Al + Al_2(SO_4)_3 + 6H$$

When an analysis is made, the percentage of aluminum and of sulphuric anhydride, SO_3, show that the formula $Al_2(SO_4)_3$ is correct.

75. An important property of the alums and $Al_2(SO_4)_3$ is their reaction with water. Taking Al_2SO_4 and water we have

$$Al_2(SO_4)_3 + 6H_2O = 2Al(OH)_3 + 3H_2SO_4.$$

Since H_2SO_4 is a very strong acid, and $Al(OH)_3$ is a very weak base, the solution of $Al_2(SO_4)_3$ is distinctly acid in character. This property is made use of in breaking up a soap, as in precipitating rosin size, in setting dyestuffs where a weak acid reaction is wanted, and in clarifying water, where the cloudlike structure of the aluminum hydrate, $Al(OH)_3$, which is insoluble in the presence of a little lime, catches impurities and carries them down in settling. In coloring paper, this aluminum hydrate $Al(OH)_3$ adheres firmly to the fibers, and attaches to them the insoluble color compounds it makes with many dyestuffs. Such compounds are called *lakes*.

QUESTIONS

(1) How much NaOH by weight will be required to neutralize 25 grams of SO_2 as H_2SO_3? *Ans.* 31.25 grams.

(2) What is (*a*) the composition of air? (*b*) is it a definite chemical compound? (*c*) can you prove this?

(3) What is (*a*) a gram molecule? (*b*) what is the weight of a gram molecule of sodium chloride (common salt)?

(4) Why does alum give an acid reaction?

(5) If the liberation of hydrogen is characteristic of acids, how is it that sodium, a strong base, liberates hydrogen when sodium is in contact with water?

LIST OF ELEMENTS, THEIR SYMBOLS AND ATOMIC WEIGHTS

Element	Symbol	Atomic Weight	Element	Symbol	Atomic Weight
Aluminum	Al	27.1	Molybdenum	Mo	96.0
Antimony	Sb	120.2	Neodymium	Nd	144.3
Argon	A	39.88	Neon	Ne	20.2
Arsenic	As	74.96	Nickel	Ni	58.68
Barium	Ba	137.37	Niton	Nt	222.4
Bismuth	Bi	208.0	Nitrogen	N	14.01
Boron	B	11.0	Osmium	Os	190.9
Bromine	Br	79.92	Oxygen	O	16
Cadmium	Cd	112.40	Palladium	Pd	106.7
Cæsium	Cs	132.81	Phosphorous	P	31.04
Calcium	Ca	40.07	Platinum	Pt	195.2
Carbon	C	12.00	Potassium	K	39.10
Cerium	Ce	140.25	Praseodymium	Pr	140.6
Chlorine	Cl	35.46	Radium	Ra	226.4
Chromium	Cr	52.00	Rhodium	Rh	102.9
Cobalt	Co	58.97	Rubidium	Rb	85.45
Columbium	Cb	93.5	Ruthenium	Ru	101.7
Copper	Cu	63.57	Samarium	Sa	150.4
Dysprosium	Dy	162.5	Scandium	Sc	44.1
Erbium	Er	167.7	Selenium	Se	79.2
Europium	Eu	152.0	Silicon	Si	28.3
Fluorine	F	19.0	Silver	Ag	107.88
Gadolinium	Gd	157.3	Sodium	Na	23.00
Gallium	Ga	69.9	Strontium	Sr	87.63
Germanium	Ge	72.5	Sulphur	S	32.07
Glucinum	Gl	9.1	Tantalum	Ta	181.5
Gold	Au	197.2	Tellurium	Te	127.5
Helium	He	3.99	Terbium	Tb	159.2
Holmium	Ho	163.5	Thallium	Tl	204.0
Hydrogen	H	1.008	Thorium	Th	232.4
Indium	In	114.8	Thulium	Tm	168.5
Iodine	I	126.92	Tin	Sn	119.0
Iridium	Ir	193.1	Titanium	Ti	48.1
Iron	Fe	55.84	Tungsten	W	184.0
Krypton	Kr	82.92	Uranium	U	238.5
Lanthanum	La	139.0	Vanadium	V	51.0
Lead	Pb	207.10	Xenon	Xe	130.2
Lithium	Li	6.94	Ytterbium	Yb	172.0
Lutecium	Lu	174.0	Yttrium	Yt	89.0
Magnesium	Mg	24.32	Zinc	Zn	65.37
Manganese	Mn	54.93	Zirconium	Zr	90.6
Mercury	Hg	200.6			

ELEMENTS OF CHEMISTRY

PART 1

EXAMINATION QUESTIONS

(1) How would you describe alum and salt, using sight and taste?

(2) What is the difference between a chemical change and a physical change?

(3) What are (*a*) the properties of water? (*b*) the chemical components of water?

(4) How can iron oxide be used to confirm the composition of water?

(5) What successive reactions take place in quicklime that is stored in a moist atmosphere exposed to chimney gases?

(6) How many pounds of calcium carbonate can be produced from 56 pounds of quicklime? Ans. 100 lb.

(7) What effect does (*a*) "hard" water have on rosin size? (*b*) on steam boilers? (*c*) what is the difference between "temporary" and "permanent" hardness?

(8) What is the difference between elements and compounds?

(9) If 24 pounds of sodium were treated with 35.5 pounds of chlorine, why would a pound of sodium be left over? How many grams of salt can be produced with 10 grams of sodium? Ans. 25.4 + grams.

(10) What do you understand by atomic weight?

(11) What do you understand by (*a*) the law of definite proportions? (*b*) the law of multiple proportions?

(12) Why is oxygen more active chemically when just liberated from hydrogen peroxide (or from bleaching powder solution)?

(13) What is meant when it is stated that carbon is a reducing agent? Name two other reducing agents, and two oxidizing agents.

(14) Of what compounds is a soapy taste characteristic?

(15) How would you define (*a*) an acid? (*b*) a base? (*c*) how is litmus paper used to distinguish between acids and bases?

(16) (*a*) What alkaline compound is called a hypothetical metal? (b) show by an equation its likeness to sodium in its chemical action.

(17) What are soaps? name a soap much used in paper making.

(18) What are (*a*) salts? (*b*) acid salts?

(19) Is paper maker's alum a true alum? How does it act with water? Name some of its uses.

ELEMENTS OF CHEMISTRY

(PART 2)

PRACTICAL CHEMISTRY

SULPHUR AND ITS COMPOUNDS

76. Remark.—Enough of the theory has now been given to permit the discussion of some very important compounds in commercial use, and to see how the principles of chemistry affect their action and control manufacture. Any additional principles and theories can be regarded as incidental.

There are three processes employed in preparing fibers for making paper:

1. Grinding or other mechanical action
2. Cooking or digesting
3. Bleaching

The first does not require attention at this time, but the other two require the application of chemistry and involve a knowledge of the properties of several important elements and their compounds, such as sodium (which has already been touched upon), sulphur, calcium, magnesium, and chlorine.

77. Sulphur; Symbol, S; Atomic Weight, 32.07.—Sulphur is one of the elements that occurs native; that is, it is found and can be obtained in an uncombined state. It differs in this respect from the majority of the elements, which can be obtained only from compounds. Sulphur is also widely distributed in certain compounds, the principal ones being:

Sulphide of iron, FeS_2, called iron pyrites or "fool's gold," from its color, which resembles gold;

Sulphide of lead, PbS, called galena;

Sulphide of copper, $CuFeS_2$, called chalcopyrite or bornite;

Sulphate of calcium, $CaSO_4$, called gypsum.

For making sulphite pulp, the native sulphur is chiefly used, the greater part of the supply of which comes from two localities:

Sicily and Louisiana. Formerly, the chief source of supply for this country was Sicily, but this has been recently superseded by the deposits of Louisiana. The latter had been known for years, but they could not be worked, because they lay deep under a stratum of quicksand. A method for getting the sulphur out was devised by Dr. Frasch, and may be explained as follows: Sulphur melts between 115°C. and 120°C., a little above the boiling point of water, and corresponding to the temperature, approximately, of steam when the pressure is about 30 lb. per sq. in., absolute, about 15 lb. per sq. in., gauge. By sinking a double pipe, one within the other, through the surface to the sulphur and passing superheated water down the outer pipe, the sulphur was melted in place and forced up through the inner pipe, discharging in a fluid state. The principle was simple, but the practical difficulties to be overcome were enormous. Much of the sulphur used on the Pacific Coast comes from Japan.

78. Some Properties of Sulphur.—The density of sulphur is about twice that of water, its specific gravity varying between 1.90 and 2.05. Sulphur has the property possessed by several of the elements of existing in more than one physical form; this is called **allotropy.** Thus, sulphur does not always crystallize in the same form, and these different crystalline forms have different melting points; this also accounts for the variation in density. When sulphur is heated, it first melts (about 120°C.); as the temperature increases, it boils and becomes a vapor. On cooling, it does not assume a liquid state, but condenses to a powder (solid). Substances that act in this manner, changing from a vapor to a powder without becoming liquid are said to **sublime,** and the process of thus producing the powder is called **sublimation.** When molten sulphur is quickly cooled, it becomes a rubber-like, plastic mass, which gradually hardens; this is illustrated by the following experiment:

EXPERIMENT 10.—Place about ¾ in. of powdered sulphur in a test tube and heat slowly to melting, in which state, the sulphur is a pale-yellow, mobile liquid. Now raise the temperature, and note that the color changes, the liquid becoming dark and viscous. Remove from the source of heat and allow the mass to cool slightly; pour it into water and note that on handling it appears quite plastic, even when cool. Sulphur in this state is said to be **amorphous** (without form), that is, it has no crystalline form.

Sublimed sulphur is commonly called **flowers of sulphur,** and when produced in a pulp mill, it is because of the high temperature

in the burner without sufficient air supply or because the vapors cooled so rapidly that they failed to combine with the air. It may then deposit in pipes and cause much trouble, which can best be avoided by careful and scientific control of burning conditions.

79. Vulcanizing Rubber.—The pure rubber gum that is obtained by evaporating the sap of certain trees that grow in warm countries is not permanently water proof or flexible. Two independent workers, Thompson in England and Goodyear in the United States, found that sulphur in varying quantities had amazing effects on rubber gum, with the result that products could be obtained varying all the way from flexible, elastic tubes and sheets to hard substances that could be turned in a lathe and could be used for knife handles, combs, and many other things of a practical or decorative nature. The process of blending sulphur with rubber is called **vulcanization,** and rubber so treated is said to be **vulcanized.** This term should not be confused with the vulcanizing of paper, which will be taken up later.

80. Oxides of Sulphur.—When investigating the chemical properties of an element, one of the general methods is to find out how it behaves with oxygen. Burning—or **combustion,** as the word is commonly known—is a case of some substance combining with oxygen. The first step, then, is to ascertain if the element burns readily; sulphur does burn readily—otherwise, there would be no sulphite-pulp industry. The next step is to find out if it combines with oxygen in more than one proportion. In the case of sulphur, there are two very important oxides: sulphur dioxide, SO_2, and sulphur trioxide, SO_3. These oxides are the anhydrides of two very important acids; thus, adding a molecule of water to a molecule of the oxides, the result may be expressed as

$$SO_2 + H_2O = H_2SO_3, \text{ called sulphurous acid}$$
$$SO_3 + H_2O = H_2SO_4, \text{ called sulphuric acid}$$

A pause will be made here to consider the subject of how chemical names of compounds are formed, which goes under the rather high-sounding title of *chemical nomenclature.*

81. Chemical Nomenclature.—When two elements unite to form a compound, the name of the compound so produced ends in *ide;* in most cases, apart from the oxides, these compounds do not contain oxygen. Note the formation of the names of the following compounds:

Iron sulphide, FeS; iron disulphide, FeS_2; sodium chloride, $NaCl$; carbon disulphide, CS_2; calcium carbide, CaC_2; silicon hydride, SiH_4; etc. Note that the *ide* becomes a part of the second or last element; if SiH_4 were written H_4Si, it might be called hydrogen silicide.

The salts of acids that do not contain oxygen end in *ide*, as potassium chloride, KCl; which is a salt of hydrochloric acid; but when oxygen is present in the acid, its salts end in *ate* or *ite*. For example, sulphur dioxide unites with water to form sulphurous acid, H_2SO_3, whose salts are known as sulph*ites*; and sulphur trioxide unites with water to form sulphuric acid, H_2SO_4, whose salts are known as sulph*ates*. Note that compounds whose names end in *ate* always have more oxygen than those whose names end in *ite*. This distinction should be kept in mind in connection with the "sulphite" process, which uses calcium bisulphite, and the "sulphate" or "kraft" process, which uses sodium sulphate. Also keep in mind the meaning of the prefixes mon or mono, uni, bi or di, etc. as defined in Art. **46**.

The names of acids whose salts end in *ite* have the termination *ous*; thus, sulphites are formed from sulphurous acid. The names of acids whose salts end in *ate* have the termination *ic*; thus, sulphates are formed from sulphuric acid, and nitrates are formed from nitric acid.

Sometimes there are more than two salts containing oxygen; in that case, a change is made at the beginning of the word instead of giving it different endings, as shown in the following table:

Acid		Typical salts	
Hydrochloric acid..........	HCl	Potassium chloride.........	KCl
Hypochlorous acid.........	$HClO$	Potassium hypochlorite....	$KClO$
Chlorous acid	$HClO_2$	Potassium chlorite........	$KClO_2$
Chloric acid...............	$HClO_3$	Potassium chlorate.........	$KClO_3$
Perchloric acid............	$HClO_4$	Potassium perchlorate......	$KClO_4$

There is no hydrochlorous acid, but there is a chlorous acid, and the acid that contains one more oxygen atom than chlorous acid is called chloric acid. Since there is another acid containing one atom less of oxygen than chlorous acid, it is called hypochlorous acid, the prefix *hypo* meaning *under* or *less;* and since there is also another acid containing one more atom of oxygen than chloric acid, it is called perchloric acid, the prefix *per* mean-

ing *over* or *more* or *greater*. The prefix *hyper* has the same meaning as *per*, and this acid might have been called hyperchloric acid. Note that the names of the salts follow the rule in regard to the endings *ite* and *ate*.

The endings "ic" and "ous" indicate either the relative amounts of oxygen in the molecules of compounds or the fact that they are produced by introducing oxygen into a compound of lower oxygen content or expelling oxygen from a compound of higher oxygen content; in the former case, the compound is said to be oxidized, and in the latter case, to be reduced. (See Art. **52.**) It will be seen later that other elements than oxygen may have a similar effect. Oxidation may cause an element to increase in valence and reduction to decrease the valence, as an example, stannous (tin) chloride, $SnCl_2$, which contains no oxygen, becomes stannic chloride, $SnCl_4$, under oxidizing conditions.

82. Sulphur Dioxide.—**Sulphur dioxide (sulphurous anhydride)** is a colorless gas having a molecular weight of 64.07, which is usually taken as 64. It is soluble in water, a matter of considerable moment to makers of sulphite pulp, 1 volume of water dissolving about 80 volumes of gas at 0°C.; but at 20°C., which is about the temperature of water supplied in summer, 1 volume of water dissolves only about 39 volumes of the gas. For this reason, the gas from sulphur burners should be cooled and passed into water that is as cool as possible. Note that the atomic weight of sulphur is 32.07, say 32, which is the same as the molecular weight of oxygen, O_2 $(2 \times 16 = 32)$; hence, 1 pound, say, of sulphur requires 1 pound of oxygen to convert it into sulphur dioxide, SO_2. Now, since oxygen is only about 23.21% of the atmosphere by weight, 1 lb. of sulphur requires about 4⅓ lb. of air for its complete combustion. If it does not get that much air or a little more, flowers of sulphur will get into the cooling systems and towers and will cause trouble. By volume, it takes about 60 cu. ft. of air to 1 lb. of sulphur for complete combustion to sulphur dioxide.

83. Sulphur Trioxide.—At high temperatures and in the presence of water vapor, sulphur dioxide annexes to itself another atom of oxygen and becomes **sulphur trioxide,** SO_3, otherwise called **sulphuric anhydride,** which has a molecular weight of 80.07, say 80. Therefore, when burning sulphur for pulp mills excessively high temperatures and the presence of water vapor

are to be avoided. Wet sulphur should not be used in any case; because the steam arising from this source when the sulphur is heated combines with the sulphuric anhydride, SO_3, to make sulphuric acid, as shown by the equation of Art. **80**, which is far more corrosive than sulphurous acid and has the added disadvantage of not being volatile. Wherever spray from the tower exhaust falls, there it stays and gets to work on wood, metal or paint.

A humid atmosphere contains large amounts of water vapor; consequently, the air going to the sulphur burners should, if practicable, be cooled, to remove excess water vapor by condensation. Sometimes the acid towers are exhausted by steam, to pull the gases through and in case of fan trouble. The hot steam, coming into contact with the waste gases (which often contain sulphur dioxide in more or less quantity) in the presence of oxygen, acts as a catalyzer, producing sulphuric anhydride, SO_3, and the neighborhood is covered with a fine spray of sulphuric acid. In sulphur burning, other more complex sulphur, oxygen, and hydrogen compounds are produced, all tending to increase the sulphur consumption per ton of pulp, without helping the production. Burners, therefore, should be carefully watched, should not get intensely hot, should have enough oxygen, but not a great excess, and should be kept as free from moisture as is possible.

84. How Sulphuric Acid is Made.—Sulphuric acid is made commercially in large quantities. Two processes are used that differ very widely, considered chemically, and it will be worth while to discuss them briefly. Iron pyrites, disulphide of iron, FeS_2, which might be called ferric sulphide, is largely used as the source of sulphur in the manufacture of the acid. The pyrites are roasted, with a supply of air, in furnaces that contain devices for stirring mechanically, and the sulphur and iron are both oxidized. The oxide of iron thus formed is not volatile and remains in the furnace, except for a small amount that floats about as a fine dust. The sulphur is oxidized to sulphur dioxide, a gas, for which reason, pyrites are sometimes used as a source of burner gas in sulphite mills. The iron oxide dust, however, has been found troublesome.

After the sulphur has been oxidized to SO_2, the problem of how to cause it to take up the extra oxygen atom and become SO_3 has been solved in two important ways. In the older process,

known as the **chamber process**, sodium nitrate, $NaNO_3$, (frequently called *Chili saltpeter*) is used to provide the extra oxygen atom; it is known as a "carrier," and is another instance of a small amount of raw material doing a large amount of work. The action is as follows: When the sodium nitrate is heated, it breaks up, and oxides of nitrogen are formed, one of which is nitrogen peroxide, sometimes called nitrogen tetroxide, NO_2. In the presence of steam and sulphur dioxide, the peroxide gives up one atom of oxygen and becomes nitric oxide (also called nitrogen dioxide), NO, according to the equation

$$NO_2 + SO_2 + H_2O = H_2SO_4 + NO$$

The nitric oxide, on contact with air, takes up one atom of oxygen again and obligingly hands it over to the sulphur dioxide, which then becomes sulphur trioxide, SO_3, and this unites with water to form sulphuric acid. This process appears to go on indefinitely, the oxygen atom being delivered by the carrier and another taken on and again handed over. There is always a leakage of the nitrogen oxides, so the nitrate has to be replaced from time to time.

The second and more modern process is known as the **contact process**; its action depends upon the action produced by a catalyst. A catalyst was defined in Art. **30**, which should now be re-read. The following incident will serve to impress the meaning more firmly on the mind:

During a street railway strike, the company got some strike breakers and assembled them in one of its car barns. Outside, a crowd collected, mostly composed of strike sympathizers. Inside, the strike breakers had waited for some time and were getting "jumpy." A small boy, out of curiosity, climbed up to peek in the window; on being perceived a hose was turned on him and a part of the crowd outside was soaked. The result was that all the hospitals soon had hurry calls for ambulances. The "reaction" was sudden and complete. The small boy had nothing to do with the affair; he was neither a striker nor a strike-breaker; his mere presence started the ruction; he was a catalyst.

In the contact process, sulphur dioxide is prepared as before, but care is taken to make it dry and dust free. It is then mixed with dry air, and the mixture is passed over asbestos that has been covered with finely divided platinum. The reaction is started by heating the platinized asbestos, which needs no subse-

quent heating, because the gases coming in contact with the platinum combine directly ($SO_2 + O = SO_3$) and liberate enough heat in so doing to continue the reaction. The platinum does not enter into the reaction, but acts like the small boy in the ruction mentioned above—as a catalyst.

A small amount of platinum is thus the agent for producing a large amount of sulphuric acid, and it would so act indefinitely were it not that impurities creep in with the gases and "poison" the catalyst. Just as a small quantity of matter may act as a catalyst for large amounts of matter, so a minute quantity of some foreign element sometimes stops a catalytic action altogether; such substances are called *catalytic poisons.*

The presence of traces of selenium encourages the formation of SO_3; hence, selenium should not be present in sulphur for pulp mills.

LIME AND MAGNESIA

85. Calcium; Symbol, Ca; Atomic Weight, 40.07; Valence 2. This element oxidizes so readily that it is never found in the free state, but it has been produced pure in the electric furnace. Its oxide, CaO, called **lime,** is not found native either, because it is alkaline and combines with carbon dioxide to form carbonate of lime, $CaCO_3$, which is properly termed calcium carbonate. As calcium carbonate, it is found chiefly as limestone in immense deposits that form mountain ranges at times. When the carbonate is very pure and is very finely crystallized, it is known as *marble. Chalk* is calcium carbonate that has been formed and deposited in more recent geologic time. There is an almost infinite variety in the forms of carbonate of lime, varyingfrom beautiful, clear, water-white and tinted crystals to the dark-gray, opaque rock of which macadam roads are made.

86. Limestone ranks as the second in importance in the list of raw materials used by sulphite-pulp mills. High towers are filled with broken limestone through which water trickles. Sulphur dioxide gas goes in at the bottom of the tower, and in rising, comes into contact with the water trickling downward, producing two reactions:

First, $CaCO_3 + SO_2 + H_2O = CaSO_3 + H_2CO_3$

Second, $CaSO_3 + H_2O + SO_2 = CaH_2(SO_3)_2$

In the first reaction, calcium sulphite is formed, which is converted by the second reaction into *calcium bisulphite*, called the *acid* by the pulp maker. It is from calcium bisulphite that the sulphite process is named.

The physical condition of the limestone is of first importance; if coarse and crystalline, it may be very troublesome, easily breaking into granular particles and choking the towers, while a dense, fine-grained stone may dissolve so slowly that an insufficient amount of "combined" acid is produced.

It was noted that two salts were formed in succession when sulphur dioxide was admitted to the towers: calcium bisulphite, the final product, which is an acid salt, and calcium sulphite, also called calcium monosulphite. Calcium bisulphite exists only in its solutions; it has never been isolated. When attempts are made to obtain it by evaporation, it releases its hold on the sulphurous acid and becomes calciun sulphite, which is almost insoluble in water; thus,

$$CaH_2(SO_3)_2 = CaSO_3 + H_2O + SO_2.$$

If, therefore, the cooking solution is overheated or is deficient in free SO_2, there is a precipitation of calcium sulphite in the digester, and "lime," so called, is found in the pulp.

87. Lime Burning.—Limestone, when heated to a high temperature (825°C.) in a kiln, breaks up into quicklime, CaO (calcium oxide) and carbonic acid (carbon dioxide) gas, CO_2, according to the equation $CaCO_3 = CaO + CO_2$. Quicklime has a great liking for water, and when water is poured on it, there is a violent reaction, which produces slaked lime and liberates a great amount of heat; thus:

$$CaO + H_2O = Ca(OH)_2$$

Slaked lime, $Ca(OH)_2$, is known chemically as *calcium hydroxide* or *calcium hydrate,* and is frequently called **hydrated lime;** it is slightly soluble in water, the solubility increasing as the temperaturo of the water decreases, the solution being called **lime water;** and when the lime water contains undissolved particles of hydrated lime, it is called **milk of lime.** Hydrated lime, used in rag boiling, is produced as a fine powdery substance when there is no excess of water during the reaction; and when in this form, can be transported to the best advantage. It does not absorb carbonic acid gas from the air and revert to its former condition of a carbonate, as lime does ($CaO + CO_2 = CaCO_3$).

When water is added to quicklime, it is said to be **water slaked;** but when it is exposed to the air and combines with the carbon dioxide of the air, it is said to be **air slaked.** If hydrated lime be heated to a temperature above 450°C., the water leaves it, and it becomes quicklime again; thus, $Ca(OH)_2 = CaO + H_2O$.

In the so-called milk of lime system for making sulphite cooking-acid, sulphur dioxide gas is bubbled through lime milk and produces calcium sulphite and calcium bisulphite in two stages, thus:

$$Ca(OH)_2 + SO_2 = CaSO_3 + H_2O$$
$$CaSO_3 + SO_2 + H_2O = CaH_2(SO_3)_2$$

For this purpose the lime should be relatively free from magnesia.

It is to be understood that when lime slakes in air, the process takes place slowly; this is what happens in the case of lime mortar, used in laying brick, stone, and in plastering, and which causes the mortar to harden, but slowly. It sometimes takes years before the mortar of a building is completely carbonated. The sand used merely acts as a filler.

Practically pure calcium carbonate is used as a filler in some papers.

88. Precipitates.—When, on mixing solutions of two substances, a chemical reaction produces a new, non-volatile substance that is not soluble in the solution, it appears as a solid of some kind and is called a **precipitate.** When finely divided slaked lime is mixed with water and sodium carbonate is added, the reaction is expressed by the equation

$$Na_2CO_3 + Ca(OH)_2 = 2NaOH + CaCO_3$$

The sodium hydroxide is in solution; but the calcium carbonate is insoluble and is precipitated. The lime is said to *causticize* the sodium carbonate (**soda ash**); that is, it converts the carbonate of soda into caustic soda (NaOH), which may be obtained by filtering or drawing off, after the precipitate ($CaCO_3$) has settled. This process is called **precipitation,** meaning "a throwing down," and is frequently used in chemical operations. Sometimes a compound may be soluble in some solutions or under certain conditions and not in others; hence, the same compound may appear as a precipitate in some solutions and not in others.

89. Boiler Scale.—Mention was made of scale-forming compounds that occur in water, one of which was carbonate of lime. This compound is slightly soluble in water containing carbonic

acid; but when the water is heated, the carbonic acid (CO_2) is driven off and the insoluble carbonate of lime is precipitated. (This action is similar to that which takes place when calcium bisulphite is heated, liberating sulphur dioxide and calcium sulphite, as described in Art. **86**.) The calcium carbonate thus precipitated is one of the scale-forming compounds.

90. The Soda Process.—In soda pulp mills, the spent soda solution (black liquor), after cooking the chips, is dried and burned, to get rid of the woody matter; the "black" ash is composed largely of sodium carbonate (soda ash), which, is run into water, forming a solution of sodium carbonate. This solution is pumped to tanks wherein it is treated with lime. A reaction takes place according to the equation of Art. **88**; the soluble caustic lime becomes insoluble carbonate of lime, and the carbonate of soda becomes caustic soda. The insoluble calcium carbonate is allowed to settle or is filtered out, and the caustic soda solution is ready for use in cooking. In course of time, there is a loss of soda by leaks, by incomplete washing of the pulp, and in fumes going into the air.

91. The Sulphate Process.—The sulphate process takes its name from the fact that sodium sulphate, Na_2SO_4, also called sulphate of soda and **salt cake,** is used to make good the loss of soda, instead of using soda ash, as in the soda process. Salt cake is a by-product that is obtained in the manufacture of hydrochloric acid from salt by the use of sulphuric acid, as shown by the equation

$$2NaCl + H_2SO_4 = 2HCl + Na_2SO_4$$

The characteristic reaction of this process is the reduction of the sodium sulphate by the carbon from the woody matter in the "black" ash from the spent liquor, producing sodium sulphide, Na_2S, which becomes an active agent in the cooking liquor

$$Na_2SO_4 + 2C = Na_2S + 2CO_2$$

The unreduced salt cake, along with the carbonate, is also causticized by lime, producing calcium sulphate (only slightly soluble), and caustic soda, according to the equation

$$Na_2SO_4 + Ca(OH)_2 = 2NaOH + CaSO_4$$

The calcium sulphate is removed by settling and filtration along with the calcium carbonate produced in the main reaction.

NOTE.—*Niter cake* is sometimes used to produce salt cake. Sodium nitrate (called **niter**) is heated with sulphuric acid to produce nitric acid, according to the equation

$$2NaNO_3 + H_2SO_4 = Na_2SO_4 + 2HNO_3$$

The sodium sulphate thus obtained (also a by-product) is called **niter cake.** The reaction does not proceed so completely as when salt cake is formed; consequently, some acid sulphate, $NaHSO_4$, is present and is a source of danger, unless carefully handled, as will be explained in the Section on Sulphate Pulp Manufacture.

92. Plaster of Paris.—Calcium sulphate, also called *sulphate of lime,* is found native in many localities in a crystalline form called **gypsum,** its molecular formula being $CaSO_4{\cdot}2H_2O$. Gypsum is soft, white, satin-like, dense (at times), and contains water of crystallization when in its natural state, a part of which may be driven off by heat as in the following experiment:

EXPERIMENT 11.—Put a few grams of gypsum (dry crown filler will do) in a test tube and heat while holding the test tube nearly horizontal. Note the water condensing on the cooler end of the test tube.

Plaster of Paris is gypsum with most of the water of crystallization driven out by heating. When water in proper quantity is mixed with plaster of Paris, it is absorbed, crystallization again occurs, and the plaster *sets,* as it is termed.

93. Crown Filler.—Sulphate of lime is used as a filler in making paper. For this purpose, it is prepared artificially and the product is called **crown filler.** The reaction is shown by the following equation:

$$Na_2SO_4 + CaCl_2 = 2NaCl + CaSO_4$$

Calcium chloride and sodium sulphate are both salts that are soluble in water; when the solutions are mixed, the radicals SO_4 and Cl_2 change places, forming common salt and calcium sulphate. The calcium sulphate is only slightly soluble in water and is precipitated, while the sodium chloride being readily soluble, is washed out with water. The calcium sulphate is filtered from the water, and forms tiny slender crystals, $CaSO_4{\cdot}2H_2O$, of the same molecular formula as gypsum. When received at the mill, it should not have more than about 10% of moisture in addition to the water of crystallization. If a gram of it be put in a crucible and heated to redness, it should not lose more than about 30% of its weight; this is called *loss on ignition,* and it includes both the water of crystallization (about 20%) and the excess of moisture (about 10%). Thus, the molecular

weight of the crystalline form is about $40 + 32 + 4 \times 16 + 2 \times 18 = 172$; if when received, 10% of the weight is additional moisture, the molecular weight may be considered to be $172 \div (1 - .10) = 191$; hence, if ignition drives off this ten per cent and the water of crystallization also, the molecular weight of the amorphous sulphate will be $40 + 32 + 4 \times 16 = 136$, and the weight lost will be $191 - 136 = 55$, or $\frac{55}{191} = .288 = 29\%$ very nearly.

94. Magnesium; Symbol, Mg; Atomic Weight, 24.32; Valence, 2.—Magnesium is a light, silvery white metal, which does not combine readily with oxygen at ordinary temperatures, and may therefore be kept for a time in air. It burns with an intense, white light that has considerable actinic power, a property of certain forms of light whereby chemical action is caused in the sensitive compounds used for photographic work. Flash-light photography is accomplished by igniting powdered magnesium. On exposure to air, magnesium slowly combines with oxygen, for which reason it is not found native.

Magnesium occurs as a carbonate, called **magnesite,** $MgCO_3$, in a number of localities, notably California and Quebec in America and along the Grecian coast of the Ægean sea, the latter deposit being the purest. Magnesite was formerly used by Ekmann, one of the pioneers of the sulphite industry, in place of limestone; it is not largely used on account of its unsatisfactory physical behavior in towers, and because of the wider distribution of limestone. The hydroxide $Mg(OH)_2$ is only slightly soluble.

Dolomite, which is a combination of the carbonates of calcium and magnesium, $CaCO_3 \cdot MgCO_3$, is very widely distributed. Some deposits contain much magnesium and little calcium; these are known as **magnesium dolomites.** In other deposits, the calcium predominates, and these are known as **calcium dolomites,** also called **magnesium limestones.** The latter are largely used for the sulphite process, and are said to produce a whiter pulp than the straight limestones. Dolomites are usually coarsely crystalline and crumble too easily in the tower, causing trouble by choking and channeling.

Magnesium sulphate, $MgSO_4$, differs from calcium sulphate in that it is readily soluble in water. In the crystalline form, $MgSO_4 \cdot 7H_2O$, it is known as **Epsom salts,** and is of interest to

the paper maker as well as to other mortals on account of its medicinal properties.

Magnesium sulphite, *$MgSO_3$, is also soluble in water, and its precipitate does not cause trouble similar* to that caused by the precipitation of sulphite of lime in the digester.

THE HALOGEN GROUP

95. The Halogens.—The four elements, *chlorine, bromine. iodine,* and *fluorine,* have such close relations in their chemical characteristics that they form a chemical family called the **halogens,** which means sea-salt producers; they are thus called because they unite directly with hydrogen to form acids, which, in turn, unite with bases to form salts that are similar in appearance to common salt, and also because the first three form salts that occur in sea water. The following table exhibits some of their leading properties.

THE HALOGENS

Name	Symbol	Atomic weight	Appearance at ordinary temperature	Valency
Fluorine....	F	19.00	Pale, yellowish gas	1
Chlorine......... . .	Cl	35.46	Greenish-yellow gas	1
Bromine.............	Br	79.92	Dark brown liquid	1
Iodine..............	I	126.92	Purplish-black solid	1

These elements are all monads—their atoms have but one bond—and unite with hydrogen atom for atom to form acids that contain no oxygen, known as **hydracids.** The acids so formed are: hydrofluoric, HF; hydrochloric, HCl; hydrobromic, HBr; and hydriodic, HI.

96. Fluorine; Symbol, F; Atomic Weight, 19.—This element is a gas, and had never been isolated until a few years ago. While it is one of the most active of the elements, there is no known compound of fluorine and oxygen. A compound of calcium and fluorine, CaF_2, calcium fluoride, known as **fluor-spar,** is a widely distributed mineral that is found in large quantities, particularly in Illinois. Other minerals containing compounds of fluorine, called *fluorides,* are also found in large quantities.

Hydrofluoric acid, HF, which might also be called hydrogen fluoride, is a weak acid, but has a great liking for silica and sili-

cates, which are readily dissolved by it; it cannot, therefore, be kept in glass vessels, because glass is a mixture of various silicates. Even the fumes from the acid will attack glass. Advantage is taken of this property to etch letters and designs on glass. The glass is first covered with a thin coating of wax (which is not affected by the acid); then with the aid of a sharp stylus, the design is drawn on the waxed surface, the stylus removing the wax where it touches. The hydrofluoric acid is then applied in any convenient manner, sometimes only the fumes are used; it eats into the glass, thus making the design permanent, after which, the wax is removed.

97. Chlorine; Symbol, Cl; Atomic Weight, 35.46.—At ordinary temperatures, chlorine is a greenish-yellow gas that is extremely irritating to the membranes of the nose and throat. When inhaled in diluted form, it causes a condition similar to that caused by a "cold in the head." In concentrated form, it was the first "poison gas" used by the Germans in the recent war, and its effects were terrible and lasting. It is a very heavy gas, one liter weighing 3.1674 grams, while a liter of air weighs only 1.2928 grams.

Chlorine can be readily prepared by treating manganese dioxide with hydrochloric acid, the reaction being expressed by the following equation:

$$MnO_2 + 4HCl = Cl_2 + MnCl_2 + 2H_2O$$

EXPERIMENT 12.—Place in the bottom of a test tube about 2 grams of manganese dioxide and add a few drops of hydrochloric acid; this will be sufficient at first to produce the characteristic smell. Having noted this, add about 5 c.c. of 1 to 1 hydrochloric acid (this means a solution composed of equal parts distilled water and "concentrated" hydrochloric acid of 1.2 specific gravity), heat the tube, and collect the gas as it comes off. This latter may be done by displacing the air from a jar that stands with its mouth up, the heavier chlorine falling down and forcing the air out. The delivery tube, which connects the test tube with the jar, should go down to the bottom of the jar. If the jar is clean and the glass clear and colorless, place a piece of white paper behind it, and the chlorine may be seen rising in the jar.

Chlorine may also be prepared from common salt by mixing the salt with manganese dioxide and treating the mixture with sulphuric acid, in accordance with the equation

$$MnO_2 + 2NaCl + 2H_2SO_4 = Cl_2 + Na_2SO_4 + MnSO_4 + 2H_2O$$

There are, in reality, two reactions: the first is between the sulphuric acid and the salt, forming sodium sulphate and hydro-

chloric acid; the second is between the hydrochloric acid and the manganese dioxide, as before. There are also intermediate reactions: the liberation of oxygen from the manganese dioxide and its reaction with hydrogen to form water is the reaction that actually liberates the chlorine; the hydrogen is taken from both acids, but the sulphuric anhydride is not volatile while the chlorine is; so sulphates are formed and the chlorine is liberated in the free state. Take note of the greenish appearance of the chlorine; the odor will manifest itself without any effort on your part. Merely sniff it; otherwise you will regret it.

98. Bromine; Symbol, Br; Atomic Weight, 79.92.—Apart from its use in medicine in the form of bromides, bromine is not of much interest in this Section. It is used in investigations on cellulose and in organic chemistry. At moderately low temperatures, it is a dark brown liquid, which being slightly raised in temperature becomes a brown gas that is very irritating to the nose and throat, the effects being similar to those produced by chlorine. Bromine should not be handled by inexperienced persons; if the liquid comes in contact with the flesh, it destroys the tissues and makes painful, slowly-healing burns. If any should be spilled or splashed on the skin, dilute ammonia, washing soda, or other dilute alkaline solution, even soap, should be applied at once.

99. Iodine; Symbol, I; Atomic Weight, 126.92.—Iodine is a violet-black substance, with a metallic appearance. When heated, it melts to a dark brown liquid, which almost immediately changes to a violet vapor, and which condenses to a brown stain on cool objects held in it. If the vapor is dense and is suddenly cooled on porcelain, it forms fine flakes and crystals of sublimed iodine. This property is taken advantage of when preparing pure iodine from the crude compounds in the ash of certain seaweeds, which form the chief commercial sources of iodine. Iodine is intensely antiseptic, and is used in surgery for that reason. It is not soluble, or only very slightly soluble, in water; but it can be readily dissolved by water in the presence of about an equal amount of potassium iodide, which is, therefore, a part of the chemical equipment of a sulphite mill, as a solution of iodine is used for determining sulphurous acid.

100. Use of Iodine in Testing Bleach.—Chlorine ions readily liberate iodine ions from compounds such as potassium iodide,

KI, and this fact affords a ready means of determining if the chlorine has been removed in the washing of bleached pulp. This test depends upon the fact that iodine in contact with starch produces a blue coloration. So a thin starch paste, about one part starch to 200 parts water, is made up, and to it is added 2 or 3 parts of potassium iodide. Strips of paper are soaked in this mixture, and are dried in an atmosphere free from chlorine. These strips are white; but when they are dipped in a beater or tub containing even a trace of free chlorine, they turn blue. The paper strips should be kept in a dark, air-tight bottle, preferably, in a dark place.

QUESTIONS

(1) Why is it that some elements are found free and native while others are found only in compounds?

(2) How many cubic feet of air are required per pound (453.59 grams) of sulphur in the reaction $S + O_2 = SO_2$? Ans. 53.3 cu. ft.

(3) (*a*) What is causticizing? (*b*) How much lime will react with 500 lb. of soda ash? Ans. (*b*) 264+ lb.

(4) What chemical compound is characteristic of the cooking liquor for (*a*) the sulphite process? (*b*) the soda process? (*c*) the sulphate process?

(5) Why are calcium compounds objectionable in boiler waters?

PRINCIPLES OF BLEACHING

101. Chlorine not a Direct Bleaching Agent.—The chief use of chlorine in the paper industry is in bleaching. The chlorine itself, however, does not bleach; it acts only to release oxygen, which is the real bleaching agent. The chlorine reacts with compounds containing oxygen and hydrogen, forming hydrochloric acid and liberating oxygen. Thus, in the presence of water or water vapor, chlorine reacts in accordance with the equation

$$2Cl + H_2O = 2HCl + O$$

To prove that chlorine does not act directly as a bleaching agent, the following experiment may be tried:

EXPERIMENT 13.—It is first necessary to obtain pure chlorine, that is, chlorine free from water and hydrochloric acid. This may be done by means of the apparatus shown in Fig. 7. The flask *A* to which heat is

applied, contains manganese dioxide and hydrochloric acid (see Experiment 12); jar *B* is partly filled with water, and jar *C* is partly filled with sulphuric acid. The chlorine is collected in jar *D* by displacing the air. As the chlorine gas is generated, it passes into jar *B* and bubbles up through the water, which absorbs any hydrochloric acid that may be present; it then passes into jar *C* and bubbles up through the sulphuric acid, which absorbs any water that may be present; it finally passes into jar *D* free from water or hydrochloric acid.

If, now, a piece of highly colored cloth, thoroughly dry, be placed in jar *D*, it can be left there indefinitely without changing color; but if the cloth be first moistened with water and then placed in the jar, the color will soon bleach out, and the cloth will be white.

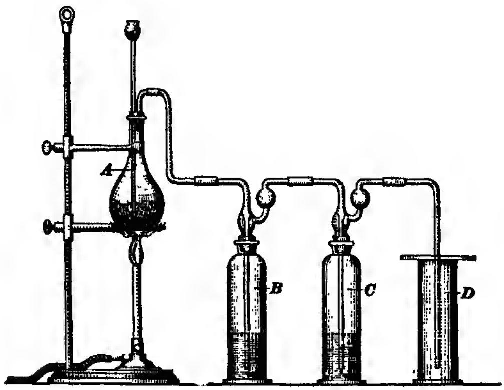


FIG. 7.

The question now arises: if it is the oxygen that does the bleaching, why doesn't the oxygen of the air act in the same way? It does, but very slowly. A great deal of the bleaching of textiles and other substances has been done for ages by exposing the goods to the weather; but with the aid of bleaching agents, effects have been achieved in an hour or so that would have taken months by the older methods.

102. A number of substances may be used as bleaching agents; these either liberate oxygen from their own compounds or they cause other compounds to do so. And this brings up another question: why does the oxygen liberated in such cases act so much more rapidly than the free oxygen of the air? Because the oxygen is in the nascent state. (See Art. **49.**) When

oxygen is liberated by bleaching agents and within a mass, such as might be found after running paper, rag stock, or wood pulp around a beater or agitating in a tank, it combines quickly with the most readily oxidizable matter present. In the case of wood pulp, those substances are the non-fibrous compounds of the woods, which have survived the cooking process and the subsequent washing. Rag stock contains colors used in dyeing the cloth, and most of these colors are substances that can be chemically changed by oxidation. Since 19 out of 20 of these substances are white or colorless when oxidized, it is seen that the chances are 19 to 1 that the color will be bleached by the nascent oxygen.

If an attempt were made to use the oxygen at some place other than where it was generated, it would be but little more efficient for bleaching than ordinary air, because its atoms would by that time have united to form molecules of oxygen, and the greater part of its chemical activity would be gone.

The compounds formed by oxygen in the bleaching of halfstuff are largely soluble, and they are therefore removed by washing, leaving behind the cellulose, which is not readily oxidized at ordinary temperatures and is insoluble. Cellulose is white; so the whitening of stock on bleaching and washing is due to the elimination of substances surrounding the fibers in addition to the formation of colorless compounds. Sometimes paper stock will not bleach because of the presence of insoluble colored pigments; in such cases, the addition of more bleach will not help, nor will the use of steam.

Bleaching may be done by: (1) agents containing oxygen, which are unstable and release the oxygen very readily; (2) agents that are stable and whose oxygen is liberated by other compounds; (3) agents that set oxygen free by indirect means from other substances. These three classes of agents will be considered in order.

103. Bleaching Agents of the First Class.—The unstable agents of this class readily break down under the influence of slight changes of temperature and liberate oxygen. One of the most powerful of these unstable agents is *ozone* (Art. 51), which, it will be remembered, is represented by the molecular formula O_3; it is very unstable, gives up the extra oxygen atom on the slightest provocation, and reduces to O_2, the stable form of the oxygen molecule.

The condition of things may be represented diagrammatically as in (a),

$$\begin{array}{ccccccccccc}
0 & 0 & & 0 & 0 & & 0 & & 0 & & \\
0 & 0 & = 2O_3 & \multicolumn{2}{c}{00} & = 3O_2 & \multicolumn{3}{c}{0\text{–}x\text{–}0} & = 2O_2 + xO_2 & \\
0 & 0 & & 0 & 0 & & 0 & & 0 & & \\
\multicolumn{2}{c}{(a)} & & \multicolumn{2}{c}{(b)} & & \multicolumn{3}{c}{(c)} & &
\end{array}$$

which represents two molecules of ozone. In (b), the two molecules of ozone become three molecules of oxygen, the two extra molecules uniting as indicated. If some molecule, which may be indicated by x and having two bonds, should come between the two molecules of (a), it might grab one molecule of oxygen with each bond, and thus become oxidized, as indicated in (c).

A number of bleaching processes have been devised to make use of the above principle, but with little success. Ozone can be liquefied by cold and pressure. It has a boiling point of $-119°$C. ($-182.2°$F.), that is, it changes into a gas at this temperature. Consequently, trade compounds called *liquid* ozone are likely to be fakes.

104. A more useful compound of this class is *hydrogen peroxide*, H_2O_2. Its usual method of preparation is by the action of dilute sulphuric acid on barium peroxide, thus:

$$BaO_2 + H_2SO_4 = BaSO_4 + H_2O_2$$

The insoluble barium sulphate is precipitated and the hydrogen peroxide is held in solution (the sulphuric acid was diluted, remember, and the water used to dilute the acid absorbs the hydrogen peroxide). The reaction must take place at a low temperature and the solution must be kept cold. The excess of sulphuric acid is neutralized by careful addition of barium hydroxide. The solution of hydrogen peroxide is filtered away from the precipitated barium sulphate; it can be used for very delicate fabrics, as it leaves no corroding residue. The writer knows of one case in which hydrogen peroxide was tried as a bleach on sulphite pulp; it was too expensive, and in the case cited, there seemed to be a marked tendency to go back in color on exposure to light.

105. Sodium peroxide, Na_2O_2, has been used at times for bleaching. When this substance is added to water, sodium hydroxide is formed and oxygen is liberated; thus,

$$Na_2O_2 + H_2O = 2NaOH + O$$

Sodium hydroxide is a very active and caustic compound, and it must be used with great care. The sodium hydroxide formed is neutralized with sulphuric acid, and any excess of that must in turn be neutralized by a weaker alkali or a volatile one, such as ammonia.

106. Bleaching Agents of the Second Class.—To this class belong compounds that contain oxygen, are quite stable, and yield oxygen more slowly. *Potassium permanganate*, $KMnO_4$, is one of these, and many attempts have been made to use it on paper making fibers, but without any marked success; its own cost is one obstacle, and the need for reducing the brown manganese hydroxide that is left in the stock is another. This latter is effected with sodium bisulphite, thus adding to the cost of handling and control.

107. Bleaching Agents of the Third Class.—This class is best represented by **bleaching powder.** These compounds do not liberate oxygen from their own molecules, but bring about the liberation of oxygen from other compounds.

The exact composition of bleaching powder is not definitely known. By analysis, it seems to have the formula $CaOCl_2$, but in its chemical reactions, it seems best represented by the formula CaClOCl or CaCl(OCl), in which case, it might be termed *calcium oxychloride* or *calcium chlorohypochlorite*, formed from calcium hypochlorite $Ca\begin{array}{l}\diagup OCl \\ \diagdown OCl\end{array}$, in which one OCl group is replaced with Cl, thus producing a compound having the formula $Ca\begin{array}{l}\diagup OCl \\ \diagdown Cl\end{array}$. The molecular weight is about $40 + 16 + 2 \times 35.5 = 127$, of which the chlorine forms $\frac{71}{127} = .559 = 55.9\ \%$ and it would naturally be expected that this amount of chlorine would be available after a chemical reaction. As a matter of fact, only from 30 to 35% is available in practice. Bleaching powder appears to be a mixed salt, a calcium salt of hypochlorous and hydrochloric acids; it is also commonly called **chloride of lime.**

108. Bleaching powder is made by passing chlorine over slaked lime. There is likely to be present not only free chlorine but also free lime; and as slaked lime has an excess of water up to 2 or 3 per cent, some hydrochloric acid is formed from the water

and combines with the lime to form calcium chloride, which has no bleaching value. Thus,

$$2Cl + H_2O = 2HCl + O$$
$$2HCl + Ca(OH)_2 = CaCl_2 + 2H_2O$$

If the conditions were ideal, the entire reaction would be expressed by the equation

$$Ca(OH)_2 + Cl_2 = CaCl(OCl) + H_2O$$

The strength of bleaching powder is expressed as "per cent of available chlorine," that is, in terms of the amount of chlorine set free to liberate oxygen.

When the compound CaCl(OCl) is treated with water, two molecules reassemble into one molecule of calcium chloride and one molecule of calcium hypochlorite, $Ca(OCl)_2$; thus,

$$CaCl(OCl) + CaCl(OCl) = CaCl_2 + Ca(OCl)_2$$

109. For use, bleaching powder is stirred in water for a short time and is allowed to settle; the excess of lime goes to the bottom, and the clear liquid contains the calcium hypochlorite. There appears to be some free chlorine in the solution, and this chlorine starts the reaction; thus,

$$2Cl + H_2O = 2HCl + O$$

The hydrochloric acid thus formed acts on the calcium hypochlorite, liberating hypochlorous acid; HClO; thus,

$$Ca(OCl)_2 + 2HCl = CaCl_2 + 2HClO$$

Hypochlorous acid may be considered as being produced from the anhydride, Cl_2O, by the addition of water; thus,

$$Cl_2O + H_2O = 2HClO$$

In action, the hypochlorous acid breaks up into hydrochloric acid and oxygen; thus,

$$HClO = HCl + O$$

From the foregoing, it will be seen that the two atoms of chlorine in the 2HClO from the initial two molecules of bleaching powder which contained four atoms of chlorine are able to liberate two atoms of oxygen, while its valence would indicate a replacing power equal only to one atom of oxygen; it parts with its own oxygen and separates one atom more from water.

110. Determining Strength of Bleach.—In determining the available chlorine of bleach, *i.e.*, the chlorine which is present in such form that it takes an active part in bleaching, the figure

arrived at indicates the chlorine equivalent of the oxygen liberated. The strength of bleach is determined by reagents that act as reducers, such as sodium thiosulphate or arsenious acid. By using standard strength solutions of them it is easy to measure the oxygen they use up. It is necessary to keep in mind that oxidation and reduction are simultaneous. A reducing agent is simply a substance that combines more readily with oxygen than the substance to be reduced.

Arsenious oxide, As_2O_3, in the presence of alkalis, takes up oxygen to form arsenic oxide, As_2O_5; consequently, a solution of arsenious oxide in sodium carbonate is used for testing bleach. This and other methods for determining the strength of bleaching powder and liquids are given in another Section (Bleaching of Pulp).

If the tub, bleacher, or other container in which bleaching is done, be too hot, the oxygen will be liberated too rapidly; some of it will be lost in the air, while the free chlorine, and the hydrochloric acid resulting from its action, will react upon the fibers, making them tender and brittle, and entailing a loss of fiber substance. It is generally conceded that a temperature of 38°C. to 40°C. (say 100°F. to 105°F.) need not be exceeded to obtain the maximum economical result.

111. Acids Used to Accelerate Bleaching.—Sulphuric acid (vitriol), hydrochloric (muriatic) acid, and acetic acid (the acid of vinegar) are sometimes used to accelerate bleaching; the reaction with chloride of lime is that of combining with calcium and liberating the hypochlorous acid; thus,

(1) Sulphuric acid— $H_2SO_4 + Ca(OCl)_2 = CaSO_4 + 2HClO$

(2) Hydrochloric acid—$2HCl + Ca(OCl)_2 = CaCl_2 + 2HClO$

(3) Acetic acid— $2(H{\cdot}C_2H_3O_2) + Ca(OCl)_2 = Ca(C_2H_3O_2)_2 + 2HClO$

The molecular formula for acetic acid is $C_2H_4O_2$, but when written as above, it shows how the hydrogen separates from the radical $C_2H_3O_2$; the compound $Ca(C_2H_3O_2)_2$ is called *calcium acetate*.

The safest acid of the three is acetic acid; the other two are liable to attack the fibers, besides corroding and perforating the tubs, drum washers, and other metal parts, leaving soluble metallic residues that are likely to cause discoloration and, later, adversely affect sizing and coloring work. Alum can also be

used for the above purpose, since it has an acid reaction when in solution. (See Art. **75.**)

112. Antichlors.—Sometimes, owing to the need for obtaining rapid results or to get a brighter shade of white, an excess of bleach is used. To get rid of this excess when it is not feasible to wash it out, certain substances that are known as antichlors are used. *Sodium thiosulphate*, $Na_2S_2O_3$ (frequently, but incorrectly, called *hyposulphite* and shortened to *hypo*) is one of these; another is *sodium bisulphite*, $NaHSO_3$. The prefix "thio" is from the Greek word *theion*, which means sulphur. When prefixed to the name of a chemical compound, it usually indicates that a part or all of the oxygen of another compound has been replaced by sulphur. Thus, sodium sulphate has the molecular formula Na_2SO_4, while sodium thiosulphate has the formula $Na_2S_2O_3$; here one atom of oxygen in the sodium sulphate is replaced by one atom of sulphur. As another example, sulphuric acid has the formula H_2SO_4; while thiosulphuric acid has the formula $H_2S_2O_3$. The formula for carbonic acid is H_2CO_3, and that for thiocarbonic acid is H_2CS_3.

The antichlors, sodium thiosulphate and sodium bisulphite, combine with the chlorine of the excess bleach to form hydrochloric acid and thus stop the chain of reactions of Art. **109** in accordance with the equations:

$$Na_2S_2O_3 + 4Cl_2 + 5H_2O = Na_2SO_4 + H_2SO_4 + 8HCl$$
$$2NaHSO_3 + 2Cl_2 + 2H_2O = Na_2SO_4 + H_2SO_4 + 4HCl$$

The acid products of these reactions must be washed out carefully. The expense items are thus increased by the cost of the excess of bleach and the cost of the antichlor. Good bleaching practice in paper mills should render unnecessary the use of antichlor.

113. Other Substances used in Bleaching.—Sulphur dioxide, SO_2, sodium sulphite, Na_2SO_3, sodium bisulphite, $NaHSO_3$, or other sulphites and bisulphites are used at times for bleaching, especially on ground wood. The solid salts are merely the vehicles for the gas, sulphur dioxide, which is the agent producing the bleaching or blanching effect, and which is largely used in bleaching wool. The action in these cases is the direct reverse of that which occurs when chlorine is used. Chlorine is an oxidizing agent, while the use of sulphur dioxide is a reducing process. Some colored oxygen compounds become colorless

by the removal of a part or all of their oxygen. But the resulting compounds are very susceptible to the action of oxygen, even in the air; and they therefore "go back" or become yellowed by exposure, especially to the sunlight. If bleaching be done by sulphur dioxide in any form, thorough washing is essential, not only for the foregoing reason but because the oxidation of sulphur dioxide produces sulphur trioxide, SO_3, the anhydride of sulphuric acid, which will combine with water to form sulphuric acid and destroy the material.

114. Bleach Prepared by Electrolysis.—The practice of preparing bleach liquors from common salt, sodium chloride ($NaCl$), by electrolysis is becoming quite general. This is referred to as *electrolytic bleaching*, but the term is not a proper one, since the bleaching is performed in the same way chemically as when powder is used. The electric current, under proper conditions, liberates chlorine from salt, and the chlorine is passed into towers, going in at the bottom. A stream of milk of lime flows in at the top and meets the ascending chlorine. The chlorinated lime flows to the settling basins, where it is recovered and used in the same way as bleaching powder; it is, in fact, a solution of bleaching powder, which is sometimes called *electrolytic bleach.*

Under certain conditions, sodium hypochlorite is formed directly from the salt, and this might properly be termed electrolytic bleach. It bleaches more efficiently than bleaching powder, but it costs more to produce.

QUESTIONS

(1) How is it that starch mixed with potassium iodide is a test for the presence of free chlorine?

(2) What is the difference in principle between bleaching with chlorine and bleaching with sulphurous acid or sulphites?

(3) What chemical property is common to all anti-chlors? If sodium sulphite were used, what compounds would be formed?

(4) Why is "electrolytic bleach" an improper expression?

METALS OF INTEREST TO PAPERMAKERS

115. Some General Characteristics of Metals.—It was previously stated that the distinction between a metal and a non-metal is sometimes very slight. In general, however, a **metal** is an element that will replace the hydrogen of an acid to form a

salt; see definition of acid in Art. 62. Metals combine with the group OH, called hydroxyl, to form bases, and the alkali metals form very characteristic bases, as has already been noted. In general, the metals have a metallic luster, a characteristic color (usually white), and possess hardness and certain other physical characteristics that enable them to be recognized. Considered electrically, the metals are, in general, electropositive, while the nonmetals are electronegative.

The following table gives a list of some of the principal metals, which have been grouped in accordance with their electrical properties, the most strongly electropositive being at the top, and the others following in order, the bottom ones being least electropositive. Any metal in this list is less electropositive than any above it and more electropositive than any below it. It will also be noted that the oxides of those metals that are most strongly electropositive are the most difficult to reduce, while those at the bottom of the list are reduced very readily.

Oxides cannot be reduced completely to metal, even in a current of hydrogen.	Sodium, potassium (alkalis) Barium, calcium (alkaline earths) Magnesium Aluminum Manganese
Oxides readily reduced.	Zinc Chromium Cadmium Iron Cobalt Nickel Tin Lead Hydrogen Copper Arsenic Bismuth Antimony
Oxides break up by simple application of heat.	Mercury Silver Palladium Platinum Gold

Any metal will displace from their salts metals below it in this list and will be displaced from its own salts by metals above it. Iron, for example, will displace copper from copper sulphate; this can be demonstrated very readily by dipping the blade of a knife in a solution of copper sulphate, the displaced copper being deposited on the blade and sulphate of iron produced. Or let a piece of copper (wire or sheet) stand for a time in black ink; then place a knife blade in the ink, and in a short time it will be coated with copper. All the metals in the above list, down to copper, oxidize with ease on exposure to the air; those below copper either do not oxidize in air or do so very slowly. While hydrogen is not metallic, it displaces and can be displaced by metals, which is the reason for including it in the above list.

IRON, MANGANESE, NICKEL, COBALT

116. The four metals, iron, manganese, nickel, and cobalt, are closely related chemically, as might be inferred from their atomic weights; thus,

Manganese...	54.93
Iron.........	55.84
Nickel.......	58.68
Cobalt.......	58.97

Of this group, cobalt is of least interest to the papermaker, though one of its compounds, smalts, a cobalt-potassium-silicate, is used as a paper pigment. This pigment is of great permanence, but expensive, and it has a low coloring power as compared with organic dyes. It can be readily detected even after the paper is burned, because the blue color remains in the ash and is not destroyed by alkalis or acids, as is the case with ultramarine blue.

All of the above metals exhibit the property of magnetism or of susceptibility to magnetic influences. They combine in two series with acids, forming two series of salts, and they have two oxides. In one series, they appear to be divalent (to be dyads) and in the other series trivalent (to be triads). For instance, there is

Iron protoxide, FeO, or ferrous oxide,	and iron sesquioxide, Fe_2O_3, or ferric oxide;
Manganous oxide, MnO,	and manganic oxide, Mn_2O_3;
Nickelous oxide, NiO,	and nickelic oxide, Ni_2O_3;
Cobaltous oxide, CoO,	and cobaltic oxide, Co_2O_3;
Ferrous chloride, $FeCl_2$, or iron protochloride	and ferric chloride, Fe_2Cl_6, or iron perchloride.

117. Further Remarks Concerning Chemical Nomenclature.—In connection with chemical nomenclature, Art. **81**, an explanation was given of the suffixes "ous" and "ic" and the prefix "per;" and in Art. **46**, the meaning of "mon" or "mono," "bi" or "di," etc. was explained, when used as prefixes. The prefix "proto," as used above, is derived from the Greek, and it has the same meaning as "mono," which is derived from the Latin. The prefix "sesqui" is derived from the Latin and means one and one-half; as used above, it indicates that there are $1\frac{1}{2} = \frac{3}{2}$ times as many oxygen atoms as there are of iron, manganese, etc. Ferrous oxide might also be called iron monoxide, and thus three different names may be used to designate the same compound, FeO.

118. Iron; Symbol, Fe; Atomic Weight, 55.84.—Pure iron is practically unknown, and it can be obtained only with great difficulty. It is a silvery metal, having a specific gravity of 7.86 and melting at 1505°C. (2741°F.) and is soft, ductile, and malleable. The compounds of iron are very widely distributed, but it is seldom found native, except in the case of meteorites, which have been found to contain metallic iron.

Iron is readily soluble in all acids, even when they are dilute; hence, it should not be used to make containers for transporting or storing fluids of acid character, such as sulphite liquors. An exception, however is concentrated sulphuric acid, free from water, which does not attack iron, and may, therefore, be shipped in iron drums. In paper mills, while the corrosion itself may be so slight as to be negligible, a small amount of iron in the furnish has much effect on the color of the paper. Sulphur combines directly with iron in the presence of oxygen, forming iron sulphide FeS, as may be proved by the following experiment:

EXPERIMENT 14.—Mix well and place in a dry test tube $5\frac{1}{2}$ grams of fine iron filings and 3 grams of powdered sulphur. Heat gently at first, but raising gradually to a dull red heat. There will be noted about this time or a little sooner a bright glow, starting at the bottom and running through the entire mass. This internal increase of heat denotes the progress of the reaction indicated by the equation

$$Fe + S = FeS$$

The point at which the reaction begins is called the *critical temperature*. After cooling, it is necessary to break the tube in order to remove the fused mass, which, on being tested with a magnet, will be found to be non-magnetic, except possibly a few fragments that did not combine. When a few particles of the mass are treated with 1 : 1 sulphuric acid, hydrogen sulphide,

H_2S, is given off, and may be recognized by the familiar smell of rotten eggs, which is caused by the same compound, and is noticeable in the vicinity of some sulphate mills.

As an illustration of the foregoing, the writer has seen the cast-iron impeller of a fan used in blowing hot burner gases changed almost entirely to iron sulphide. It was observed to run out of balance, and when the case was opened, the blades were found to be about twice as thick as when installed. Thinking that the blades were coated with some deposited material, the workman struck them with a hammer, when they immediately broke and crumbled to pieces. This result would occur only in the presence of free sulphur, caused by overheating and an insufficient air supply in the burner.

119. Iron Ores.—The most important ores of iron are two oxides; ferric oxide, Fe_2O_3, called **hematite**, and magnetic iron oxide, Fe_3O_4, called **magnetite**; the latter appears to be a combination of FeO and Fe_2O_3. Carbonate of iron, $FeCO_3$, called **spathic iron**, and bisulphide of iron, FeS_2, called **iron pyrites** or **fool's gold**, are also important sources of iron. When a metal is found in any mineral in sufficient quantity to make its extraction profitable from a commercial standpoint, it is called an **ore**; the minerals mentioned above are all iron ores, though the sulphides are not used directly for reduction to iron, being first employed for the production of sulphur for sulphuric acid. When heated with free access of air, their sulphur combines with the oxygen to form sulphur dioxide, SO_2, and the iron is left as ferric oxide, Fe_2O_3, which may be reduced to iron by another process. The commercial processes for extracting metals from their ores and the scientific control of such processes is called **metallurgy.**

120. Metallurgy of Cast Iron.—The metallurgy of iron is typical of the general practices with metals—reduction of oxides and elimination of impurities by fluxes. If the metals are found in combinations other than oxides, they are brought to that condition (oxides) by first heating in air, and then the reduction is undertaken.

Iron ores are reduced in what are known as **blast furnaces,** a sectional view of one being shown in Fig. 8, and so called from the fact that a blast of air is blown into the base of the furnace through a series of holes supplied by pipes called *tuyeres;* one of these holes is shown in the figure and marked *A*. These furnaces are from 70 to 80 ft. in height, and are made of steel and lined

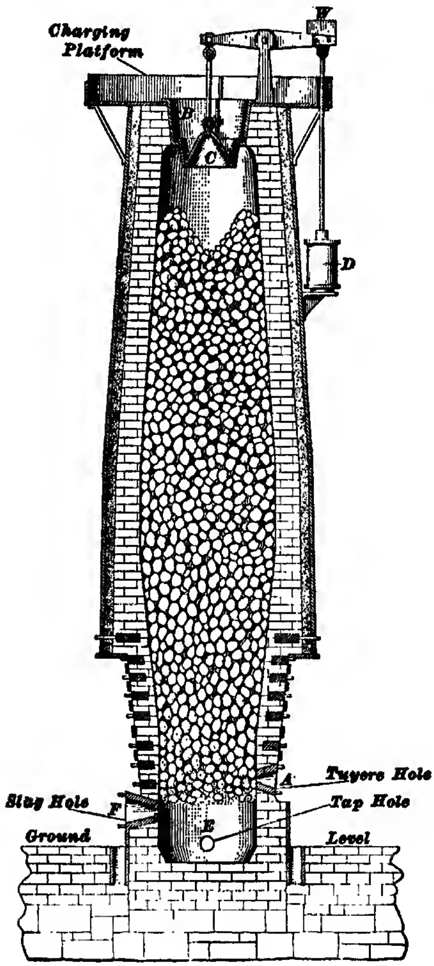

Charging Platform
W
B
C
D
A
Tuyere Hole
Slag Hole
F
Tap Hole
E
Ground
Level

FIG. 8.

with fire-brick. When once started, a furnace is kept in continuous operation. The furnace is filled with a mixture of coke (which is almost pure carbon), iron ore, and limestone in the following manner:

At the top of the furnace is a charging platform to which the material is hoisted and dumped into the hopper *B*, which is closed at the bottom by the bell *C*. This bell is raised and lowered by various means; that used in the present case is a cylinder *D*, the piston of which is operated by steam or air. The materials, the amount of which has been carefully calculated and weighed, are dumped into the hopper in the following order: first, the coke, then the iron ore, and lastly the limestone. The bell is then dropped, and the materials fall into the furnace. This process is carried out until the furnace is filled. When first starting the furnace, materials for firing are first placed in the bottom; but this is not again necessary after the furnace has once been started, as it is never again entirely empty. The air, which is supplied by a blowing engine, enters through the tuyeres and is forced up through the furnace. The melted iron, after reduction, accumulates in the bottom, and is discharged at regular intervals through the tap hole *E*. The slag, which is lighter than the iron and floats on its top, is run off from time to time through the slag hole *F*.

The reaction is quite simple in principle, but in practice, there are several stages of reduction and oxidation. The amount of air admitted is carefully calculated, and is only sufficient to oxidize the coke to carbon monoxide; thus,

$$C + O = CO$$

The heat is so great, however, that the reaction proceeds to the second stage and forms carbon dioxide, the oxygen for this reaction coming from the iron ore, in accordance with the equation

$$3CO + Fe_2O_3 = 3CO_2 + 2Fe$$

The limestone is called the **flux**; it combines with the impurities and forms a molten mass known as the **slag**. These terms, ore, flux, and slag, are used in connection with all metallurgical operations. The ore is the source of the metal taken from the mine; if it is a sulphide, it is first *roasted*, that is, heated in the air, to remove the sulphur and get it into the form of an oxide. The oxides are then heated with a reducing agent, usually carbon,

and a flux of such nature that it will combine with the impurities to form a liquid slag, through which the metal sinks as it is formed. In the case of iron, the metal is run off through the tap hole in the bottom of the furnace into little previously prepared ditches, which reminded early workers of a sow with a litter of pigs; so they called the iron bars thus formed **pigs**, and the iron itself was called and is still called **pig iron.** It will be readily seen that pig iron is likely to contain (must necessarily contain) impurities and that one of these will be carbon, which is likely to be present in pig iron to the extent of 3% or 4%. The differences between cast iron, wrought iron, and steel are due more to the amount and condition of the carbon present than to any other cause.

121. Impurities in Iron Ores and Iron.—In addition to the oxide of iron, the ores contain varying amounts of silicon, phosphorus, sulphur, and manganese compounds, and are classified commercially by the amounts of these impurities present, the metal obtained from them being much influenced by such compounds. In general, the effects produced are:

Silicon—hardens and strengthens; it is not considered very injurious, except when present in considerable amounts, when the metal becomes very hard and brittle.

Phosphorus—very undesirable; it makes iron and steel *cold short*, that is, renders it liable to break when cold.

Sulphur—also undesirable; it makes iron *hot short*, that is, renders it liable to break when heated.

Manganese—helps to offset the effects of sulphur and increases the tensile strength. In steel, in quantities over 1%, it makes the metal brittle under shock.

122. Effect of Carbon.—Carbon may be regarded as a constituent of iron, rather than an impurity. It is present in two forms: as carbon in the form of graphite, in which case, the iron and carbon constitute a mixture; and as a chemical combination with iron, forming what is really a carbide of iron, in which case, it is referred to as **combined carbon,** the first case being called **graphitic carbon.**

Steel is iron containing a small amount of combined carbon, and the value of steel lies in the condition and amount of its carbon. Mild steel contains under 0.05% carbon. From about 0.05% to 1.50% the strength and hardness of steel increases with the increase in carbon.

When pig iron is cooled as it lies in the molds, the cooling takes place slowly, and the carbon it contains crystallizes out in tiny flakes resembling black specks, which make up the graphitic carbon, also called *free carbon.* Pig iron must be treated in various ways before the metal is suited to industrial purposes. Depending on the manner of treatment, the result is cast iron, wrought iron, or steel, and there are many varieties of each.

Cast iron is made from a form of pig iron known as gray iron, which is produced when certain proportions of the fluxes are used. With the exception of valve wheels, drain pipes, machine frames, boiler fronts, and parts of foundations for pumps and machinery not subjected to stresses other than compression, cast iron will not be often met with in the paper mill.

123. Steel.—When iron is heated and quickly cooled by plunging it into water, the carbon has no time to crystallize, and it remains in combination with the iron; the resultant product is **steel,** a metal that is tougher and stronger than cast iron. There are many varieties of steel, some of which will now be considered.

Bessemer steel is iron that has been melted under proper conditions in a suitable container called a **converter,** air being blown through the molten iron, thus oxidizing the impurities or enabling the slag present to do so. The sulphur and carbon are oxidized to gases, escaping through the mouth of the converter, and the phosphorus goes into the slag, forming phosphate of lime. If the converter is lined with firebrick or other siliceous material, the resulting metal is called **acid steel**; but if it is lined with calcined dolomite or magnesia brick, the resulting metal is called **basic steel.** In the former case, the lining is of an acid nature, which prevents the removal of sulphur and phosphorus, hence, steel produced by this process can be made only from ores that are low in these elements. With the basic lining and the use of a little lime thrown into the converter, it is possible to remove most of the phosphorus in the slag, as calcium phosphate. This basic process can be used to produce a milder steel than can be obtained by the acid process.

Cast steel is now largely used in place of both cast iron and wrought iron. The molten steel is produced by the *open hearth process* or the steel is melted in the electric furnace. There are many forms of such furnaces, but the heating is usually done by placing the metal in an electric circuit. The furnace conditions

can be very closely controlled, and steels of very high quality and of varying degree of hardness and malleability are produced.

124. Wrought iron is made by heating pig iron or scrap iron in what are called *reverberatory* or *puddling furnaces*, a diagrammatic section of such a furnace being shown in Fig. 9. This type of furnace, which is much used in metallurgical operations, consists of two main parts that are partially separated by a low wall *D*. The part *A* contains the fire, and part *B* contains the metal or mass to be melted or treated. The flames from *A* are reflected to *B* by the slope of the roof, whence the name, reverberatory.

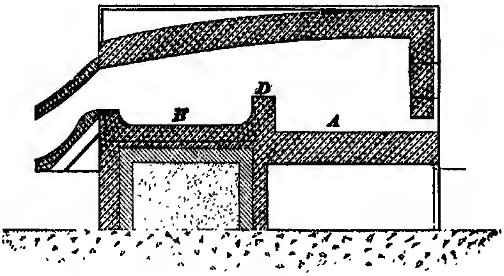


FIG. 9.

In making wrought iron, the metal (with or without slag producing additions) is melted and puddled (stirred) with long steel bars, with the object of oxidizing the impurities, especially carbon, and getting rid of gas so produced. The metal thus obtained is free from flakes of graphitic carbon and also from blowholes, and it may be welded, hammered, or forged. Bolts, rods, and bars are generally made of wrought iron or mild steel (steel containing less than 0.05% carbon). Pulleys, shafting, and machine parts generally are now usually made of steel.

125. Recalescence.—Steels having special qualities are obtained either by tempering in various degrees or by the addition of some other metal. When pure iron is heated to about 900°C. and allowed to cool slowly, the fall in temperature is in simple proportion to the time, for the most part; there are, however, three points where the regular rate of cooling is retarded. The temperatures at which these effects are noticed are 825°C., 720°C., and 650°C., and the effects, which are called **recalescence,** are due to molecular changes in the iron, which cause it to give out

heat at these points. If much carbon is present, recalescence is noted at only one temperature, 670°C. When iron is slowly heated, there are similar points where the rise is retarded, the temperature is about 30° higher than the four temperatures that were noted when the iron cooled. Close attention to these facts and a knowledge of their relation to the coloring of steels when tempering (the temper colors) make possible the production of steels of widely varying physical properties. The older workers in steel did not use thermometers, and they tempered their steel by color. A very pale yellow is the first color to be observed; this becomes darker, shading into brown, then red, purple, and finally blue. The usual way to obtain these colors is to heat the steel to a cherry red, then dip in water for a few seconds, the length of time depending upon the size of the steel object. After removing from the water, a strip of the part quenched is polished, to remove the scale, and the colors are then noted as they appear. The outside surface, which was cooled by immersion in the water, is heated by the hot interior part, gradually becoming hotter as the various colors appear. Consequently, the temperature at which the first color appears is the lowest, and the blue is the highest. These colors are really due to thin layers of different oxides, which serve as good indicators of temperature. With a knowledge of the behavior of carbon in the recalescence of iron, and the use of pyrometers (instruments for measuring high temperatures) very uniformly tempered steel can be produced, suited to various uses.

126. Annealing.—When metal is worked or is heated and quickly cooled, the molecular structure is set in a state of strain, and it may crack under quick changes of temperature or under sudden blows. If, however, the heating and cooling are done slowly and regularly, or if the metal is maintained for some time at the temperature of treatment and then slowly cooled, the metal is toughened and does not crack; this method of treatment is called **annealing.** Not alone metals, but glass, porcelain, and other substances that solidify on cooling are annealed.

127. Alloy Steels.—The properties of steel are greatly modified in respect to hardness, strength, toughness, etc. by the addition of small amounts of certain other elements—principally metals—such as chromium, manganese, nickel, tungsten, vanadium, or molybdenum; such steels are called **alloy steels,** and they are severally termed chrome-steel, manganese-steel, nickel-steel, etc.

128. Protective Coatings for Iron.—**Galvanized iron** is made by first dipping the iron in a vat containing dilute sulphuric acid, to remove surface impurities and oxides; this is called *pickling*. The iron is then dipped in molten zinc, which adheres to the cleaned surface and forms a coating over it. The action is not electrical, as the term "galvanized" indicates; in fact electricity tends to cause destruction of the iron; because, wherever the zinc coating is imperfect (commonly referred to as *pinholes*), so that spots of iron are uncovered, an electrolytic action is set up between the iron and the zinc, in the presence of moisture. This breaks the bond, and the protection that should be afforded by the zinc coating is destroyed. Zinc is readily soluble in dilute acids, so galvanized iron has a very short life in or around a sulphite-pulp plant. Good sheet iron that is free from rust and that has been painted is much more serviceable.

Tin plate is sheet iron covered with tin, the process being much the same as that for galvanizing iron.

Nickel-plated iron is iron having a coating of nickel, the nickel being applied by a process known as *electroplating*. The plating is a protective coating, as nickel is not readily affected in air by water and carbonic acid. It is, however, corroded by and is soluble in fumes of other acids, and it therefore protects the iron from such only for a short time. Nickel plating is largely used as a matter of ornament; with proper care, it preserves the metal under it, while maintaining a highly decorative appearance.

It may be remarked that it has recently been found that sheet iron containing small amounts of copper in its composition resists the action of the weather much longer than when the copper is absent.

129. Some Iron Compounds.—There are not many compounds of iron that are of direct interest to the paper maker. The ferrous compounds are usually light-green in color; they have a tendency to change into ferric compounds in the presence of air or oxidizing agents generally. If paper stock containing iron compounds (as it does at times) were bleached with sulphur dioxide, it would, as a rule, shortly "go back." Ferric compounds are reduced to ferrous compounds by sulphur dioxide; but exposure to oxygen will very soon cause a reversion to the ferric condition again; consequently, sulphur dioxide is not practi-

cal as a bleaching agent for paper stock, where an off color is caused by iron. Iron in water is very detrimental to the making of white or delicately colored papers. It darkens or dulls the shade.

130. Iron oxides and hydroxides are found in many places associated with clay; these compounds are known as **ochers,** and are used as pigments for coloring purposes. If the proportion of iron is low, the color is light, forming *yellow ochers,* the darker shades containing more iron. When the ochers as found are subjected to heat, a great variety of shades is produced, from pale yellowish-brown to deep purple, depending on the temperature, length of time heated, and the character of the furnace—whether open (oxidizing) or closed (reducing). These pigments are used in coloring paper. Compared with organic dyes, they have low coloring power; but on account of the clay present, they also act as a filler. Owing to their high specific gravity, they have a tendency to sink on going over the wire, so that the under side of the sheet is more highly colored than the upper side. It would help to remedy this defect somewhat if the ocher and stock were put into the beater and well circulated before adding size, and if the stock color and size were again well circulated before adding the alum. By so doing, the color and fiber are intimately associated, and the flocculent rosin-alum compounds are precipitated around them as binders. These pigments occasionally form a part of the ink in old paper, and then give trouble in bleaching. A little hydrochloric acid helps to remedy this, but it should be carefully used and quickly washed out.

131. Iron combines with sulphuric acid to form ferrous sulphate, which forms crystals with 7 volumes of water, the molecular formula being $FeSO_4 \cdot 7H_2O$. All the compounds of metals with sulphuric acid were formerly called **vitriols,** ferrous sulphate being called *green vitriol,* copper sulphate, *blue vitriol,* and zinc sulphate, *white vitriol,* because of the color of the compounds. When ferrous sulphate was heated highly, the sulphuric acid was driven out, condensed, and called *oil of vitriol* or, simply, *vitriol.*

132. Compounds containing the radical CN, as HCN, KCN, etc. are called **cyanides,** and the radical itself is called **cyanogen.** The rather complicated substances formed by combination of iron and alkali metals with cyanogen are important, as they are

the sources of several blue shades of the Prussian blue type. One method of getting this blue on paper stock is to add nitrate of iron to the stock as a mordant (see Section on Coloring), and then potassium ferrocyanide, also called yellow prussiate of potash, $K_4Fe(CN)_6$, the reaction giving a beautiful blue. This blue may be bought as a paste blue and used directly. The term Prussian is due to the compound being derived from Prussic acid, the chemical name of which is *hydrocyanic acid*, HCN, one of the most active and deadly of poisons. All the cyanides are extremely poisonous, and must be handled with exceptional care; even their fumes are very poisonous.

133. Writing Inks.—Iron is one of the chief constituents of writing inks; and, until the advent of the coal tar dyes, it was the principal constituent of all black writing inks. The effectiveness of iron in ink depends upon its reaction with *tannic acid* to form tannate of iron, a deep black precipitate that is so finely divided it does not settle for a long while; in fact, under the conditions of modern ink manufacture, it is almost a solution. It is colloidal,* and the addition of a small amount of gum arabic or sugar renders its suspension practically permanent. The bark of the oak tree is one of the chief sources of tannin in commerce. A small boring insect has a way of laying eggs in parts of the oak, which cause swellings on the twigs and branches; these are known as *nut galls*, and they contain a high percentage of tannic acid.

The older way of preparing ink was first to prepare an infusion of tannic acid by crushing the nut galls, soaking them in water, straining off the resulting liquor, and adding to this an adequate amount of ferrous sulphate, when the black color was produced. The outline produced with this ink is not black at first, but becomes very dark in time. This drawback is overcome by adding a strong blue dye, so that when first written, the ink appears blue, but later becomes black; this is the principle of the so-called *blue-black inks*.

What are known as **invisible inks** are made in various ways, one being to make use of the iron-tannic acid reaction. Thus, write on paper with a solution of ferrous sulphate, which leaves no marks on the paper. When it is thoroughly dry, dip the paper

* A *colloidal substance*, or **colloid**, is one whose particles are so finely divided that suspension of them in a liquid forms what is practically a solution. Some colloidal solutions will even pass through an ordinary filter. Colloids settle very slowly if at all.

into a pot of tea that has stood for some time, and thus contains a solution of the tannic acid that is present in the tea leaves; the tannic acid combines with the ferrous sulphate of the writing and turns it black.

134. Nickel; Symbol, Ni; Atomic Weight, 58.68; Sp. Gr., 8.9. Nickel is found in only a few places, the two chief sources being New Caledonia (an island in the South Pacific Ocean) and the Sudbury district of Ontario, Canada. The ore is a sulphide of nickel, with iron and copper and small amounts of gold and platinum. The ore is roasted, and the resultant mass, known as *matte*, is reduced and refined. Nickel is chiefly used as an alloy for steel and for nickel plating, although a small amount is used in coinage. Its compounds are not used in the paper industry.

135. Manganese; Symbol, Mn; Atomic Weight, 54.93; Sp. Gr., 7.39.—Manganese is found as a hydrated oxide called *bog manganese* and as the dioxide, *pyrolusite*. It occurs in many localities in North America, but not in very large quantities. Its use in steels has already been referred to. The dioxide, MnO_2, is used in glass making to correct the greenish tinge due to iron present as an impurity.

Manganese is interesting in compounds with sodium and potassium; in combination with oxygen, it forms negative ions, with the result that two acids are generated—manganic acid, H_2MnO_4, and permanganic acid, $HMnO_4$. The former has not been isolated, but the latter forms the salt *permanganate of potash* (*potassium permanganate*), $KMnO_4$, the use of which in bleaching has already been mentioned. This compound or the analogous *sodium permanganate*, $NaMnO_4$, is used in sanitation as a deodorizer. Both salts contain a large proportion of oxygen that is not very strongly bound, and which readily combines with organic matter. The presence of this loosely bound oxygen makes potassium permanganate a very useful reagent in volumetric analysis.

QUESTIONS

(1) Do all metals liberate hydrogen from acids? How would you prove your answer?

(2) What effect may free sulphur in burner gases have on an iron fan? Write the equation for the reaction and suggest a remedy.

(3) How is it that concentrated sulphuric acid may be safely handled in iron drums, but dilute sulphuric acid cannot?

COPPER AND LEAD

136. Copper; Symbol, Cu; Atomic Weight, 63.57; Sp. Gr., 8.93. The symbol Cu is from *Cuprum*, a corruption of Cyprus, an island in the Mediterranean Sea where copper was found. Copper is found native in a number of localities. Its chief ores, however, are the sulphides *chalcopyrite* and *bornite*. It is also often found as a carbonate in *malachite*, which is a beautiful green mineral of silky appearance, and in *azurite*, an equally beautiful blue mineral. Copper forms two series of compounds, cuprous and cupric; in the first, it is univalent (a monad), and in the second it is divalent (a dyad).

Copper is used in the metallic form in many familiar ways; for instance, it is used to make pipes for paper stock, as a lining for beaters, to prevent discoloration by iron, and in many other forms, one of the principal ones being in the form of copper wire for conveying electric currents.

Copper is not directly attacked by acids, its salts being formed from the oxides. Nitric acid appears to attack it vigorously, but this is due to the strong oxidizing action of the acid. Hydrochloric acid does not dissolve copper except in the presence of oxidizing compounds. For this reason, in addition to the others that were given in the discussion of bleaching, it is not desirable to heat the stock more than necessary when bleaching in the beater, because the hydrochloric acid and oxygen formed will shorten the life of the wire in the drum washer by attacking the copper, which in most mills is more susceptible on account of second-hand machine wire being used. The reason that dilute acids attack machine wire is because it contains other metals (impurities) in addition to copper.

Sulphuric acid attacks copper only when concentrated and hot. Copper pipe and apparatus may be used with sulphite liquors, since they are reducing, not oxidizing, agents. When hot and under pressure, these liquors sometimes attack copper, probably on account of the action of sulphurous acid on the impurities present. Digester fittings are made of acid resisting bronze.

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QUESTIONS

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sulphur), with the result that after roasting in air some lead oxide, PbO, is formed and some of the sulphide, PbS, is left. When the air is shut out, the heat causes the sulphur of the remaining sulphide to combine with the lead oxide that was formed, according to the equation

$$PbS + 2PbO = 3Pb + SO_2$$

It would appear to be a very economical process; and so it would be, except that impurities such as antimony, zinc, and silver may be present and must be eliminated, and it is not easy to separate lead and antimony. Fortunately, however, lead containing antimony is much harder than pure lead, and it does not collapse as lead does when made into pipes. This has been found to be of great value in the sulphite industry for the construction of coolers, eliminating in a large measure the leaks and other troubles incidental to the collapsing and "creeping" of soft lead pipe. About 10 per cent of antimony gives an excellent material.

139. Another important series of lead alloys are the bearing metals or Babbitts. One in use for general purposes has the following composition: Sn, not below 6½%; Cu ½–¾%; Sb 14–16%; Pb 77–79%; P .025–.110%; impurities under .5%.

140. Lead is a heavy, soft metal, which when freshly cut, is bright lustrous blue-gray; but the surface soon becomes dull on exposure to the air, because of the formation of a sub-oxide that later becomes a carbonate. Pure lead is not readily attacked by cold acids, but if it contain much oxide, it may then become spongy from corrosion. Lead is largely used for piping and for sheet-metal work, where its softness and pliability make for easy working and for facility in conforming to the surfaces to which it is attached. In the early days of the sulphite industry, it was employed to line digesters, and until recently, it was used almost exclusively for household piping and gutters. In cases where rain water or very pure spring water is used for drinking, there is danger of lead poisoning when the water stands in lead pipes of containers, because, if any carbonic acid be present in the water, it dissolves the lead to some extent. If a little carbonate of lime or other substance that makes hard water be present, a protective coating of insoluble salts is formed in the pipe. All lead salts are poisonous, and the effect is cumulative; that is, the lead is not eliminated by the kidneys or bowels, but is retained, behaving

in a dangerous, disagreeable way, and eventually causing death. The sulphate of lead is insoluble; therefore, if lead has been taken internally, the treatment suggested (*indicated*, as the doctors say) is to change the lead compound into the insoluble sulphate by giving the patient a sulphate, such as Epsom salts, which is magnesium sulphate.

The valency of lead is rather puzzling; it is usually 2 or 4, but there are five different oxides, Pb_2O, PbO, PbO_2, Pb_2O_3, and Pb_3O_4, of which the second, third, and last are of special interest here.

141. *Lead monoxide*, PbO, called **litharge**, is largely used as a constituent of cement in digester linings, and is also used in glass making. Its reddish-yellow color and other physical properties render it useful in paints.

Lead peroxide, PbO_2, also called *lead dioxide*, is a brown powder, and acts as an oxidizing agent; it also forms an important link in the chain of reactions upon which the lead accumulator, or storage cell, is based, and which will now be considered.

The elements of a storage cell of the lead peroxide type are:

(*a*) two plates of lead, corrugated or perforated so that

(*b*) a paste of litharge may be spread on them and held in place;

(*c*) a container filled with 20% sulphuric acid (about 1.142 sp. gr.).

The litharge coated plates are immersed in the acid solution, and the litharge, PbO, is converted into lead sulphate, $PbSO_4$. A current of electricity is passed through the cell, and, as it will be remembered, when an electric current is passed through water containing sulphuric acid, the water is decomposed, giving up hydrogen at one pole and oxygen at the other (Experiment 1), and this is what happens here; but the hydrogen instead of passing off as a gas, takes the place of the lead in the lead sulphate and forms hydrogen sulphate (sulphuric acid). The lead is deposited as a black spongy mass on the plate that acts as the cathode. The oxygen at the other plate, the anode, secures the help of the sulphuric acid and forms lead persulphate, $Pb(SO_4)_2$, which is quickly hydrolyzed by the water, in accordance with the equation

$$Pb(SO_4)_2 + 2H_2O = PbO_2 + 2H_2SO_4$$

and a dark brown coat is laid on the anode. The cell has been "charged."

As was previously noted, sulphuric acid is composed of hydrogen ions and sulphate ions, and when the acid is in solution, these ions are more or less free. The charged cell contains two plates, one coated with finely divided lead and the other with finely divided lead peroxide, immersed in a solution of sulphuric acid. Now, when the charging current is cut off and a new circuit is made that connects the two plates, the hydrogen ions run over to the lead peroxide, which is reduced back to litharge again, thus

$$PbO_2 + H_2 = PbO + H_2O$$

and the two atoms of hydrogen have given up two positive charges at that pole. The sulphate ions, SO_4, travel to the other plate and form lead sulphate with the spongy lead coating, giving up two charges of negative electricity for each sulphate ion, thus

$$Pb + 2SO_4 = Pb(SO_4)_2$$

Consequently, one plate has a positive charge and the other a negative charge; and when the two plates are connected by a circuit outside the cell, a current will flow and will continue to flow until the brown lead peroxide is all gone, when the cell is said to be "discharged." Railroad cars and automobiles are lighted by storage batteries, and they can be equipped to utilize some of their motive power in recharging the cells.

Red lead, Pb_3O_4, when mixed with oil, is used to make tight joints in pipefitting; white lead and litharge are equally serviceable for the same purpose.

142. Lead Salts.—Lead salts are mostly insoluble in water, though lead nitrate and lead acetate are soluble. The acetate is commonly called *sugar of lead;* it has a sweetish taste, but like all the lead salts is very poisonous.

Chrome yellow is lead chromate, $PbCrO_4$, and is produced in the furnish by first adding to the beater either lead nitrate or lead acetate, which is allowed to circulate for a time, and then a solution of sodium bichromate, $Na_2Cr_2O_7$, when the insoluble lead chromate is precipitated in the fibers and the soluble sodium nitrate or sodium acetate is subsequently washed out. As the result of the reaction between the sodium bichromate and lead nitrate, lead bichromate is formed, thus:

$$Na_2Cr_2O_7 + Pb(NO_3)_2 = 2NaNO_3 + PbCr_2O_7$$

But lead bichromate is unstable in water, and the resulting yellow

color is that of lead chromate, $PbCrO_7$, which is probably formed directly from lead oxide in some intermediate reaction.

White lead, or basic carbonate of lead, $2Pb(CO_3)_2 \cdot Pb(OH)_2$, is used as a pigment; it will be referred to briefly in the discussion of paints.

143. Zinc; Symbol, Zn; Atomic Weight, 65.37; Sp. Gr., 7.10; Melts at 419.4°C.—Zinc is found most frequently as a sulphide, ZnS, called *zinc blende*. This mineral is roasted to eliminate the sulphur, and is then further heated until the metal runs out. In this form, the metal is impure and is known as *spelter*. When cooled, pure zinc is brittle and breaks with a beautiful crystalline fracture.

Pure zinc is a bluish-white metal; when heated to 120°C. to 150°C., it can be rolled into sheets, which remain flexible when cooled; it is brittle at 200° to 300°C. It melts at 419.4°C. and boils at 918°C. In pure dry air it exists for a while without change, and in most cases it will resist weather for a long time; it is attacked by acid fumes in the air, however. As before mentioned, zinc is largely used in the form of galvanized iron.

Ordinary yellow brass is an alloy of zinc and copper, 2 parts copper to 1 part zinc, while red brass has 90% copper. There are also several alloys of zinc and aluminum.

Alloys exhibit many astonishing properties. For instance, a certain alloy of zinc and aluminum contains 95% zinc and 5% aluminum, or 19 parts zinc and 1 part aluminum. The specific gravity of zinc is 7.1 and of aluminum 2.7; it would naturally be expected that the specific gravity of the alloy would be $\frac{19 \times 7.1 + 1 \times 2.7}{19 + 1} = 6.88$, assuming the alloy to be a mixture, but in reality it is only 2.8, only a trifle more than that of aluminum. As another example, there is an alloy called Wood's metal, which consists of 4 parts bismuth, 2 parts lead, 1 part each of tin and cadmium; the melting points of these metals are, respectively, 269.2°, 327°, 231.9°, and 320°C., and it could reasonably be expected that the melting point of the alloy, assuming it to be a mixture, would be

$$\frac{269.2 \times 4 + 327 \times 2 + 231.9 \times 1 + 320 \times 1}{4 + 2 + 1 + 1} = 285\tfrac{1}{3}°\text{C.},$$

very nearly, but as a matter of fact, the melting point is 65.5°C. (150°F.); that is, it will melt in water that would

not scald one's finger. In fact, alloys can be produced that will melt at almost any desired temperature, a fact that is of great importance in the manufacture of sprinkler heads in fire extinguishing systems and of fusible plugs in steam boilers.

144. Tin; Symbol, Sn; Atomic Weight, 119; Sp. Gr., 7.29; Melting Point, 231.9°C.; Boils at 2270°C.—Tin is found in abundance in only a few places, Straits Settlements, England, and Bolivia being the chief sources.

What is called *sheet tin* is usually sheet iron plated with tin by dipping clean sheet-iron plates in molten tin. Block tin is the name given to material composed entirely of tin. Helical pipes of pure tin are used as condensers for distilled water, as tin is not affected by air or water or both at ordinary temperatures, and dilute acids act on it only very slowly.

145. Tin forms two series of compounds, in one of which it is divalent (a dyad) and in the other quadrivalent (a tetrad); these compounds are called stannous and stannic, respectively. Stannous chloride, $SnCl_2$, is used as an accessory in some coloring processes, and is referred to as *tin salts* or tin crystals.

146. Mercury; Symbol, Hg; Atomic Weight, 200.6; Sp. Gr., 13.6; Melts at −38.8°C.; Boils at 356.7°C.—Pure mercury is a white, silvery liquid at ordinary temperatures; it is commonly called **quicksilver**, first, from its silvery appearance and, second, from the fact that it moves, the word *quick* here having a meaning opposite to that of the word *dead.*

A familiar use of mercury is in the bulbs of thermometers, barometers, and manometers. Since it freezes at −38.8°C. (−37.84°F.) and boils, vaporizes, at 356.7°C. (674°F.), it cannot ordinarily be used beyond these temperatures; but by filling the space in the tube above the mercury with an inert gas, like nitrogen, the gas will be compressed as the mercury rises, which increases the pressure on the mercury, raises its boiling point, and makes it possible for a mercurial thermometer to indicate temperatures considerably higher than 674°F. (356.7°C.).

The bright red pigment known as *vermillion* is mercuric sulphide, HgS, but it is expensive, for which reason red lead and organic coloring matters are much used instead. Mercury occurs as a sulphide in the ore called *cinnabar,* and it may be obtained from this mineral and also from mercuric oxide simply by heating.

147. Amalgams are alloys in which one of the constituents is mercury. *Gold amalgam,* for example, is the alloy obtained when gold is dissolved in mercury. *Sodium amalgam* plays an important part in one type of electrolytic cell used in making bleach; in contact with water, the sodium acts as if it were free, forming sodium hydroxide (NaOH) and liberating hydrogen. Amalgams of tin or silver are used for the backs of mirrors.

Corrosive sublimate is *mercuric chloride,* $HgCl_2$, commonly called *bichloride of mercury.* It is a white, finely crystalline powder, intensely poisonous, and is a powerful disinfectant and germicide. It is soluble in water, and is used in surgery as an antiseptic. **Calomel** is *mercurous chloride,* HgCl, commonly called *chloride of mercury,* and is used in medicine; while not poisonous, it should not be taken except as prescribed by a physician. As used, it is a white powder, which should not be exposed to sunlight, since the light affects it, liberating chlorine and forming some mercuric chloride, $HgCl_2$; the latter has been used for calomel by mistake, with fatal results.

148. Gold; Symbol, Au; Atomic Weight, 197.2; Sp. Gr., 19.33.—Gold is readily soluble in mercury, and much of the finely disseminated gold in gold ores is recovered by dissolving it out with mercury and then vaporizing the mercury. The gold is left behind, as it does not even melt until a temperature of 1062.4°C. (1944°F.) is reached; the mercury vapor is collected, condensed, and may be used over again, the process being repeated indefinitely. This method of obtaining metallic gold is called the *amalgamation process.*

149. Aluminum; Symbol, Al; Atomic Weight, 27.1; Sp. Gr., 2.65; Melts at 658.5°C. (1217.3°F.).—This metal occurs universally, and ranks third in the list of elements that compose the earth, being exceeded only by oxygen and silicon. Having a strong affinity for oxygen, it is never found native, but exists chiefly as a silicate in different kinds of clay, all of which are silicates of aluminum, with varying amounts of impurities and water of constitution. These will be described in connection with the element silicon. Aluminum is prepared commercially from *bauxite,* a mineral containing a large proportion of **alumina,** which is the common name for aluminum oxide, Al_2O_3. A familiar compound of aluminum is *emery,* which is a common form of the mineral *corundum;* it is extremely hard and is much used as

an abrasive. With traces of other substances, such as manganese and chromium, corundum forms such precious stones as the ruby, sapphire, oriental amethyst, and oriental topaz.

Bars, sheets, and wires of aluminum are not much affected by exposure to the atmosphere, because a film of oxide quickly forms on the outer surface and protects it. This fact, coupled with a conductivity nearly as high as copper and a specific weight only about one-third that of copper, makes aluminum wire a very desirable substitute for copper in transmitting electric currents.

150. The most interesting salt of aluminum in paper making is *aluminum sulphate*, $Al_2(SO_4)_3$, also called *sulphate of alumina* and referred to as **papermaker's alum.** (See Art. **75.**) It crystallizes with 18 molecules of water as $Al_2(SO_4)_3 \cdot 18H_2O$, the molecular weight of which is $2 \times 27.1 + 3(32.1 + 64) + 18 \times 18 = 666.5$; hence, the crystal contains $324 \div 666.5 = .486 = 48.6\%$ water.

The alums proper are *double salts* that contain alkalis as well as aluminum, all of which crystallize with a large proportion of water of crystallization. For instance, there is sodium alum, $Al_2(SO_4)_3 \cdot Na_2SO_4 \cdot 24H_2O$; also, iron and chromium sulphates form alums without aluminum, but crystallizing in the same form with 24 molecules of water. From this it will be seen that the alum family is a large one.

PAINTS AND PAINTING

151. Paints may be considered as being made up of a pigment and a vehicle, the **pigment** being the coloring material and the **vehicle** the liquid that holds it so it can be applied to the surface.

152. Pigments are usually dry, powdered mineral or metallic oxides, carbonates, sulphates, or silicates. The ochers—brown, yellow, red, and purple—are oxides of iron, prepared by heating the natural mineral, either to dry it for grinding and sifting or to produce varying shades, which depend upon temperature and furnace conditions.

153. White Lead.—**White lead** is basic carbonate of lead (Art. **142**), $2Pb(CO_3)_2 \cdot Pb(OH)_2$, and is prepared from metallic lead by two general processes: the older "Dutch" process, and the more recent *quick process*.

The **Dutch** or **old process** takes about 3 months to complete. It consists of submitting buckle-shaped pieces of lead to the action of acetic acid (vinegar) and carbon dioxide, which is produced in the same chambers from fermenting tan bark. Eventually, a hydrated carbonate of lead, $2Pb(CO_3)_2 \cdot Pb(OH)_2$, is produced.

In the **quick** or **new process** the acetic acid solution of lead is acted upon directly by carbonic acid (carbon dioxide) gas or an alkaline carbonate. According to some authorities, the old process produces a better pigment.

154. Zinc Oxide.—Zinc oxide is used as a pigment, and is known as **zinc white.** It is preferred to white lead because it does not blacken under the action of sulphurous fumes, as lead does; this is because lead sulphide is dark brown or black, while zinc sulphide is white. There are other technical points in favor of zinc: it dries harder than white lead, and is thus used in getting enamel effects; but it may produce a brittle coat; hence, where a lasting color is not necessary and long protection is required, a mixture of the two pigments is very satisfactory.

155. Other White Pigments.—There are other white pigments, such as *whiting, barytes, barium sulphate,* and *lithopone.* The last is a mixture of zinc sulphide and barium sulphate, produced by adding a solution of zinc sulphate to one of barium sulphide, thus:

$$ZnSO_4 + BaS = ZnS + BaSO_4$$

The reaction products are both insoluble white precipitates, and they have no paint value in this form. But, an Englishman named Orr discovered a method of so treating them that they are now used very largely in paints, and have been found to have even more covering and hiding value than zinc oxide. This paint is also known as *Orr's white, ponolith, Beckton white,* and by several other names.

156. Extenders.—It has been found that perfectly pure pigments, like white lead, lead sulphate, zinc oxide, lithopono, do not give such good service as they do when diluted with what are known as **extenders.** These are inert silicates, china clay, talc, etc., which not only increase the life of the paint but also make the matter of repainting more satisfactory. Yet only a few years ago they were considered adulterants, and efforts were made to keep the paint makers from using them.

157. Whiting.—**Whiting** is carbonate of lime, and while it is deficient in covering power when used pure, it is not deserving of the poor opinion some paint people have of it. It serves to correct any acidity that might occur in a paint, and some paint makers use a little of it in all their products. Whiting is generally prepared from chalk, sometimes from marble, by grinding and bolting (screening) through a 200-mesh sieve. The marble dust, and similar material, has what is technically termed *tooth,* and is therefore a good element in priming coats; it also leaves a better surface for repainting. The tooth is due to the angular crystalline edges of the fine fragments.

158. Gypsum.—*Calcium sulphate,* otherwise called **sulphate of lime** and **gypsum,** familiar in paper mills as **crown filler,** etc., is another of the white pigments around which much discussion has centered. If not completely dehydrated, it acts somewhat as in plaster of Paris, taking water from its neighbors to complete its molecule in the crystalline form, $CaSO_4 \cdot 2H_2O$, which causes a hardening or setting of the paint when stored. Neither whiting nor gypsum should contain any free lime, and since some artificial preparations of both are not free from this defect, it is better to use the natural material.

159. Colored Pigments.—The white pigments, extenders, and ochers form the bulk of the pigment material. Coloring materials, such as smalts, ultramarine, prussian blue, red lead, litharge, vermillion, and some of the organic colors, are used to produce the many shades called for.

160. The Vehicle.—The vehicle is the liquid part of paint; it may consist of three parts: *oil, drier,* and *volatile thinner.* Sometimes gums are added, but these are ground with the pigment.

161. Oils Used in Vehicles.—The oils used are of the class known as *drying oils,* that is, oils that when spread in thin films and exposed to the air combine with oxygen to form a rubbery or hard surface. When paint is thus exposed, the surface of the vehicle is multiplied tremendously by being spread over the particles of a pigment mixed with the oil, and the film produced becomes hard and dry in a day or so.

The oil most commonly used for paint is **linseed oil,** obtained by pressing flax seed, which contains 35 to 40 per cent of oil. After linseed oil has been stored for some time, it shows a sediment of flocculent material known as **foots.** Some linseed oils, on

heating to about 350°F., exhibit what is called a **break,** that is, they become cloudy by the separation of mucilaginous matter. Such oils, and those containing foots, are not suited to some purposes, and are refined. This is done by agitating with 1% of its weight of sulphuric acid and subsequently washing with water; or by heating moderately, and stirring with fullers' earth, followed by filtration. Oil for use in paints should be free from foots, but an oil that breaks may be used for paint making.

Linseed oil, when heated with oxides of lead, manganese, or cobalt, exhibits greatly increased drying properties; such oil is called **boiled oil,** though, properly speaking, it is not boiled, since decomposition follows heating at too high a temperature. Sometimes the metallic oxides are heated with rosin and then mixed with the oil; these are called **resinate boiled**; but, if excessive amounts of rosin are used, the oil is inferior. Those oils in which the metallic oxides are heated with linseed oil are said to be *linoleate boiled.*

Other oils are used for paint, such as *China wood oil,* also called *tung oil,* and *soya bean oil,* and these are being increasingly consumed.

162. Driers.—If boiled oil is used, the paint dries too soon to a hard, brittle coat; therefore, a certain amount of raw oil is used to produce a slower drying and a more elastic coat. In fact, it is believed by quite competent authorities that raw oil will make as good paint as boiled oil, though in such cases a little Japan drier should be added. **Japan drier,** or **Japans,** as they are usually called, are made by heating oil with metallic oxides and adding turpentine or a light petroleum oil. Consequently, they are called at times *turpentine driers.*

163. The Thinner.—The thinner most employed and most preferred by paint makers is **turpentine,** which is distilled from the resins of the southern pines. Another variety is distilled from the wood itself, by steam, but it is not so uniform. Of this class would be any turpentine that might be obtained as a by-product in the pulp industry where a resinous wood is used, especially, in making sulphate pulp from jack pine. Another much used thinner is a light petroleum distillate that is similar to gasoline. Coal tar furnishes another light-oil distillate that is used to some extent as a thinner.

The requirements of a good thinner are that it shall be com-

pletely volatile and a good solvent; the latter term meaning that it shall not cause any flocculation or separation of the materials, its function being to thin the paint for increased covering power or to improve its working quality under the brush.

164. Better service and a saving of time in repainting would result in many cases by coöperation between the painter and the paint chemist. The atmospheric conditions, reflection and absorption of light, speed of drying, etc., are all points worthy of careful study, and they cannot be economically left to chance.

Much practical information on oils, paints, and varnishes is concisely given in Circular No. 69, of the United States Bureau of Standards, reference to which is hereby acknowledged by the writer.

CARBON AND SILICON

165. Two General Classes of Carbon Compounds.—Every living thing, animal or vegetable, and everything derived from things that are alive or have once lived contain carbon, and all such substances are termed **organic substances.** Many other substances that are closely allied to these in their chemical composition are classified as organic for this reason, though they are not organic in the sense of having once formed a part of the animal or vegetable kingdom; for this reason, the term **carbon compounds** is frequently used instead of the word *organic*, and the longer term, chemistry of the carbon compounds, is employed instead of organic chemistry, when one wishes to be precise.

The compounds of carbon included under the head of organic chemistry are almost innumerable, and some of these will be considered later. There are, however, a few (comparatively speaking) compounds of carbon that belong to inorganic chemistry, and these, together with the element itself, will be considered here.

166. Carbon; Symbol, C; Atomic Weight, 12; Valence, 2 and 4.—Carbon occurs in three well recognized forms, two of which are crystalline and the other is amorphous (without form). The diamond is a crystal of practically pure carbon; it is the hardest substance known. Graphite is another crystalline form of carbon, but is widely different in its physical characteristics from

the diamond; the so-called "lead" in a lead pencil consists principally of graphite. Coal, coke, and charcoal constitute varieties of the amorphous form of carbon.

167. Coal.—Every coal bed is made up of what was once a forest, the transition from a tree to coal being somewhat as follows: first, a bog, then peat, next lignite, then, in succession, semi-bituminous, bituminous, semi-anthracite, and anthracite coal.

Anthracite is the oldest and purest form of coal; it contains a very high percentage of carbon, and is frequently called *hard coal.* Age, enormous pressure, and temperature have combined to remove nearly all the volatile and liquid impurities, with the result that when it burns there is very little flame and practically no smoke. This is the chief domestic coal, but the dust and very fine sizes are burned in power plants that are especially equipped for that purpose. To burn this coal successfully, the fires should be thin, constantly fed, not "worked" too much, the grates should have small openings, and there should be a good draft.

Steam coal has several subdivisions, according to locality where mined and the use to which it is put. Among these are semi-anthracite, which (as the name indicates) tends to the anthracite class, and gas coal, which is used for the preparation of illuminating and heating gas. This coal contains impurities, mainly volatile, and yields a large volume of gaseous material when heated.

168. Gas Making.—In the manufacture of gas, the coal is heated in closed chambers, called **retorts.** As the gas comes off, it contains water vapor, ammonia, and tarry matter. It is then washed by "scrubbing," as the process is called, in towers, the gas going in at the bottom of the tower and leaving at the top, being washed in the meantime by a current of water trickling down over baffle plates, which removes the ammonia. The tarry matter is collected in traps, and a part of it is recovered also from the ammoniacal water. Many valuable chemicals, such as benzene, carbolic acid, naphthalene, etc., are obtained by distillation of the tar. The residual tar is used for roofing and for other waterproofing purposes. The chemicals that are recovered from the tar form the basis of the great coal-tar industry and of many compounds used in medicine and photography.

After removing the tar and ammonia, the gas still contains

sulphur, which must be removed to prevent odor when burning and to prevent blackening the paint in houses. This is accomplished by passing the gas over iron oxide, spread out in large flat pans. The oxide combines with the sulphur to form sulphide of iron. The spent, or used, oxide, after its sensitiveness has been destroyed by absorption of sulphur, may be renovated by exposing it to the air, as the sulphide of iron that is formed in this manner is very unstable. The washed gases are dried over quicklime, which also absorbs any carbonic acid that may be present, and are finally stored in large inverted tanks, open at the lower end and sealed by water; these tanks are called *gasometers*, and are a familiar feature of city skylines.

After the gas and other volatile substances are driven off from the coal, the solid carbon (with its combined ash) is left in the retort in fragments that have much the same general shape as the coal fragments from which the gas was derived; this material is relatively hard, and is called **coke.** Coke is an excellent fuel, being smokeless, and it does not clinker; it is not readily ignited, and it requires a strong draft, burning with an intense, steady heat.

A recent method for firing coal for industrial purposes is to grind it very fine, so fine that when mixed with fuel oil it is almost in a colloidal state; this mixture is then sprayed into the firebox. This method has been employed successfully in oil-fired boilers.

169. Graphite.—Carbon is found in many places in a form variously known as **graphite, plumbago,** and **black lead.** Graphite is both crystalline and amorphous, the flake graphite being crystalline. The crystals, however, have an entirely different shape from those of the diamond. Graphite can be burned only with difficulty, for which reason it has been used as a lining to protect crucibles from the action of molten masses that have been heated in them. Flake graphite, with or without oil, is used as a lubricant for heavy bearings on slow-moving shafts. It has no grit, is unaffected by any increase in temperature due to friction, and is superior to oil for many purposes.

An important use for the amorphous graphite is in the manufacture of carbons of electric-arc lights; also, as anodes for electric furnaces and electrolytic cells for bleaching plants. Artificially prepared graphite is used in making anodes, because it is more uniform than the gas-retort carbon that was formerly used.

Gas-retort carbon is a form of graphite that is deposited inside the retorts in gas works; it is very dense, and when ground and compressed it made fairly good electrodes for arc lamps. It was found, however, that occasional impurities caused local action in the electrodes and impaired their usefulness.

170. Bone Black and Lamp Black.—**Bone black,** which is sometimes called **animal charcoal,** is made by heating bones and animal refuse in closed vessels that do not admit air. A number of volatile substances are first driven off by the heat, and these are sometimes condensed and recovered. Eventually, there are left the charred bones (and the carbonized residue of the animal refuse), which consist of carbon and calcium phosphate, and are sometimes used as a pigment when finely ground. A high-grade black is prepared by dissolving the phosphate with hydrochloric acid, washing, drying, and grinding the unaffected carbon remaining, which is known as **bone black.** Bone black has the power of absorbing colors and odors from solutions that may be filtered through it; it is used, for example, in sugar refining.

When oils, rosin, turpentine, or kindred substances burn, they give off a heavy, black smoke. The smoke is unconsumed carbon, and a very old practice is to cause this smoke to be deposited, like soot, on a cold surface, from which it is removed and used for paints and inks, for example, in India ink. It is known as **lamp black,** from the familiar phenomenon of blackening the chimneys of oil-burning lamps.

171. Charcoal.—**Charcoal** has been used for fuel and other purposes from ancient times. It is prepared from wood by heating the wood to a high temperature, without access of air, and driving out the volatile substances. This is done in various ways, one being to use retorts in practically the same manner as when making coke in gas manufacture, in which case, the valuable volatile products are condensed and saved. Both coke and charcoal are extensively used in the manufacture of steel and in other reduction processes. Charcoal is very useful for filtering purposes, though not as effective as bone black in special cases.

172. Oxides of Carbon.—Carbon forms two oxides, the monoxide, CO, in which it is divalent, thus, $C = O$, and the dioxide, CO_2, in which it is quadrivalent, thus, $O = C = O$. In the first case, carbon is a dyad and in the second case, a tetrad; in most of its chemical relations, it acts as a tetrad.

Carbon has a strong affinity for oxygen, and is, therefore, of great use in metallurgy, as was previously mentioned, since it takes oxygen from metallic oxides (ores) and reduces them to metals, the carbon combining with the oxygen. Carbon monoxide, CO, is formed when organic matter, coal, or other forms of matter containing carbon is burned with an insufficient air supply. Carbon dioxide is usually formed first in this process; but it has to divide its oxygen with the excess of carbon present, reducing to the monoxide, in accordance with the reaction

$$CO_2 + C = 2CO$$

This reaction occurs in producer-gas plants, which are essentially stoves or furnaces in which coal is burned with only enough air to form CO instead of CO_2. Its presence in flue gas from a boiler plant indicates insufficient oxygen in the fire box.

173. Gas Engines.—When the CO (produced as just described) is not used immediately, as in glass works, it is cooled and stored in gasometers for future use, as in the cylinders of gas engines, which are also called *internal combustion engines*, and which operate in a manner very similar to the engines of automobiles. In the case of a gas engine, a mixture of air and gas is admitted to the cylinder, where it is ignited. The resulting combustion, which is so rapid as to be called an explosion, increases the temperature of the gas mixture to a very high point, causing it to expand and force the piston to the other end of the cylinder. By a suitable arrangement of valves, etc., a regular series of explosions, on one side or on both sides alternately, of the piston, causes it to move to and fro in the same manner as in the case of a steam engine. The burned gases are forced out of the cylinder on the return stroke in the same manner that the exhaust steam is forced out of the steam-engine cylinder.

174. Reason for Using Carbon Monoxide.—If carbon could be readily volatilized, it would not be necessary to use CO; but since it becomes a gas only at temperatures above 3500°C. (6332°F.), it must first be burned to the first stage of oxidation in order to get it into a gas form. It might be asked, "why is it not sufficient to use carbon as a fuel? Why convert it into a gas?" This is done mainly because the process is more economical. When coal is burned under a boiler to produce steam, so much of the original energy of the coal (heat units generated by the combustion) is lost that it is quite common to find only 25

per cent of total heat energy of the coal is delivered for useful work in the engine cylinder, the remaining 75 per cent being dissipated in the heat carried up the stack in the escaping hot gases, unconsumed carbon in the smoke, heat required to raise the temperature of the air used for combustion, an over supply of such air, heat losses from radiation, etc., etc. But when the carbon is converted into carbon monoxide and used directly in the engine cylinder, there is a great saving in heat units.

175. Carbon Monoxide.—Carbon monoxide is very poisonous when inhaled; it is not soluble in water. It enters into direct combination with oxygen, chlorine, and some other elements, one of these compounds being $COCl_2$, the chemical name for which is *carbonyl chloride*, but is commonly called *phosgene gas*, which was one of the poison gases used during the late war.

176. Carbon Dioxide and Carbonic Acid.—Carbon dioxide, CO_2, which is commonly, but erroneously, called carbonic acid, is formed whenever carbon is burned and a sufficient amount of air (or oxygen) is present during the combustion. The amount present in flue gases is an indication of the efficiency of the boiler house. As before stated, it is carbonic anhydride, the anhydride of carbonic acid; its density is 1.5 times that of air, and it is soluble in water, 1 volume of water dissolving 1 volume of the gas at 15°C. and atmospheric pressure; the resulting solution has a biting, pungent taste, somewhat like a weak acid solution. Carbon dioxide is used in the manufacture of soda water and other effervescent drinks and portable fire extinguishers; a comparatively small quantity in the air will smother a fire by displacing or diluting the oxygen. This may be demonstrated by breathing gently on a small match flame.

Carbonic acid is so very unstable that it is known only in the form of a very dilute solution; any attempt to isolate it results in its being broken up into carbonic anhydride and water, in accordance with the reaction

$$H_2CO_3 = CO_2 + H_2O$$

This acid forms a number of very important salts. Since it has 2 replaceable hydrogen atoms, it forms both normal and acid salts (see Art. **71**); but, because it is such a weak acid, both the normal and the acid carbonates of the alkalis have a marked alkaline reaction. The salts, carbonate of soda (Na_2CO_3) and

carbonate of lime ($CaCO_3$), which are the two most important carbonates in the paper industry, have already been discussed.

177. Silicon; Symbol, Si; Atomic Weight, 28.3; Valence, 4.—This element is, next to oxygen, the most abundant in nature; it makes up 28 per cent of the earth's crust. It is never found native, but always in compounds, either as an oxide or as a silicate. The oxide SiO_2 is usually called **silica**. Quartz is silica, and sands are composed largely of quartz. Practically every kind of rock contains silica to some extent.

Silicon, like carbon, is a tetrad; it has four bonds. Like carbon, it exists in three allotropic forms—two of them crystalline and one amorphous. Theoretically, at least, and with much support from practical results, various series of compounds of silicon, just as intricate, varied, and numerous as those of carbon, may be constructed.

Silicon combines very readily with oxygen, but only one oxide is known; this is commonly called silica, the chemical name being either *silicic oxide* or *silicon dioxide*, and has the structural formula $O = Si = O$. The element, which is a non-metal, may be isolated by electric reduction, by which process it is found as a brown powder or as fine, dark-gray scales that are similar to graphite; consequently, this form is called *graphitoid silicon*. The alloy, *ferrosilicon*, finds an important application in the metallurgy of iron.

Carborundum is the name given by its discoverer, Dr. Acheson, to *silicon carbide*, SiC, which is prepared in the electric furnace; it is almost as hard as the diamond, for which reason it has wide use as an abrasive, taking the place of emery and corundum.

Glass is a compound containing a wide range of substances, according to the purpose for which it is to be used; its chief constituent, however, is silica in the form of silicates of lime, lead, and soda. A great many important substances are silicates; thus, talc, asbestos, and mica are silicates of magnesia, and so is slate.

Clays, including china clay, are silicates of alumina, differing from one another by reason of the presence of various impurities or, owing to the presence of more or less water of combination, having different physical properties. If alkalis, like soda, potash, or lime, or metals like iron and lead, are present, the clay will not be refractory, that is, it will not withstand very high temperatures, because these substances form silicates that fuse at lower temperatures. If the amount of water of combination

is small or if it is driven out by heating, the clay cannot be used for modeling; it is said to lack plasticity. The *plasticity*, or stickyness, of clay makes clay of importance to the paper maker as a **filler**, *i.e.*, a substance which fills the pores between the fibers of the paper and produces a flat, smooth surface that takes ink well in printing; clay, talc, etc., when thus used, are also called *loading*.

Silicate of soda, otherwise called *water glass*, is used as a binder for cements and pastes that have to withstand high temperatures. It is also largely used in making paper boxes (cementing the layers of laminated boards), and to some extent to give stiffness to paper. Water glass mixed with litharge (yellow oxide of lead, PbO) is used as a cement in sulphite digester linings.

QUESTIONS

(1) If the wire of a paper machine be attacked by the paper stock, what is likely to be the cause?

(2) Why is antimony present in lead for sulphite mills?

(3) What compounds of lead are useful in the pulp or paper mill, and for what is each used?

(4) What is the percentage of aluminum sulphate in $Al_2(SO_4)_3 \cdot 18H_2O$?

Ans. 51.4%.

(5) Is boiled linseed oil a suitable lubricant? Explain your answer.

(6) What substance is used to make lead pencils? What is the principal element present?

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ELEMENTS OF CHEMISTRY

(PART 2)

EXAMINATION QUESTIONS

(1) What are the chief causes for the presence of flower sulphur in cooling systems of paper mills?

(2) How did the low melting point of sulphur bring about a great change in the market?

(3) How does the fact that sulphur combines readily with oxygen affect the sulphite pulp industry?

(4) How is it that sulphuric acid is present to some extent in burner gases?

(5) What is the difference between compounds ending in *ite* and those ending in *ate*?

(6) (*a*) What does the reaction expressed by the following equations refer to?

$$CaCO_3 + SO_2 + H_2O = CaSO_3 + H_2O + CO_2$$
$$CaSO_3 + H_2O + SO_2 = CaH_2(SO_3)_2$$

(*b*) How many kilograms of limestone containing 95% $CaCO_3$ will produce 10,000 liters (1 liter weighs 1 kilogram) of cooking acid containing 1% of combined SO_2 as $CaSO_3$? *Ans.* (*b*)87.7 Kg.

(7) What is the reaction when quicklime is slaked with water?

(8) (*a*) How is sulphate of lime obtained for "crown filler"? (*b*) How many pounds of the sodium compound and of the calcium compound are required to make 100 pounds of crown filler containing 10% moisture in addition to the water normally present as water of crystallization? See Art. **93**.

Ans. { Sodium compound, 74.3 lb.
Calcium compound, 58.1 lb.

(9) What is the objection to the use of dolomite in the tower system for making sulphite liquor?

(10) (*a*) What two members of the halogen group are of interest in the pulp and paper industry? (*b*) What is the valency of the members of the halogen group?

(11) (*a*) When chlorine or its compounds are used for bleaching, is chlorine the active (direct) agent? (*b*) What is there in common between bleaching by chlorine and bleaching by exposure to air?

(12) What substances likely to be present in paper stock do not become white when treated with a solution of bleaching powder?

(13) What is meant (*a*) by the term "available chlorine"? (*b*) by antichlors?

(14) How is it that the two atoms of univalent chlorine in the anhydride Cl_2O are apparently able to liberate two atoms of divalent oxygen in the reaction incidental to bleaching?

(15) (*a*) Into what two general classes may elements be divided? (*b*) Can this distinction be made in all cases?

(16) Why does copper appear on a steel knife blade when dipped into a solution of copper sulphate?

(17) What is the function of iron in writing ink?

(18) What is the meaning of the suffixes *ous* and *ic* in such words as ferrous and ferric?

(19) (*a*) What is the difference between cast iron and steel? (*b*) What is meant by annealing?

(20) Why do ochers tend to produce paper that is darker on the wire side?

(21) How can copper sulphate be used as a test for the presence of very small amounts of water?

(22) (*a*) What is "half-and-half" solder? (*b*) What principle is involved in the action of sprinkler heads?

(23) What is the difference between corrosive sublimate and calomel?

(24) (*a*) What is the difference between whiting and white lead? (*b*) Why is zinc white paint preferable to white lead paint in a sulphate pulp mill?

(25) (*a*) What is meant by a "drying oil"? (*b*) Why is linseed oil boiled?

(26) What are some of the by-products of making coke?

(27) Mention two properties of (*a*) carbon monoxide; (*b*) carbon dioxide.

ELEMENTS OF CHEMISTRY

(PART 3)

ORGANIC CHEMISTRY

ALIPHATIC COMPOUNDS

GRAPHIC FORMULAS

178. Empirical and Constitutional Formulas.—A formula like H_2SO_4 is called an **empirical formula,** because all that it shows is the number and kind of atoms in a molecule; it indicates neither how the atoms are arranged nor how the molecule splits up when taking part in a chemical reaction. If, however, the formula be written $H_2{\cdot}SO_4$ it is immediately inferred as soon as the formula is seen that, under proper conditions, the two hydrogens may be replaced by a bivalent element or radical. The formula may also be written $H{\cdot}H{\cdot}SO_4$, indicating that H_2 may be replaced by two univalent atoms or one atom of a dyad, and that metals of higher valence would require more than one molecule of sulphuric acid. (See Art. **75.**) Such formulas are called **constitutional formulas.** Thus, ferrous sulphide, FeS, contains 1 atom of iron and 1 atom of sulphur, both of which are dyads in this case; consequently, the reaction indicated by the following equation may be expected

$$FeS + H_2{\cdot}SO_4 = Fe{\cdot}SO_4 + H_2S$$

and experiment shows this to be the case.

179. Stereochemistry.—As previously stated, carbon has a valence of 4, that is, it has 4 bonds; hence, to satisfy it completely, each bond of a single carbon atom must be united to a univalent atom or to a univalent radical. In order that the resulting molecule may be stable, *i.e.*, balanced, it was assumed by Van't Hoff that the carbon atom occupied the position of the center of gravity of a regular tetraedron (a solid whose four sides are all

equilateral triangles, as shown in Fig. 10) and the other four atoms or radicals or combinations of atoms and radicals were situated at the four vertexes, as illustrated in Fig. 10. This explanation of the structure of a carbon-hydrogen molecule may now be regarded as a theory, not enough being known as yet to enunciate it as a law. Referring to the figure, a carbon atom is shown at the center of gravity of a regular tetrahedron, and if there were a hydrogen atom at each vertex, the whole would represent a molecule having the formula CH_4. All the full lines, which represent the bonds, are supposed to be of equal length. It is to be noted that the molecule itself is not a solid, but the atoms are so arranged that planes may be passed through them (assuming them to be points), which will form a solid. That branch of chemistry that deals with the arrangement of atoms (and radicals) in space is called **stereochemistry,** *stereo* being the Greek word for solid.

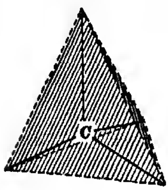


FIG. 10.

In the case of a radical, whether simple or compound, one (or more) of the bonds will be left dangling; that is, it will not have an atom or radical at both ends of the line. Thus, the radical called hydroxyl, HO, has the structural formula H—O—. Here one bond of the oxygen atom (which is bivalent, a dyad) is attached to a hydrogen atom, while the other is free, thus leaving the hydroxyl radical in position to unite with a monad atom or radical; the hydroxyl is therefore univalent.

180. Structural Formulas.—It is difficult to represent solids on a plane surface in a satisfactory manner; consequently, the molecule CH_4 is usually expressed by

$$\begin{array}{c} H \\ | \\ H\text{—}C\text{—}H \\ | \\ H \end{array}$$

Here each of the lines joining C to an H is a bond, there being four in this case, since carbon has 4 bonds. An expression of this kind is called a *structural formula.*

In any structural formula, the number of lines leading from any atom or radical indicates the valence of that atom or radical; thus, in the above structural formula, one line leads from each H, showing that H is univalent, and since 4 lines lead from C, C is tetravalent (quadrivalent). The structural formula

for hydrochloric acid is H—Cl, which shows that both H and Cl are univalent; the structural formula for water may be written

$$\mathrm{H{-}O{-}H} \quad \text{or} \quad \begin{array}{l}\mathrm{H}\diagdown\\ \quad\;\;\mathrm{O}\\ \mathrm{H}\diagup\end{array}$$

either formula showing that H is univalent; O is divalent, there being one line leading from each H and 2 lines from O.

The structural formula for sulphuric acid, the empirical formula for which is H_2SO_4, may be written

$$H_2 = SO_4 \qquad \text{or} \qquad \begin{array}{c}\mathrm{O}\diagdown\diagdown\quad\diagup\mathrm{O{-}H}\\ \mathrm{S}\\ \mathrm{O}\diagup\diagup\quad\diagdown\mathrm{O{-}H}\end{array}$$

The first form shows the relations between the two radicals, while the second form shows the relations between the atoms. The properties of the acid indicate that the hydrogen is less firmly held than the other elements, because the hydrogen is given off free when the acid attacks metals. The first of the above structural formulas indicates two univalent hydrogen atoms and a bivalent sulphate radical; the second shows sulphur to be sexivalent, oxygen bivalent, and hydrogen univalent.

The structural formula for carbonic acid, H_2CO_3 (which may be written $CO\cdot(OH)_2$) is

$$\begin{array}{l}\mathrm{H{-}O}\diagdown\\ \qquad\quad\mathrm{C} = \mathrm{O}\\ \mathrm{H{-}O}\diagup\end{array}$$

which shows that C, having 4 lines leading from it, is tetravalent, O is divalent, and H is univalent. Note that each of the three O's has two lines leading from it, each line indicating a bond.

Note that in constitutional formulas, the various radicals are separated by dots (in some cases by parentheses), while in structural formulas they are separated by lines. Enough concerning these two classes of formulas has now been given to enable the reader to understand everything that follows.

HOMOLOGOUS SERIES OF HYDROCARBONS

181. Hydrocarbons.—As was previously stated, organic chemistry deals with the carbon compounds, an enormous number of which are known, and the number theoretically possible is almost infinite. There is an exceedingly large number of compounds containing only carbon and hydrogen, for which reason, they are called **hydrocarbons,** the simplest molecule of which is CH_4, the

structural formula for which was given in Art. **180.** The hydrogen atoms of this molecule may be replaced, either single atoms or two or more, by other atoms of different elements or by radicals, thus yielding new compounds; also, like molecules may combine among themselves to form new compounds, one of the H's being replaced with the radical CH_3, as shown herewith:

$$
\begin{array}{c}
\mathrm{H} \\
| \\
\mathrm{H{-}C{-}H} \\
| \\
\mathrm{H} \\
CH_4
\end{array}
\qquad
\begin{array}{c}
\mathrm{H\ \ H} \\
|\ \ | \\
\mathrm{H{-}C{-}C{-}H} \\
|\ \ | \\
\mathrm{H\ \ H} \\
C_2H_6
\end{array}
\qquad
\begin{array}{c}
\mathrm{H\ \ H\ \ H} \\
|\ \ |\ \ | \\
\mathrm{H{-}C{-}C{-}C{-}H}, \\
|\ \ |\ \ | \\
\mathrm{H\ \ H\ \ H} \\
C_3H_8
\end{array}
\quad \text{etc.}
$$

Here the H on the right end has been replaced in each case by the combination $\begin{array}{c}\mathrm{H}\\|\\ \mathrm{{-}C{-}H}\\|\\ \mathrm{H}\end{array}$. Note that each carbon atom has all four of its bonds engaged, for which reason the molecule is said to be **saturated.** Note also the chain-like arrangement and the fact that it may apparently be extended indefinitely. It is because of this feature that these compounds are called **open-chain compounds.** All compounds that may be represented by an open chain, whether their molecules are saturated or not, are called **aliphatic compounds,** the word *aliphatic* having reference to the open-chain arrangement. There is another class of hydrocarbons whose compounds may be represented by a closed chain; these are called *closed-chain,* or *aromatic, compounds.* For the present, only the aliphatic compounds will be considered.

182. It will be observed that when the three compounds CH_4, C_2H_6, and C_3H_8 are written in consecutive order as here printed, the difference between two consecutive compounds, subtracting atoms from atoms, is CH_2, and if the chain were extended, this common difference would always be obtained; thus the next link in the chain would make the empirical formula C_4H_{10}, and the next C_5H_{12}, etc., as may be seen by adding to the above chains and counting the atoms. The compounds thus form a regular series, each term of which differs from the preceding by a constant quantity, in this case by CH_2. Any series in which the terms are

related to one another in this manner is called a **homologous series.**

The second term of the above series, C_2H_6, may be expressed by the constitutional formula $CH_3 \cdot CH_3$, the third term by $CH_3 \cdot CH_2 \cdot CH_3$, the fourth term by $CH_3 \cdot CH_2 \cdot CH_2 \cdot CH_3$, etc., all the terms except the end terms being CH_2.

183. If the number of atoms of carbon in any term of the above homologous series be denoted by n, the number of atoms of hydrogen in the same term will be $2n + 2$; for instance, in the third term, C_3H_8, $n = 3$ and $H = 2 \times 3 + 2 = 8$; in the fifth term, C_5H_{12}, $n = 5$ and $2 \times 5 + 2 = 12$; in the first term, CH_4, $n = 1$ and $2 \times 1 + 2 = 4$; etc. Hence, any term in this series may be expressed by $C_n + H_{2n+2}$. There are many such series of hydrocarbons, but only two will be considered here. Some of the other series are: C_nH_{2n}, the term containing the lowest known value of n being C_2H_4; C_nH_{2n-2}, the term containing the lowest known value of n being C_2H_2; C_nH_{2n-4}, the term containing the lowest known value of n being C_5H_6; C_nH_{2n-6}, the term containing the lowest known value of n being C_6H_6; etc. In all these series, the difference between any two consecutive terms is CH_2.

THE METHANE, OR PARAFFIN, SERIES

184. Methane.—The compound denoted by the molecular formula CH_4 is a gas, the chemical name of which is **methane**; it also has several other names, as *marsh gas*, *fire damp*, and *light carburetted hydrogen.* It is called **marsh gas** because it is found at the bottom of stagnant pools, bogs, marshes, etc., due to the slow decay of vegetable matter; it is also found in considerable quantities in coal mines, where it forms a dangerously explosive mixture with the oxygen of the air, which is readily ignited by a flame of any kind, thus causing the loss of many lives. For this reason it is called **fire damp,** the word *damp* in this case meaning gas; it is the most dreaded of the dangers that the miner has to face. It is called **light carburetted hydrogen,** because it is the lightest of all the hydrocarbons, being only 8 times as heavy as hydrogen. Thus, the molecular weight of CH_4 is $12 + 4 = 16$; the weight of a molecule of hydrogen, which contains two atoms, is 2; and $16 \div 2 = 8$. C_2H_4 is also a gas; it is sometimes called *heavy carburetted hydrogen,* because it is

much heavier than CH_4, a molecule being 14 times as heavy as a molecule of hydrogen.

185. It was stated in Art. **181** that the methane molecule was saturated, all the four bonds of the carbon atom being engaged. Assuming the atoms to be arranged as in Fig. 10, it is to be expected that methane would be a very stable compound, and such is the case; it can be made to enter into combinations only with difficulty. All the compounds in the series C_nH_{2n+2} are saturated, as will be evident after consulting the structural formulas of Art. **181,** and they are also very stable. For this reason, this series is called the **paraffin series,** and the different compounds included in it are called the **paraffins.** The word paraffin comes from two Latin words that mean *little affinity,* implying thereby that these compounds do not have much affinity for other compounds.

The names of the first few compounds of the paraffin series and their empirical formulas are shown in the subjoined table.

Methane...	CH_4
Ethane.....	C_2H_6
Propane....	C_3H_8
Butane.....	C_4H_{10}
Amane.....	C_5H_{12}
Hexane....	C_6H_{14}

The name of every compound in this series ends in *ane.* *Amane* is also called *pentane,* and this and all subsequent compounds of the series are formed by adding *ane* to the Greek form of the word denoting the number of atoms of carbon, as pentane, hexane heptane, octane, etc. The mineral lubricating oils are chiefly hydrocarbons of the paraffin series, which is also frequently called the *methane series.*

186. In connection with the foregoing compounds of the paraffin series, the first three, methane, ethane, and propane, are gases at ordinary temperatures; beginning with butane and extending to $C_{15}H_{32}$, they are liquids; from $C_{16}H_{34}$ to $C_{60}H_{122}$, the highest compound known of this series, the compounds are solids. In general, the lower in the series the compound is (the smaller the number of atoms in, and the less the molecular weight of, the molecule) the lower the boiling point; also, the higher the boiling point, the thicker and more viscous is the liquid. The first solids are waxes, which gradually become denser and harder as the molecular weight increases. They are called **paraffin waxes,** and are graded according to melting point. Commercial waxes contain several members of the paraffin series. Their principal use in the paper industry is for waxing and sizing paper to render it water resistant.

187. Radicals.—It was previously stated that the hydrogen atoms of methane and other paraffins may be replaced by other elements or radicals, though with difficulty; this is particularly true of the halogens, chlorine, bromine, and iodine. Indeed the halogens are usually the best means of breaking into the paraffin molecule. For example, all the following compounds, which may be considered as formed by replacing the hydrogen of methane by halogens, are known:

CH_4	CH_3Cl	CH_2Cl_2	$CHCl_3$	CCl_4
CH_4	CH_3Br	CH_2Br_2	$CHBr_3$	CBr_4
CH_4	CH_3I	CH_2I_2	CHI_3	CI_4

To show that these results are possible, theoretically at least, the molecular formula for methane might be written CH_3H, in which case, the radical CH_3 is called **methyl.** Some of the important radicals are given here; others will be mentioned later. Nearly all organic acids will be found to contain the characteristic radical COOH, the structural formula of which may be expressed as

$$\begin{array}{l} \quad O \\ \quad \| \\ -C-O-H, \end{array}$$

which shows that there is one free carbon bond for uniting with another radical.

Methyl	CH_3	—Characteristic of paraffins
Hydroxyl	OH	—Characteristic of alcohols
Carboxyl	COOH	—Characteristic of acids
Amidogen	NH_2	—Characteristic of amines
Nitroxyl	NO_2	—Characteristic of explosives

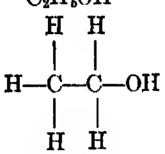
The **amines** are organic bases, which are derived from ammonia, NH_3, by replacing an H with a radical, such as CH_3. Thus, methylamine may be written NH_2CH_3 (omitting the dot), in which case, the structural formula would be built up from the ammonia radical NH_3 by substituting the methyl radical (which is, of course, univalent) for one atom of hydrogen. It is likewise possible to make two and even three such substitutions, and thus obtain $NH(CH_3)_2$ and $N(CH_3)_3$, as indicated by the following structural formulas:

$$\begin{array}{llll} H & CH_3 & CH_3 & CH_3 \\ | & | & | & | \\ N-H & N-H & N-CH_3 & N-CH_3 \\ | & | & | & | \\ H & H & H & CH_3 \\ \text{Ammonia} & \text{Methylamine} & \text{Dimethylamine} & \text{Trimethylamine} \end{array}$$

188. Radicals are formed from the other paraffins in the same way that methyl is formed from methane, *i.e.*, by taking out one atom of hydrogen; and their names are formed by changing the *ane* in the name of the compound to *yl*. For example, *methyl*, CH_3, from methane, CH_4; *ethyl*, C_2H_5, from ethane, C_2H_6; in a similar manner are formed *propyl*, C_3H_7, *butyl*, C_4H_9, and *amyl*, C_5H_{11}. Ethane may therefore be written as C_2H_5H, propane as C_3H_7H, etc. If one H in NH_3 be replaced by ethyl, the compound is called *ethylamine*, though strictly speaking, it should be *monoethylamine*, *mono* meaning one, and the constitutional formula is $NH_2(C_2H_5)$; if two H's are replaced by ethyl radicals, the compound is called *diethylamine*, *di* meaning two, and the constitutional formula is $NH(C_2H_5)_2$; if three H's are replaced by ethyl radicals, the compound is called *triethylamine*, *tri* meaning three, and the constitutional formula is $N(C_2H_5)_3$. The nomenclature of other compounds formed by substituting paraffin radicals, called **alkyl** radicals, for the H's in NH_3 and the constitutional formulas for the resulting compounds follow a similar rule. Evidently, the formula for an alkyl radical is C_nH_{2n+1}, there being one less H than in the paraffin molecule, whose formula is C_nH_{2n+2}.

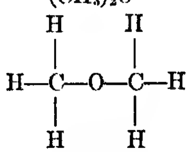
189. Isomers.—Many compounds in organic chemistry have the same number and kinds of atoms and the same molecular weights, but, nevertheless, have very different properties; for instance, the empirical formula C_2H_6O represents two very different substances: *ethyl alcohol* (ordinary grain alcohol) and dimethyl oxide, commonly called *methyl ether*. The difference is explained by their structural formulas, which show a different arrangement of their molecules, and by their constitutional formulas; thus,

C_2H_5OH



Ethyl alcohol

$(CH_3)_2O$



Methyl ether

The structural formula for methyl ether might have been written in a more condensed form as $\begin{matrix}CH_3\\ CH_3\end{matrix}\!\!>\!O$, showing at a glance that

two univalent methyl radicals are united to a bivalent oxygen atom. Compounds of this kind are called **isomers** or **isomerides**, the word isomer being derived from two Greek words *isos* and *meros*, meaning equal and parts, respectively; hence, the word isomer means *equal parts*. Such compounds are said to be **isomeric**, and when two or more substances with different properties are found to have the same elements and the same number of atoms of each element, merely differing in the arrangement or space relations of the elements or radicals, they are said to be examples of **isomerism**; they are **isomeric**.

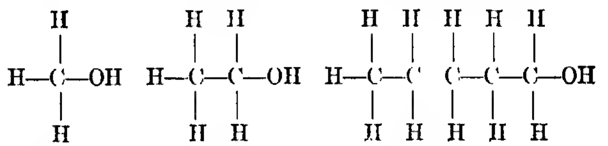
190. Polymers.—There are many compounds in organic chemistry which have the same percentage composition but different molecular weights. A striking example is acetylene, C_2H_2, and benzene, C_6H_6. Here the empirical formula, CH, would answer for both, in so far as it shows the relative weights of the carbon and hydrogen. The two substances, however, are widely different, as is well known, acetylene being a gas and benzene a liquid. Compounds of this kind are called **polymers.** It may be noted that acetylene belongs to the homologous series C_nH_{2n-2}, while benzene belongs to the homologous series C_nH_{2n-6}, both compounds being the lowest terms of their respective series. The word *polymer* comes from two Greek words, *polys* (many) and *meros* (parts), meaning *many parts*, as distinguished from isomer, which means *equal parts*. An isomer has the same percentage composition and an equal number of atoms in the molecule, while a polymer has the same percentage composition but a different number of atoms in the molecule; consequently, the molecular weights of isomers are equal, but the molecular weights of polymers are not equal. Typical polymers, familiar to the papermaker, are starch, $(C_6H_{10}O_5)_n$, and cellulose, $(C_6H_{10}O_5)_x$; they have the same percentage composition but different molecular weights.

ALCOHOLS, ACIDS, AND ALDEHYDES

191. Alcohols.—If one or more hydrogen atoms of a member of the paraffin series be replaced with the same number of hydroxyl radicals, the resulting compound is called an **alcohol**; it is really a *hydrate* of the paraffin radical. For example, if one atom of hydrogen in methane be replaced with hydroxyl, the constitutional formula for the compound so obtained is CH_3OH,

which is properly called *methyl hydrate* or *methyl hydroxide,* the common names for which are **methyl alcohol** and **wood alcohol,** now legally termed "methyl hydrate" in Canada and "methanol" in the United States. The words hydrate and hydroxide have practically the same meaning, but many writers prefer to restrict the use of hydroxide to apply only to alkalis, as KOH, NaOH, NH_4OH, etc., using the word hydrate when the hydroxyl radical occurs in compounds that are not classified as alkalis.

In the same manner are formed the formulas for **ethyl alcohol** (variously known as **grain alcohol, proof spirits,** etc.), **amyl alcohol** (the chief constituent of fusel oil), etc. Thus, from ethane, C_2H_6, is derived C_2H_5OH (ethyl alcohol); from amane, C_5H_{12}, is derived $C_5H_{11}OH$ (amyl alcohol); etc. These might be called *ethyl hydrate, amyl hydrate,* etc. Any alcohol formed in this manner, by substituting one hydroxyl for one hydrogen atom, is called a *monatomic alcohol, mon* meaning *one.* It is at once evident that the monatomic alcohols of the paraffin group have the general formula $C_nH_{2n+1}OH$. The structural formulas for the three alcohols mentioned are shown herewith. It will be



Methyl alcohol Ethyl alcohol Amyl alcohol

perceived that the alcohols of this group also form a homologous series.

192. Methyl Alcohol.—Methyl alcohol has a specific gravity of .812, its boiling point is 66°C., and its constitutional formula is CH_3OH; it is obtained as one of the products of the destructive distillation of wood, which is the reason for calling it "wood" alcohol. It will be recalled that it was previously stated that methane was found in swamps, bogs, etc., and resulted from the decay of vegetable matter. Large quantities of wood were formerly burned in kilns for the charcoal; all the other products that were formed were wasted, except small amounts of tarry matter that oozed out of the kiln, and a liquid known as *pyroligneous acid,* a crude form of acetic acid, which was obtained in the same way. While some of the wood was burned in the process, the greater part was distilled, *i.e.,* the gases and liquids were

driven off and the charcoal (solid part) was left behind. The industry of wood distillation is now carried on scientifically, and products such as methyl alcohol, acetic acid, acetate of lime, acetone, disinfectants, shingle stains, tar, and charcoal are recovered in large quantities. The best results are obtained from the hard woods, as birch, maple, etc.

193. Methyl alcohol is a good solvent for gums, such as rosin, shellac, and others used in varnishes. It is one of the agents employed for denaturing ethyl alcohol, *i.e.*, making it unfit for drinking. Another name for denatured alcohol is *methylated spirits;* it contains from 5 to 10 per cent of methyl alcohol, and should not therefore be called wood alcohol, as is sometimes done through carelessness or ignorance. Methyl alcohol is a very deadly poison, and should on no account be used in foods or beverages, in surgical or first-aid work, or for external applications; even the fumes from it are very deleterious, and they have been known to cause blindness.

194. Ethyl Alcohol.—Ethyl alcohol has a specific gravity of .806, its boiling point is 78°C., and its constitutional formula is C_2H_5OH. It is frequently called **grain alcohol**, to distinguish it from wood alcohol, because it is produced when wheat, rye, corn, barley, or rice *ferment,* which is one of the stages in their decay. It is to be noted that only certain parts of the cereal ferment, those containing sugar and starches. The boiling point of methyl alcohol is 66°C., while that of ethyl alcohol is 78°C.; it is this fact that makes wood alcohol a good denaturing agent, because the boiling points of the two are so close together that these alcohols cannot be separated by fractional distillation. When the word "alcohol" is used without qualification, it is understood that ethyl alcohol is referred to. Under proper conditions, alcohol makes an excellent fuel for internal combustion engines; it is a very useful solvent for many gums, varnishes, and resins for use in the arts, and it is a very satisfactory preservative for organic tissue.

195. What is called **proof spirit** is alcohol with only so much water mixed with it that gunpowder moistened with it will just fail of ignition when a flame is applied to it; the addition of a very little more alcohol will enable the mixture to ignite the powder on application of a lighted match. The "strength" of proof spirit is about 50 per cent alcohol, or 100 proof, the strength of

chemically pure, absolute alcohol being 100 per cent alcohol, or 200 proof. A mixture containing 90 per cent alcohol and 10 per cent water is 180 proof. If the percentage of alcohol is known, the proof is found by doubling the per cent; if the proof is known, the per cent of alcohol may be found by dividing the proof by 2; the results thus obtained will be approximately correct.

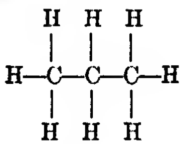
Alcohol behaves very curiously with water, with which it mixes in all proportions. If, say, 50 c.c. of each are mixed, the volume of the mixture will not be 100 c.c., but only 98 c.c., as was mentioned in Physics. Consequently, there is a difference in the value of percentage by weight and percentage by volume, and it is customary to state which is meant when reporting an analysis.

196. Alcohol is prepared by distilling it from its fermenting source. The strongest alcohol used in commerce is known as S. V. R. (*spiritus vini rectificatus*), so called because the alcohol, thought by its early discoverers to be the soul or spirit of wine, was strengthened or "rectified" by distillation and redistillation. When fruit juice ferments, it becomes a wine; when wine is distilled, brandy is produced; and when brandy is distilled (with the addition of lime), alcohol, otherwise called rectified spirits or high wine, is produced. A large proportion of all the alcohol now made is produced from wheat, barley, corn, and rye; also from molasses obtained in the manufacture of sugar. In fruits, it is their sugar that is fermented; but in cereals, their starch is first changed into sugar and then fermented.

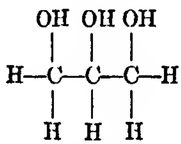
It will be shown later that woody materials are closely related to starch; because of this, it has been found possible to treat the waste liquor from sulphite pulp mills in such a way as to produce alcohol. Note that the alcohol produced by fermenting the sugars obtained from wood waste or sulphite waste liquors is ethyl alcohol, while that produced from wood merely by distillation on heating the wood is wood alcohol—methyl alcohol.

197. Amyl Alcohol.—Amyl alcohol exhibits strongly the property of isomerism, there being eight different substances having the empirical formula $C_5H_{12}O$. What is ordinarily known as amyl alcohol is usually expressed by the formula $C_5H_{11}OH$, and is the chief constituent of fusel oil. Amyl alcohol is largely used in artificial flavoring extracts, also as a vehicle for metallic paints, when combined with acetic acid as amyl acetate; painters call amyl acetate banana oil because of its odor.

198. Glycerine.—The alcohols so far mentioned have all been monatomic (also called *monohydric*), one hydroxyl replacing one hydrogen atom. There are other alcohols, however, in which two, three, etc., hydroxyls replace a corresponding number of hydrogen atoms. One of these, derived from propane, has the structural formula



Propane



Glycerine

and is usually expressed by the constitutional formula $C_3H_5(OH)_3$. This compound is commonly called **glycerine**, and the radical C_3H_5 is called **glyceryl.** The glyceryl radical is trivalent, since the three carbon atoms have $3 \times 4 = 12$ bonds and 5 of them are joined direct to hydrogen atoms and 4 more hold the carbons together; hence, the structural formula for glycerine may be written $C_3H_5\left\langle\begin{matrix}OH\\ -OH\\ OH\end{matrix}\right.$. As shown by its constitutional formula, glycerine is an alcohol; and, as may be inferred, one, two, or all three of the hydroxyls may be replaced with other radicals, in which case, the resulting compounds may be called **glycerides.**

When the hydroxyl radicals of glycerine are replaced with NO_3 radicals, *trinitroglycerine*, which is commonly called **nitroglycerine,** $C_3H_5(NO_3)_3$, is formed. It is a yellow, oily liquid and is one of the most powerful explosives known. Other explosives, as dynamite, are formed by mixing nitroglycerine with other substances—sawdust, or other absorbents.

199. Organic Bases.—It was previously stated that the hydroxyl radical was a characteristic of organic bases; it acts in the same way as in the inorganic bases, the alkalis, for instance. Thus, sodium nitrate is made by treating sodium hydroxide with nitric acid, according to the reaction, $NaOH + HNO_3 = NaNO_3 + H_2O$, the products being sodium nitrate and water. In a similar manner, ethyl nitrate may be prepared by treating ethyl alcohol with nitric acid, in accordance with the reaction

$$C_2H_5OH + HNO_3 = C_2H_5NO_3 + H_2O$$

the products being ethyl nitrate and water. Therefore, as before stated, the alcohols may be regarded as hydrates, ethyl alcohol being, properly speaking in chemical terms, ethyl hydrate. Organic salts, called *esters* (see Art. **227**), are produced when alcohols combine with acids that contain the radical COOH, called carboxyl. Amyl acetate, referred to above, which is formed when amyl alcohol combines with acetic acid, is an ester.

200. The halogens, chlorine, bromine, and iodine, readily enter into the paraffin radical, under proper conditions. Chlorine will displace successive hydrogen atoms from methane, all four even, forming in the last case *carbon tetrachloride*, CCl_4, as mentioned in Art. **187,** which is a volatile liquid that is very useful as a solvent and fire extinguisher. When three hydrogen atoms are replaced by chlorine, *trichlormethane*, or **chloroform,** $CHCl_3$, is produced. If iodine is used instead of chlorine to replace the hydrogen, **iodoform,** CHI_3, is produced. Both of these compounds are used in surgery: chloroform as an anesthetic (a name for substances that produce unconsciousness), and iodoform as an antiseptic (a name for substances that fight disease germs).

201. The replacement of hydrogen with halogens in hydrocarbons is a very important reaction in *synthetic chemistry,* which deals with the building up of compounds from atoms and molecules. The study of these processes has made it possible to handle the complex mixtures in the waste liquors of sulphite mills, sawdust, etc., so that alcohol may be produced; under proper conditions, cellulose may be made into sugar—the sugar cane does something like that.

202. Aldehydes.—When an alcohol is oxidized by replacing 2 atoms of hydrogen with 1 atom of oxygen, the resulting compound is called an **aldehyde.** The two hydrogen atoms thus withdrawn unite with an oxygen atom to form water. The reaction in the case of methyl alcohol is expressed by the equation

$$CH_3OH + O = CH_2O + H_2O$$

The compound CH_2O (which may also be written $H{\cdot}COH$ or $H{\cdot}CHO$) is called **formic aldehyde,** commonly known as **formaldehyde.** The word *aldehyde* is made up from the words *alcohol dehydrogenatum* (which mean to dehydrogenize alcohol) by using the first two letters of the first word and the first five letters of the second word; thus, al + dehyd = aldehyd or aldehyde. Formaldehyde is also known as **formalin,** and may

be produced by burning alcohol without a sufficient supply of air. It is a very powerful disinfectant, being much used to disinfect houses that have contained cases of infectious or contagious diseases. It does not bleach like sulphur dioxide, SO_2, or injure fabrics or colors, and its vapor penetrates every crevice. When inhaled, it is not, for a moment, noticeably disagreeable, but it very soon attacks the membrane of the throat, causing a dry soreness. The odor is similar to that noticed in the neighborhood of a cage of white mice.

From ethyl alcohol C_2H_5OH is derived the aldehyde C_2H_4O or $CH_3{\cdot}CHO$, which is called **acetaldehyde.** Each term of the paraffin series has its alcohol, and each alcohol has its aldehyde. The general formula for an aldehyde is $C_nH_{2n}O$, which differs from the general formula for a paraffin by having an oxygen atom in place of two hydrogen atoms; thus, $C_nH_{2n+2}-2H+O=C_nH_{2n}O$. The alcohols, therefore, form a homologous series, and the aldehydes also form a similar series. The radical COH or CHO is characteristic of all aldehydes.

203. The Fatty Acids.—If an aldehyde be oxidized by the addition of one atom of oxygen, the result is an acid that bears practically the same name as the aldehyde from which it was formed. Thus, from formic aldehyde is obtained *formic acid,* as indicated by the reaction

$$CH_2O + O = CH_2O_2 = H{\cdot}COOH$$

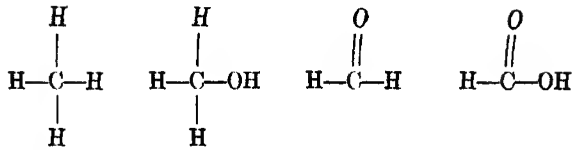
Likewise, from acetaldehyde is obtained *acetic acid,* as indicated by the reaction

$$C_2H_4O + O = C_2H_4O_2 = CH_3{\cdot}COOH$$

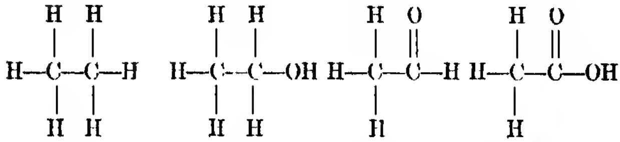
The acids, therefore, form another homologous series. Now considering the four series derived from methane—(1) the paraffins, (2) the alcohols, (3) the aldehydes, (4) the acids—it will be observed that each term differs from the next succeeding term by CH_2. For instance, the lowest acid is formic, CH_2O_2, the next higher acid is acetic, $C_2H_4O_2$, and $C_2H_4O_2 - CH_2O_2 = CH_2$.

The relations of the four series are beautifully shown by the structural formulas for the first two terms (see next page). It will be noted that the alcohols and acids contain the hydroxyl radical, but the aldehydes do not.

204. If an electric drying oven is available, one that has the wire of the heating coils exposed, such as is used for drying pulp samples, an aldehyde can be made by putting a little alcohol in a flat (shallow) dish and placing it in the oven; the aldehyde is



Methane Methyl alcohol Formaldehyde Formic acid



Ethane Ethyl alcohol Acetaldehyde Acetic acid

formed by passing alcohol over heated metals. Wood alcohol so treated will give formic aldehyde (otherwise called formaldehyde and formalin). Formaldehyde was stated to be a disinfectant, but do not add it to glue or gelatine size; the mixture will certainly "keep," because it will be impossible to use the insoluble, curdy stuff that is produced by the action of the formaldehyde.

The general formula for an aldehyde is R·COH (frequently written R·CHO), in which R represents one atom of hydrogen or one alkyl radical (see Art. **188**). Thus, formaldehyde is H·CHO and acetaldehyde is CH_3·CHO. The CHO radical characterizes the aldehydes. The lower members of the homologous series of aldehydes are gases; but, like the paraffins, the higher members are liquids and solids. When the word aldehyde is used without any qualification, acetaldehyde is always meant, just as alcohol so used always means ethyl alcohol. Some talking machine records, mouth pieces for pipes, billiard balls, etc., are made of a material called *bakelite*, produced by special treatment during reactions between formaldehyde and carbolic acid (also called *phenol*).

205. The aldehydes exhibit the phenomenon of polymerization (see Art. **190**). One of the polymers of formaldehyde is *paraformaldehyde*, $C_3H_6O_3$, which may be written $(H{\cdot}CHO)_3$; it is a white powder, and can be compressed into tablets, for use as a preservative.

206. Acetic acid, CH_3COOH, formed by oxidation of acetaldehyde, is produced in wine, cider, or other fermented fruit juices by an organism called *mycoderma aceti*, and the product is vine-

gar. The organism is a sort of a plant, like yeast, and is popularly called *mother of vinegar.*

The empirical formula for an acid of the series derived from the paraffins is $C_nH_{2n}O_2$, while the constitutional formula may be written $C_{n-1}H_{2n-1}COOH$; these acids are called **fatty acids,** because the animal and vegetable fats and oils are largely composed of these acids. The lower members of the series are liquids, but the higher ones, beginning with *capric acid,* $C_{10}H_{20}O_2 = C_9H_{19}COOH$, are solids at ordinary temperatures. The liquids are soluble in water, but after propionic acid, the solubility decreases rapidly; the solids are insoluble in water, but are soluble in alcohol and in ether. The names of some of these acids are given in the following table:

Formic acid,	CH_2O_2,	or	$HCOOH$.	Boils at 100°C.
Acetic acid,	$C_2H_4O_2$,	or	CH_3COOH.	Boils at 118°C.
Propionic acid,	$C_3H_6O_2$,	or	C_2H_5COOH.	Boils at 140°C.
Butyric acid,	$C_4H_8O_2$,	or	C_3H_7COOH.	Boils at 163°C.
Valeric acid,	$C_5H_{10}O_2$,	or	C_4H_9COOH.	Boils at 185°C.
Palmitic acid,	$C_{16}H_{32}O_2$,	or	$C_{15}H_{31}COOH$.	Melts at 62°C.
Stearic acid,	$C_{18}H_{36}O_2$,	or	$C_{17}H_{35}COOH$.	Melts at 69°C.

Palmitic acid is obtained from cocoa butter, from animal fats, and from palm oil; **stearic acid** is obtained from tallow. There does not appear to be much in common between vinegar and tallow, yet acetic acid (the acid of vinegar) and stearic acid (the acid of tallow) both have the same general formula, $C_{n-1}H_{2n-1}COOH$, n being the number of the term in the series; it also represents the total number of carbon atoms in the acid, that is, it equals $C_{n-1} + C = C_n$, the C being taken from the COOH.

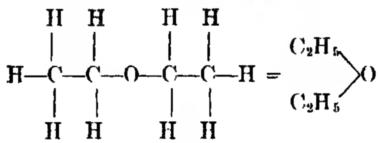
Another acid, which is quite closely related to the last two acids mentioned above, but which belongs to a different series, is **oleic acid,** which has the formula $C_{18}H_{34}O_2 = C_{17}H_{33}COOH$; it is found in olive oil and in many animal and vegetable oils.

ETHERS AND KETONES

207. Ethers.—Ethers are derived from alcohols by substituting a compound radical for the hydrogen in the hydroxyl radical of the alcohol. For example, methyl alcohol is CH_3OH; substituting CH_3 for H in OH, the result is CH_3OCH_3, which is

called **methyl ether**; also, making a similar substitution in ethyl alcohol, the result is $C_2H_5OC_2H_5$, which is called **ethyl ether.** Representing the compound radical by R, the general formula for an ether when both radicals are alike is ROR or R_2O, which corresponds to the formula for water, H_2O = HOH, when H is written in place of R. The two ethers mentioned might be written as $(CH_3)_2O$ and $(C_2H_5)_2O$, to correspond with the R_2O form. Similarly, the formula for **amyl ether** is $(C_5H_{11})_2O$, derived from amyl alcohol, $C_5H_{11}OH$.

The structural formulas for these ethers are interesting. The complete structural formula for methyl ether was given in Art. **189**, and a condensed formula was also given. The complete structural formula for ethyl ether and the condensed formula are given herewith; they show how the single oxygen atom holds the



Ethyl ether

two alkyl radicals. Since water may be termed hydrogen oxide, the ethers may be termed oxides also, and the three so far mentioned may be called *methyl oxide, ethyl oxide,* and *amyl oxide.* The constitutional formula for ethyl ether may be written either $C_2H_5\cdot O\cdot C_2H_5$ or, in a more extended form, $CH_3\cdot CH_2\cdot O\cdot CH_2\cdot CH_3$; and the constitutional formula for amyl ether may be written $C_5H_{11}\cdot O\cdot C_5H_{11}$ or it may also be written in a somewhat more extended form as $CH_3\cdot(CH_2)_4\cdot O\cdot (CH_2)_4\cdot CH_3$.

208. Ethyl ether may be prepared by cautiously mixing alcohol with strong sulphuric acid, forming the compound known as *ethylsulphuric acid,* according to the equation

$$H_2SO_4 + C_2H_5\cdot OH = HC_2H_5SO_4 + H_2O$$

Heating with an additional amount of alcohol, the ethylsulphuric acid, $HC_2H_5SO_4$, undergoes a further change, and the reaction produces ethyl ether and sulphuric acid, in accordance with the equation

$$HC_2H_5SO_4 + C_2H_5OH = (C_2H_5)_2O + H_2SO_4$$

From the fact that sulphuric acid is generally used in the preparation of ethyl ether (other acids may be used), it is frequently called **sulphuric ether.** When the word ether is used

without any qualification, ethyl (sulphuric) ether is always meant, in the same way that alcohol always means ethyl alcohol.

209. Ether is very volatile and inflammable; its boiling point is 35°C., and its specific gravity is .736 at 0° C. At temperatures below 35°C., ether is a colorless liquid of peculiar odor; it is a good solvent for oils and fats, and is used to extract these from mixtures, as in the case of rosin size. While free rosin is soluble in ether, rosin soap is not, but is soluble in water. So the white size, containing free rosin in suspension, may be shaken up with ether in a special container; the ether does not mix with the water, and it dissolves the rosin. After shaking, the size and ether are left for a short time, and the ether rises to the top (being lighter than water), carrying the dissolved rosin with it. By using a separatory funnel, the water can be run off from below the ether-rosin mixture, which is poured into a dish. The ether evaporates and leaves the rosin behind, which can then be weighed. This principle is very widely applied in analysis.

The use of ether in surgery as an anesthetic is well known.

210. Compound Ethers.—The ethers so far described may be represented by the general formula ROR or R_2O. The two radicals need not be alike, in which case, the general formula may be represented by ROR′; thus, if one of the radicals is methyl and the other ethyl, the corresponding ether is called *methylethyl ether*, and its formula is $CH_3OC_2H_5$, which may also be written $\begin{matrix} CH_3 \\ C_2H_5 \end{matrix}\!\!>\!O$. Ethers of this kind are known as **compound ethers.** Another compound ether is *ethylamyl ether*, $C_2H_5OC_5H_{11}$ or $\begin{matrix} C_2H_5 \\ C_5H_{11} \end{matrix}\!\!>\!O$. As may be supposed, there are a large number of compound ethers.

211. Ketones and Acetone.—The salt formed by the reaction of acetic acid with a base is called an **acetate.** Whenever an acetate is subjected to dry distillation, a volatile liquid called **acetone** is formed. The empirical formula for acetone is C_3H_6O, and the condensed structural formula is $\begin{matrix} CH_3 \\ CH_3 \end{matrix}\!\!>\!CO$. The latter formula shows that the difference between acetone and methyl

ether lies in the substitution of the bivalent radical CO for the oxygen atom. There are a whole series of these compounds, corresponding to the ethers, and the general formula is R·CO·R or R·CO·R′; they are called **ketones**, and acetone is the first of the series. Acetone is also called *dimethyl ketone*, *di* signifying that there are two methyl radicals. Like the ethers, the radicals may be different, giving compound ketones, as *ethylmethyl ketone*, $C_2H_5COCH_3 = \begin{matrix} C_2H_5 \\ CH_3 \end{matrix} \Big> CO.$

In connection with the distillation of wood, Art. **192**, acetate of lime was mentioned; this compound has the formula $(CH_3COO)_2Ca$, the calcium replacing the hydrogen of the acid. When acetate of lime (calcium acetate) is heated, it breaks up and forms acetone and carbonate of lime (calcium carbonate) in accordance with the equation,

$$(CH_3COO)_2Ca = (CH_3)_2CO + CaCO_3$$

Acetone is a good solvent, and very large quantities are used in making **cordite,** which is a mixture of nitroglycerine, guncotton (also called *nitrocellulose*), and vaseline, and which is a very powerful explosive. The acetone used to dissolve and blend these ingredients of cordite is afterward evaporated and recovered. It may be mentioned that celluloid is readily dissolved by acetone.

QUESTIONS

(1) What do you understand by stereochemistry?

(2) What alcohol is obtained by (*a*) fermenting waste liquor from a sulphite mill? (*b*) By distilling wood?

(3) What is the general formula of the monatomic alcohols of the aliphatic group?

(4) What element is always present in an organic base?

(5) What is chloroform?

(6) Define (*a*) a compound ether; (*b*) an aldehyde.

MINERAL OILS

212. Crude Petroleum.—The so-called **mineral oils** are chiefly hydrocarbons of the paraffin series, and have the general formula C_nH_{2n+2}. What are called the *asphaltic oils* belong in some measure to another group. The mixture of mineral oils

known as **crude petroleum** is obtained from wells, but was first found as springs in various countries. Crude petroleum is a thick, dark-greenish fluid that is a mixture of compounds that belong chiefly to the paraffin series. Some of these compounds are very volatile liquids of low boiling points; others have higher boiling points, and some are solids. The most practical way of separating the different compounds is by employing the process of **fractional distillation**, which is based on differences in their boiling points.

213. Fractional Distillation.—Fractional distillation is accomplished by boiling the mixture (the boiling point of which rises as the more volatile portions are driven off), passing the vapors through a condensing apparatus (called a **worm**), and changing the receiver for the condensed liquid (called "cutting the distillation") according to the boiling point or specific gravity desired in the *distillate* (condensed vapor). Pennsylvania crude petroleum is thus divided into naphtha, burning oils, gas and fuel oils, lubricating oils, wax, and coke (or pitch), the crude, or natural, oil being heated in a direct-fired still. It is easier to control the process by taking the specific gravity than by taking the temperature; consequently, the Beaumé hydrometer is used as a guide to the separation of the "fractions" of the distillate. The specific gravity corresponds, however, to a definite boiling point, so that, theoretically, either method of testing might be used.

The first set of fractions, obtained as above, is further separated, either by means of a special type of condenser (fractional condensation) or by redistillation, and as many as twenty products are recovered. Even these are subject to further separation, but the range of temperature is, of course, more limited. Thus, from one fractionation of crude oil might be obtained:

1. Petrolic ether.
2. Gasoline.
3. Kerosene.
4. Spindle oils.
5. Light engine oils.
6. Heavy engine oils.
7. Cylinder oils.
8. Petrolatum (vaseline).
9. Paraffin wax.
10. Pitch or coke.

It will readily be seen that the foregoing classification is arbitrary, the compounds referred to being commercial and not of the same nature as a chemical compound. For instance, there are many grades of gasoline, which would not be the case if gasoline were a definite chemical compound; and as the demand increases out of proportion to the supply, the boiling point of the mixture sold under that name is continually being raised, in order to include fractions that formerly went into burning oils. Proper blending gives a fuel of great power, but one that has a greater tendency to deposit carbon after ignition. It has recently been found, however, that the vapors of the higher boiling-point compounds can be "cracked" into compounds having lower boiling points by causing the vapors to impinge on highly heated surfaces before condensation, which increases the yield.

Sometimes, the process of condensation is used as a means of separation. The liquids are rapidly vaporized, and the various components are separated in successively cooler parts of the system. This process is known as **fractional condensation.**

FATS AND OILS

214. Animal and Vegetable Fats.—All the mineral oils are hydrocarbons; but there is another large class of oils and fats, composed of hydrogen, carbon, and oxygen, which is obtained from animals and plants, and which are commonly designated as **fats.** The fats are glycerides of the fatty acids, the acid replacing the hydroxyl. It will be remembered that the formula for glycerine is $C_3H_5(OH)_3$ (Art. **198**). The formula shows that the glyceryl radical is trivalent, as is also evident from the condensed structural formula for glycerine. By taking out a hydrogen atom from each of three molecules of a fatty acid, thus leaving one bond free, they will unite with glyceryl to form a fat; thus,

$$\underset{\text{Palmitin}}{C_3H_5\begin{cases}C_{16}H_{31}O_2\\C_{16}H_{31}O_2\\C_{16}H_{31}O_2\end{cases}} \qquad \underset{\text{Stearine}}{C_3H_5\begin{cases}C_{18}H_{35}O_2\\C_{18}H_{35}O_2\\C_{18}H_{35}O_2\end{cases}} \qquad \underset{\text{Olein}}{C_3H_5\begin{cases}C_{18}H_{33}O_2\\C_{18}H_{33}O_2\\C_{18}H_{33}O_2\end{cases}}$$

These three fats are commonly called **palmitin, stearin,** and olein, and are the principal fats of animal and vegetable origin; there is another, called **margarine,** which was formerly supposed to be intermediate between stearine and palmitin and to have

the formula $C_3H_5(C_{17}H_{33}O_2)_3$, but this has been shown to be a mixture, *margaric acid*, $C_{17}H_{34}O_2$, not being derived from animal fats in this form.

215. Soaps.—A **soap** is a salt of a fatty acid, usually stearic, palmitic, or oleic acid, in which a metal, generally sodium or potassium, replaces the glycerine of the original fat, the reaction being called **saponification.** Sodium produces hard soaps and potassium produces soft soaps. The fat and alkali are cooked in a huge kettle until the saponification is complete, when the addition of common salt causes the soap to separate and rise, while the glycerine dissolves in the water present and is drawn off at the bottom, to be recovered. The glycerine is even more valuable than the soap; it is used in pharmaceuticals, explosives, and, with litharge, as a cement for lining sulphite digesters. Soap made from oleic acid is soft and slushy; hence, it may be used in mixtures to regulate the hardness of soaps. (See Art. **67.**)

PRINCIPLES OF LUBRICATION

216. Pure Mineral Oils not Adapted to all Cases.—A glance at the list of fractions given in Art. **212** shows that most of the lubricating oils come from petroleum. For high-speed, light machinery and in ring or immersion oiling, petroleum oils may be used for lubrication without admixture. It has been found, however, that these oils are lacking in a quality which, for want of a better term, may be termed "oiliness." For heavy service, where the pressure is great, and where the "body" of the oil is relied upon to prevent the "seizing" of one metal upon the other, it has been found best to mix a small amount of animal or vegetable oil with the mineral oil; this is also necessary in some cases where moisture is present, as in the lubrication of the cylinders of steam engines. Tallow, and castor, rape, neatsfoot, and other oils have been used in such cases to mix with mineral oils.

Sometimes, animal and vegetable oils become acid and lose some of their glycerine under the influence of heat or bacterial action. The presence of the free fatty acids thus produced has usually been considered detrimental in lubricating oils; but recent investigators have found that small amounts of free fatty acids added to the mineral oils reduce the friction as much as large amounts of the original neutral oils.

The fatty acids are known to combine slowly with metals, and it is probable that this property assists in lubrication. Liquids that do not wet a surface will not act as lubricants; for example, mercury, which has a tendency to gather into globules instead of spreading out as a film. Except in those bearings in which the journals are bathed in oil, it is necessary to preserve the oil film between the contact surfaces, and this is greatly assisted by the viscosity of the oil. The specific gravity (*i.e.*, density) of the oil is of little importance in lubrication; it is chiefly valuable as a means of identification and for purposes of comparison.

The temperatures at which inflammable gases are given off from oils, referred to as the "flash" and "fire" points, relate more to fire and accident risks than to lubrication. Their determination is useful for comparative purposes, and they indicate the class of oil used and the stage of its refining.

217. Theory of Lubrication.—On the whole, there has not yet been enunciated a thoroughly satisfactory theory of lubrication. The view that lubrication (see Friction in Section on Physics) is a purely physical matter is being modified to the extent, at least, of considering the acidity of the oil and the nature of the metal or metals in contact. It may be that in the future, considera-tion will be given to the use of special oil mixtures for each metal, as well as to the pressures on, the speeds of and the heating of the lubricated surfaces.

The simplest explanation of lubrication is that the surfaces in contact are really rough, no matter how smooth they may appear to be, and the friction between them is greater than between them and the layer of lubricant that separates them.

THE CARBOHYDRATES

218. Sugars, Gums, and Starches.—A large number of compounds of the aliphatic group of hydrocarbons are called **carbohydrates,** because the proportions of hydrogen and oxygen are the same as in water, that is, there are twice as many hydrogen atoms as oxygen atoms. This class includes sugars, gums, starches, and cellulose, the chief constituent of wood fibers. The hydroxyl OH is very prominent in all these compounds.

It will be recalled that glycerine is propane, C_3H_8, from which three atoms of hydrogen have been taken and replaced with three hydroxyls, thus making the formula for glycerine $C_3H_5(OH)_3$.

the formula $C_3H_5(C_{17}H_{33}O_2)_3$, but this has been shown to be a mixture, *margaric acid*, $C_{17}H_{34}O_2$, not being derived from animal fats in this form.

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sugars are called *aldoses*. It will now be apparent that the names of all sugars end in *ose*.

Dextrose and levulose are *monosaccharides;* they can be combined, eliminating water, and the resulting sugars are called *disaccharides*, which are commonly known as **cane sugars** or **sucroses.** Maple sugar and beet sugar belong to this class, and are chemically identical. The empirical formula for cane sugar (sucrose) is $C_{12}H_{22}O_{11}$; in the presence of water, it can be split up into dextrose and levulose.

Sugars have a way of multiplying their molecules, or, rather, forming clusters of molecules (polymerization); and the compounds thus formed are called *polyoses*. **Starch,** $C_6H_{10}O_5$, is a polyose and cellulose is another. Just how many molecules are thus grouped is not known, so the general formula for a polyose is $(C_6H_{10}O_5)_x$. These polyoses are the products of natural processes of synthesis, starting with the carbon dioxide and moisture of the air and with water and soluble mineral matters from the soil. Starches may be resolved by hydrolysis into sugars, passing through intermediate stages, which yield substances of similar composition called **gums,** one of which is **dextrine.** Dextrine is used in coating and pasting paper, making adhesive paper (the back of postage stamps, for example), and sometimes as an ingredient in sizing. Starch itself has adhesive properties, and it is used for pasting and sizing, for which purpose it is frequently modified by partial hydrolysis.

CELLULOSE

219. Wood Structure.—Plants (the word includes trees and all kinds of plants) contain cellulose, ligno-cellulose, (or lignin) starches, gums, resins, coloring matter, inorganic (mineral) substances in varying proportions, and water; all these substances are manufactured by nature. The skeleton of all plant structure is the cellulose fiber, which takes many forms in different plants—short, stiff fibers in straw; long, flexible ones in spruce, and very long fibers in flax and other textile plants. Some cellulose remains in the form of cells, which are useless in paper making. The cotton fiber, which grows as hair on the cotton seed, contains very little foreign matter (it is 90% cellulose) and is readily purified.

Wood is composed of cellulose and lignin (ligno-cellulose),

with a variety of other substances of more or less acid nature, and a little mineral matter, which appears as ash when wood is burned; the ash content of spruce is only 0.3%. The principal solid constituents of spruce, according to Klason, are:

Spruce	Cellulose	53%
	Lignin	29%
	Gums, sugars, etc.	13%
	Resins, fats, etc.	4%
	Albuminates	1%

These proportions vary for different species of spruce, in fact, they vary for different parts of the same tree; but the cellulose content of dry wood may be taken as 50%. The amount of water in wood varies greatly; in green timber, it is about equal to the weight of actual wood substance.

220. Composition of Cellulose.—Cellulose is a carbohydrate, and it is related in some way to the sugars and starches in the clustering of its molecules. It is known that carbon, hydrogen, and oxygen are present in amounts that may be expressed empirically by the formula $C_6H_{10}O_5$ or by some multiple of it. This is the same basic formula as is used to express the proportions of these elements in starches, though a variety of arrangements of atoms occurs. There are also different celluloses, but their basic formula as given above is the same; the difference lies chiefly in the products derived by chemical disintegration. To the papermaker all cellulose "looks alike."

It has not been settled just how many of the basic units constitute the cellulose aggregate, so it is usually written $(C_6H_{10}O_5)_x$. Recently it has been suggested, with some support from facts, that the individual fiber may be considered as a molecule; and one argument in favor of this idea is that when cellulose is nitrated to guncotton, the fibers are not altered in appearance under the microscope, in spite of the fact that several NO_3 groups are taken into the molecule and some molecules of water are taken out.

221. Cellulose Nitrates and Acetates.—Nitrates of the cellulose aggregate are important as explosives (guncotton is chiefly hexa-nitrate) and in compounds, such as *celluloid;* they are soluble in mixtures of alcohol and ether and, as before mentioned, in acetone, and the solution is called *collodion,* which is much used by photographers.

These solutions can be evaporated to a thick consistency and pressed through fine orifices, forming a thread of any desired thickness that is denitrated upon evaporation of the solvent and immersion in a solution such as magnesium sulphide. The product is *artificial silk* or *luster cellulose*. (See Art. **55.**)

Acetic acid can be made to enter the cellulose, and the product, when dissolved in chloroform, can be recovered by evaporation as threads or as molded objects. Cellulose acetates are not explosive, and are to be preferred to the nitrates for this reason. They are largely used for movie films, threads, coating fine wire for insulation, etc. These and similar compounds are referred to as *cellulose esters*.

The cellulose aggregate is remarkable for the very drastic treatment it will survive; in fact, it requires rather drastic treatment to affect it at all. It is treated with a mixture of concentrated nitric and sulphuric acids in making guncotton, the sulphuric acid being added to remove and hold the molecules of water taken out. After nitration, in that severe reaction, it can be denitrated, and then becomes cellulose again.

222. Mercerization.—Alkali hydrates have an interesting effect on cellulose known as **mercerization,** so called after an investigator named Mercer, who first described it.

When sodium hydroxide, at a strength of 10% to 15%, is brought into contact with the cotton fiber at temperatures of about 15°C. to 20°C., the physical appearance of the fiber is altered, changing from a flat to a swollen shape. When this action takes place in the woven fibers, there is considerable shrinkage. If the fabric be held in place during the reaction, the alkali being subsequently removed by water, the fabric will have acquired certain properties of light refraction that cause a fine sheen, almost like that of silk. The cellulose is found to have taken up water, and its composition may be expressed by the formula $C_{12}H_{20}O_{10}\cdot H_2O$.

The fact that cellulose is thus acted upon by acids and alkalis is important to the papermaker, as a warning to avoid the use of strong chemicals and to remove all chemical residues. It also shows that the cellulose molecule has some faint acid and basic properties, and this knowledge is useful in understanding the sizing, coloring, and other treatment of fibers.

223. Hydrated Cellulose.—Under the pressure of the calenders in the presence of moisture or upon long beating, cellulose takes

up water in a similar way; it acquires a greasy feeling and is said to be *hydrated*. It is this reaction that produces the slow stock necessary for bond and bank-note papers. If the process be prolonged, the product becomes quite translucent, and *vegetable parchment* or *grease-proof papers* are eventually produced. Wax papers are not to be confused with grease-proof papers, as they are water-proofed only by running the paper through melted paraffin wax. The parchmentizing may be produced by treating the paper with strong sulphuric acid under special conditions of time and with great care that the acid is washed completely and quickly from the paper.

224. Oxidation of Cellulose.—Of almost equal importance with the behavior of cellulose toward acids and alkalis is its indifference to oxidizing agents, unless they are very active. Coloring matters in wood pulp or rag fibers may be destroyed completely by bleaching with oxidizing agents, and with little or no loss of cellulose. However, if care is not taken, as explained in the Section on Bleaching, the cellulose molecule is oxidized to oxy-cellulose, which is a powdery substance of no papermaking value whatever, and which is easily affected by chemicals.

225. Treating Wood for Production of Cellulose.—The principal object in the cooking of wood is the removal of the non-cellulose portions from the solution. The principle involved is *hydrolysis*, the cellulose being loosely bound to the lignin, which is the chief non-cellulose substance. The cellulose is quite easily isolated by splitting the association; but if the action is too vigorous, the cellulose molecule is more or less broken down. By continued boiling with dilute acid, it is possible to produce glucose, from which alcohol may be obtained, a product now commercially derived from sawdust.

If the pulp be intended for wrapping, board, or other paper products in which high whites or delicate shades or the permanence of the product are not essential, it is not necessary to remove the non-cellulose portions completely, while in other paper products, the nearer tho product comes to pure cellulose, the better.

226. Soda and Sulphate Processes.—Not considering mechanically prepared pulp, there are two main lines of chemical treatment—one alkaline and the other acid—and both processes are aided by heat and pressure. Both are processes of hydrolysis, accompanied to some extent by saponification, especially in the

case of resinous woods, though the soaps formed are probably decomposed in the course of the several reactions. There are two alkaline processes, known as the *soda process* and the *sulphate process*. The latter might be called a modified soda process, for the reason that the principal constituent in the cooking liquor is caustic soda. It is called the sulphate process because sulphate of soda, called salt cake, is used to replace the soda lost in the process. The burning of the solids in the cooking of liquors reduces some of the sulphate to sulphide, and this sulphide has a decided effect in resolving the non-cellulose portion of the wood.

The word **hydrolysis** does not mean *hydration;* it means the chemical decomposition of a compound that ensues when the group H_2O (water) is absorbed by it, causing the formation of new compounds.

227. Esters.—**Esters,** also called **ethereal salts,** may be defined in several ways: **(1)** An ester is a compound of an alcohol with an acid, water being liberated during the reaction; thus, the reaction between alcohol and acetic acid is expressed by

$$C_2H_5OH + CH_3COOH = CH_3COOC_2H_5 + H_2O,$$

the products being *ethyl acetate,* also called *acetic ethyl ester,* and water.

(2) An ester is formed by substituting a hydrocarbon radical for the hydrogen in the carboxyl (COOH) of an acid. In the equation above given, substituting the ethyl radical, C_2H_5, for the hydrogen in the carboxyl radical in acetic acid gives $CH_3COOC_2H_5$, the formula for acetic ethyl ester, formerly called **acetic ether.**

(3) An ester may be regarded as an ether in which one of the alkyl radicals has been replaced by an acid radical; this accounts for the term ethereal salt. Thus, the formula for ethyl ether is $C_2H_5OC_2H_5$; writing the formula for acetic acid as $CH_3CO{\cdot}OH$, and substituting the acid radical CH_3CO for the first alkyl radical, the result is $CH_3CO{\cdot}O{\cdot}C_2H_5 = CH_3COOC_2H_5$, when the dots are omitted.

228. Isolation of Cellulose from Wood.—There is no general acceptance of theories as to the processes going on to the ultimate isolation of cellulose from the complex mixture in woods. It has been found that even water, at high temperature and pressure, begins to break up the wood by hydrolysis (that is, by entry of water molecules) into esters and other compounds and

thus forming new compounds. For instance, take the case of ethyl acetate, which will combine, under certain conditions, with water, and form acetic acid and alcohol, as shown by the equation,

$$CH_3COOC_2H_5 + H_2O = CH_3COOH + C_2H_5OH.$$

This reaction is called hydrolysis. See definition, Art. **226.** Cane sugar may be similarly hydrolyzed (see Art. **216**); thus,

$$C_{12}H_{22}O_{11} + H_2O = C_6H_{12}O_6(\text{dextrose}) + C_6H_{12}O_6(\text{levulose})$$

It is evident, therefore, that in all cases, the first effect of cooking wood is hydrolysis. The presence of alkali (soda in alkaline liquors, and calcium in sulphite liquors) neutralizes any acids formed. It is evident that the action of alkali solutions containing 4% to 6% of sodium hydroxide will be much more drastic, even after a portion has been neutralized, than the action of acid sulphite of lime solutions carrying under 1½% of lime salt. The values mentioned are those in common use. Consequently, the waste liquors of the soda or sulphate processes do not yield the variety of useful compounds that are found in the sulphite liquors.

QUESTIONS

(1) What do you understand by (*a*) fractional distillation? (*b*) fractional condensation?

(2) Why do oils reduce the friction between rubbing surfaces?

(3) Name some well-known carbohydrates.

(4) Why is neutral rosin size a soap?

(5) What are the effects on cellulose of (*a*) acids? (*b*) alkalis? (*c*) oxidizing agents?

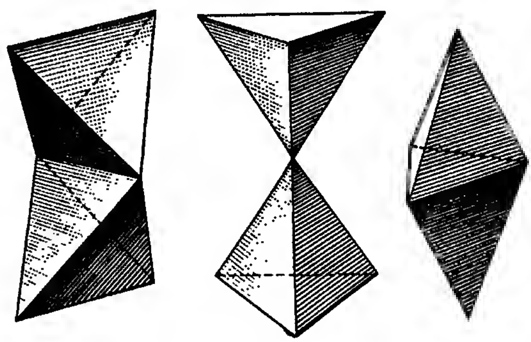
CLOSED-CHAIN, OR RING, COMPOUNDS

AROMATIC, OR BENZENE, SERIES

229. Unsaturated Hydrocarbons.—Hydrocarbons that do not have the general formula, C_nH_{2n+2}, can combine with a halogen without exchanging hydrogen atoms for halogen atoms, that is, they can add halogen atoms directly to their molecules; for which reason, such hydrocarbons are said to be **unsaturated.** Hydrocarbons of the paraffin series cannot do this, and they are therefore said to be **saturated.** All the bonds of the carbons in a saturated hydrocarbon are engaged, and before a halogen atom, chlorine, bromine, etc., can enter the molecule, a hydrogen atom must be taken out.

Consider a molecule represented by the formula C_2H_4; this may be regarded as composed of two equal radicals CH_2 and CH_2. If four of the eight carbon bonds be supposed to be engaged in holding the two carbon atoms, the structural formula for the molecule may be written $\begin{matrix} H \\ H \end{matrix}\!\!>C=C<\!\!\begin{matrix} H \\ H \end{matrix}$, and the constitutional formula may be written $CH_2:CH_2$, the double dot indicating that the two CH_2 groups are united by a double bond. Similarly, the molecule C_2H_2 may be regarded as composed of two CH groups united by a triple bond; thus, $H-C\equiv C-H$. The constitutional formula may then be written $CH\vdots CH$.

230. These assumptions are based on the following considerations: Referring to Fig. 10 and the accompanying description, the tetraedron is supposed to represent the positions of the atoms



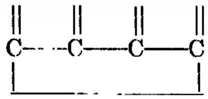
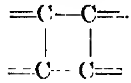
(a) (b) (c)

Fig. 11.

in the molecule of methane CH_4. The carbon atom is situated at the center of gravity of the tetraedron, and there is a hydrogen atom at each of the four corners, which are the vertexes of the four solid angles. When two such molecules are linked, they may be supposed to be joined as illustrated at (*b*), Fig. 11. This leaves three corners free, that is, there are three bonds united to hydrogen atoms that may be replaced with other atoms or radicals. The case of double linking is illustrated in (*a*), Fig. 11. Here two edges touch, instead of two corners, and two corners are left free, that is, there are two bonds united to hydro-

gen atoms that may be replaced with other atoms or radicals. The case of triple linking is illustrated in (*c*), Fig. 11. Here two sides touch, and there is only one bond united to a hydrogen atom that can be replaced with another atom or radical.

231. The compounds so far discussed have all belonged to the open-chain, or aliphatic, group; they are called **open-chain compounds** because the carbon atoms are supposed to be linked together in such a way that there are terminal carbon atoms, each attached to only *one* carbon atom. There are, however, certain hydrocarbons in which each carbon atom is attached to *two* carbon atoms, as illustrated at (*a*) in the two following diagrammatic structural formulas:

(*a*) (*b*)

Here each carbon bond is attached to two carbon atoms including the terminal carbon atoms. The arrangement may be better shown by arranging the carbon atoms in the form of a ring, as shown at (*b*) These two diagrams show only the relation of the carbon atoms; the hydrogen or other atoms or radicals will, of course, be attached to the ends of the free bonds. The reason for the terms *closed-chain* and *ring* will now be evident.

232. The Benzene Series.—The general formula for the benzene series is C_nH_{2n-6}, and the lowest term of the series is C_6H_6, which is the formula for benzene, from which the series derives its name. From the fact that some of the compounds of this series have very pleasant, aromatic scents, the series is frequently called the **aromatic series**; and since the compounds are of the closed-chain type, the name aromatic is applied to the entire group, in contrast to aliphatic, which is applied to the open-chain group.

The first four members of the benzene series are:

Benzene or benzol,	C_6H_6,	Boiling point, 80°C.
Toluene or toluol,	C_7H_8,	Boiling point, 110°C.
Xylene or xylol,	C_8H_{10},	Boiling point, 139°C.
Mesitylene,	C_9H_{12},	Boiling point, 164°C.

Benzene is chiefly derived from coal tar; it is a colorless liquid, lighter than water, and has an odor resembling that of coal gas. It solidifies at 3°C. and is highly inflammable. It is to be noted

that *benzine* (note spelling) is a mixture of several products of the paraffin series, and is obtained from petroleum; it is an entirely different substance from *benzene*, which is frequently called **benzol.**

233. The Benzene Ring.—In order to explain the structure of the benzene molecule, it is assumed that the carbon atoms are situated at the six corners of a hexagon, and that they are double-linked on one side and single-linked on the other side of each carbon atom; this leaves one bond free for attachment to a monad atom or radical. The arrangement is shown in (*a*), Fig. 12. The structural formula for benzene is shown in (*b*),

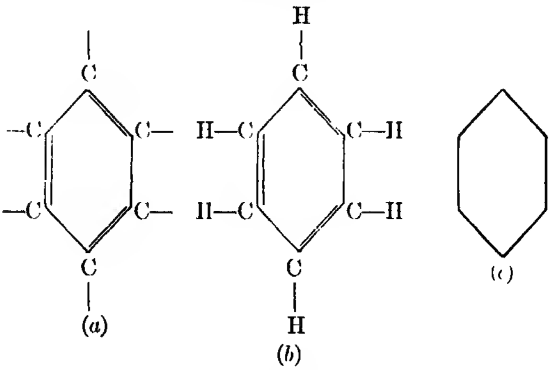


FIG. 12.

Fig. 12, and is usually called the **benzene ring;** this is quite frequently represented by a plane figure, as at (*c*), in which case, it is understood that a CH group is located at each corner, even though the letters do not appear there.

The benzene ring plays the same important part in the aromatic compounds that the structural formula for methane does in the aliphatic compounds. There are but few compounds in which the carbon atoms in the ring are replaced, and most of the reactions result in the substitution of elements or radicals (as NO_2) for the hydrogen atoms or for CH groups. This indicates a very stable inner ring of carbon atoms. The prefix *phen* is characteristic of the names of compounds containing the benzene nucleus.

234. Formation of Benzene Compounds.—It will be recalled that it was shown that methane, CH_4, may be written $CH_3\dot{H}$,

gen atoms that may be replaced with other atoms or radicals. The case of triple linking is illustrated in (*c*), Fig. 11. Here two sides touch, and there is only one bond united to a hydrogen atom that can be replaced with another atom or radical.

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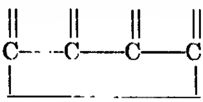
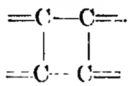


(*a*)



(*b*)

Here each carbon bond is attached to two carbon atoms including the terminal carbon atoms. The arrangement may be better shown by arranging the carbon atoms in the form of a ring, as shown at (*b*) These two diagrams show only the relation of the carbon atoms; the hydrogen or other atoms or radicals will, of course, be attached to the ends of the free bonds. The reason for the terms *closed-chain* and *ring* will now be evident.

232. The Benzene Series.—The general formula for the benzene series is C_nH_{2n-6}, and the lowest term of the series is C_6H_6, which is the formula for benzene, from which the series derives its name. From the fact that some of the compounds of this series have very pleasant, aromatic scents, the series is frequently called the **aromatic series**; and since the compounds are of the closed-chain type, the name aromatic is applied to the entire group, in contrast to aliphatic, which is applied to the open-chain group.

The first four members of the benzene series are:

Benzene or benzol,	C_6H_6,	Boiling point, 80°C.
Toluene or toluol,	C_7H_8,	Boiling point, 110°C.
Xylene or xylol,	C_8H_{10},	Boiling point, 139°C.
Mesitylene,	C_9H_{12},	Boiling point, 164°C.

Benzene is chiefly derived from coal tar; it is a colorless liquid, lighter than water, and has an odor resembling that of coal gas. It solidifies at 3°C. and is highly inflammable. It is to be noted

	Sp. Gr.	Melting Pt.	Boiling Pt.
1. Catechol (pyrocatechol)	1.344	104°C.	240°C.
2. Resorcinol	1.272	116°C.	276°C.
3. Quinol (hydroquinol)		169°C.	285°C.

It will be noted that the physical properties of these three compounds are widely different; how, then, can they be distinguished chemically? By referring to the benzene ring of Art. **235**, it will be manifest on examination that the two hydroxyls may be arranged in three different ways, and only three; thus, one may be placed at 1 and the other at 2, one at 1 and the other at 3, and one at 1 and the other at 4, as shown in Fig. 13.

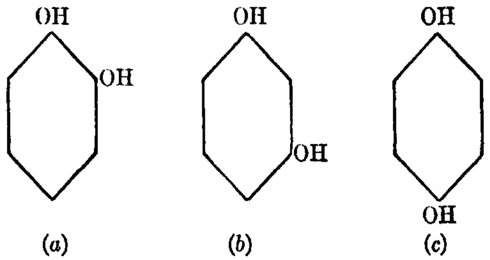


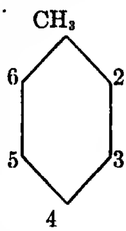
Fig. 13.

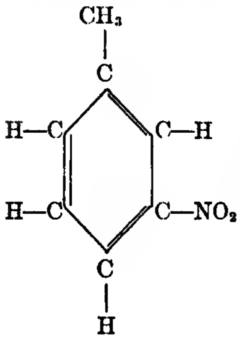
Evidently, these are the only different positions, insofar as the *relative* arrangements of the radicals is concerned. In (*a*), there is no intervening carbon atom between the two hydroxyls, and compounds of this kind are called **ortho compounds,** *ortho* meaning *straight;* in (*b*), there is one carbon atom between the two hydroxyls, and compounds of this kind are called **meta compounds,** *meta* meaning *between;* in (*c*), the arrangement is symmetrical, the two hydroxyls being opposite each other, and such compounds are called **para compounds,** *para* meaning *across.* The first arrangement applies to catechol, which is called *ortho-dihydroxybenzene, o-dihydroxybenzene,* or 1 : 2 *dihydroxybenzene.* The second arrangement applies to resorcinol, which is called *meta-dihydroxybenzene, m-dihydroxybenzene,* or 1 : 3 *dihydroxybenzene.* The third arrangement represents quinol, which is called *para-dihydroxybenzene, p-dihydroxybenzene,* or 1 : 4 *dihydroxybenzene.* The figures refer to the numbers at the corners.

237. Nitrification of Benzene.—Benzene and the other members of the series can be nitrified to produce nitrobenzol, etc.; in fact, many elements and radicals may be introduced into the

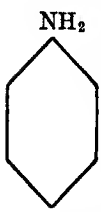
benzene ring. If one nitroxyl (NO_2) be introduced, the result will be a **mono-nitro** compound. There can be but one *mono-nitrobenzene,* because the nitroxyl may be placed at any one of the six corners, as was the case with hydroxyl.

In the case of toluol, there can be three mononitrotoluols, because if the empirical formula C_7H_8 be written $C_6H_5CH_3$, and the methyl radical be placed at, say, 1, in place of the hydrogen atom, the nitroxyl may be placed at 2 or 6, at 3 or 5, or at 4, thus giving three different combinations, and the result will be three different compounds. The same result will be obtained if the methyl radical be placed at any one of the other corners. This phenomenon, whereby different compounds are obtained by altering the position of a radical in the molecule, is called **metamerism,** and compounds that can be so altered are called **metamerides.** Metamerism is a special case of isomerism, and an example of it has been given previously in Art. **236.** The complete structural formula for meta-mononitrotoluol is shown herewith; this would usually be expressed as m-nitrotoluol. When the nitroxyl is placed at 2 or 6, the compound is called o-nitrotoluol, and if placed at 4, it is called p-nitrotoluol. If three nitroxyls are introduced, the result is the well-known explosive, **tri-nitrotoluol,** commonly called T.N.T.





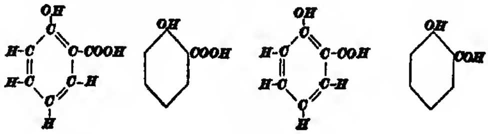
238. Some Other Benzene Compounds.—If one of the hydrogen atoms of the benzene ring be replaced with amidogen, NH_2, see Art. **187**, the formula becomes $C_6H_5NH_2$, and the compound is called **aniline,** which is the starting point for a large number of dyes. The abbreviated structural formula is here shown. It is also called *aminobenzene* and *phenylamine* because of the amidogen radical, and it may be prepared by reducing nitrobenzene with iron and hydrochloric acid, according to the equation



$$C_6H_5NO_2 + 3Fe + 6HCl = C_6H_5NH_2 + 2H_2O + 3FeCl_2$$

239. There are a whole series of acids; if but one carboxyl be made to replace one hydrogen atom in the benzene, the resulting acid thus formed is called **benzoic acid** or *monocarboxybenzene*, C_6H_5COOH. If CHO be substituted for one hydrogen atom, the resulting compound is *benzaldehyde*, C_6H_5CHO, which is the well-known fragrant oil of bitter almonds.

240. Hydroxybenzoic acid has the constitutional formula $C_6H_4(OH)COOH$, which shows that one atom of hydrogen has been replaced with hydroxyl and another atom with carboxyl. As in the case of dihydroxybenzene, there are three metamerides, the most important of which is 1 : 2 *hydroxybenzoic acid*, which is commonly called **salicylic acid.** There is also *salicyl aldehyde*, $C_6H_4(OH)CHO$. The complete structural formulas for these two compounds is shown in Fig. 14, together with the abbreviated structural formulas. Salicyl aldehyde is the odoriferous principle of the meadow-sweet spiræa.

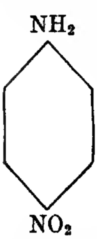


Salicylic acid Salicylic aldehyde

FIG. 14.

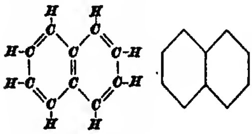
Salts formed by the reaction of salicylic acid with bases are called **salicylates,** the most important of which is **sodium salicylate,** $C_6H_4(OH)COONa$, which possesses powerful antiseptic properties, and is frequently used for the preservation of meat and other food articles. **Methyl salicylate,** $C_6H_4{\cdot}OH{\cdot}COO{\cdot}CH_3$ is the natural *oil of wintergreen.* The salicylates are much used in the treatment of rheumatism.

241. Amino Compounds.—The formula for aniline, $C_6H_5NH_2$, may be regarded in two ways; 1st, as though it were formed by replacing an atom of hydrogen in the benzene ring with amidogen, in which case, it is called *aminobenzene;* 2d, as though it were formed from ammonia, NH_3, by replacing an atom of hydrogen with phenyl, in which case, it is called *phenylamine.* The compounds formed by successive replacements of hydrogen in ammonia and the introduction of these ammonia residues into other compounds, form a very important and interesting section of organic chemistry, especially in connection with the manufacture of dyestuffs.



An important aniline compound is *paranitraniline* or *paranitro-aniline* which is used to obtain *paranitraniline red*, by an indirect process. There are also *orthonitraniline* and *metanitraniline*. All three crystallize in long, yellow needles of different shades.

242. Other series of hydrocarbons are composed of benzene in pairs and triplets, as in the case of **naphthalene** and **anthracene**, the complete and abbreviated structural formulas for which are given in Fig. 15.


Naphthalene $C_{10}H_8$

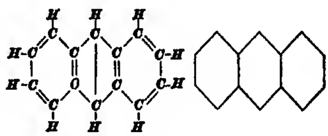
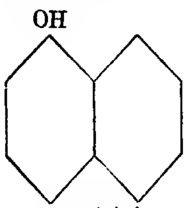
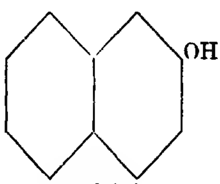
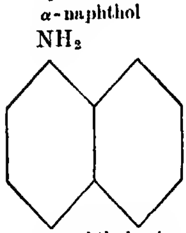

Anthracene $C_{14}H_{10}$

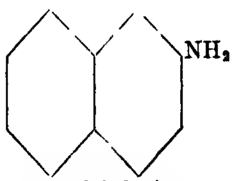
FIG. 15.

Other elements and radicals may be substituted for the hydrogen in these two compounds, thus giving rise to products that are of interest and value in dyestuffs. Thus, **naphthol** is derived by substituting hydroxyl for one of the hydrogen atoms in naphthalene, and it is to be supposed that there will be several naphthols, according to the position of the hydroxyl; two are known, which are called alpha naphthol and beta naphthol, also written *α-naphthol* and *β-naphthol*. Similarly, by replacing a hydrogen atom with amidogen, two naphthylamines are formed, viz., *alpha naphthylamine* and *beta naphthylamine*.


α-naphthol


β-naphthol


α-naphthylamine


β-naphthylamine

243. Diazo Compounds.—A **diazo compound** may be defined as one that contains two doubly-linked nitrogen atoms, each of which is linked to a monad atom or radical, of the general form R·N:N·R′, or R·N=N·R′; here the single dots indicate one bond and the double dots two bonds. The formula for *azobenzene* is $C_6H_5N{:}NC_6H_5$. The radical represented in the general formula by R′ may be replaced with a monad element or radical; thus, *diazo-benzene chloride* has the formula $C_6H_5N{:}NCl$.

The *azo* bodies are of great importance in dyestuffs. The azo colors depend largely on the reactions between the chlorides of $C_6H_5N_2$ (a group called *diazonium,* which behaves somewhat like ammonium, NH_4) and the amino bodies of the aromatic series. The simpler forms are yellow; then, by increasing the weight of the molecule by introducing paraffin or aromatic radicals, the color changes to orange, red, blue, and violet.

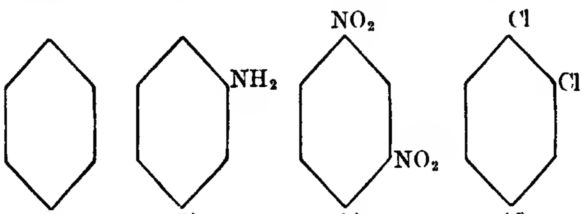
244. It is beyond the province of this Section to go farther in connection with the subject of organic chemistry. From what has been stated, it will be perceived that the chemistry of the carbon compounds is a study of fascinating interest.

QUESTIONS

(1) What is an ester?

(2) What is the principal difference between the benzene series and the paraffin series?

(3) Name the compounds represented by the following structural formulas:



(*a*) (*b*) (*c*) (*d*)

(4) What group of substances related to the benzene series is of most interest to the paper maker? What is the principal source?

(5) What is the difference between carbonic acid and carbolic acid?

(6) Besides water, what are the two chief constituents of wood?

ELEMENTS OF CHEMISTRY
(PART 3)

EXAMINATION QUESTIONS

(1) (*a*) How does the valence of carbon affect the likelihood of its forming many compounds? (*b*) What is the similarity between carbon and silicon?

(2) (*a*) What are the chief constituents of glass? (*b*) What is water glass?

(3) How does heating to redness spoil a clay for paper-mill use as a filler?

(4) (*a*) What are the allotropic forms of carbon? (*b*) What is the difference between anthracite and steam coal?

(5) What is the source of coal-tar dyes?

(6) (*a*) What are the two oxides of carbon, and under what conditions are they formed? (*b*) Why is the burning of coal under a steam boiler a wasteful procedure?

(7) (*a*) How does carbon act as a reducing agent? (*b*) What is producer gas?

(8) (*a*) What is a hydrocarbon? (*b*) Into what two large groups are hydrocarbons divided?

(9) (*a*) What is the simplest saturated hydrocarbon? (*b*) What is another name for saturated hydrocarbons? (*c*) What is marsh gas?

(10) (*a*) What is a homologous series? What radical is characteristic of (*b*) alcohols? (*c*) organic acids?

(11) What is the general formula of (*a*) the paraffins? (*b*) the ethers? (*c*) How are fatty acids formed from hydrocarbons?

(12) (*a*) What is the chief source of lubricating oils? (*b*) What effect has fatty acid in lubricating oils? (*c*) What purposes are served in determining the flash point of an oil?

(13) (*a*) What is a monatomic alcohol? (*b*) Name two alcohols that are likely to be used in the pulp and paper industry. (*c*) What is proof spirit? (*d*) What are the sources of ethyl alcohol in commerce?

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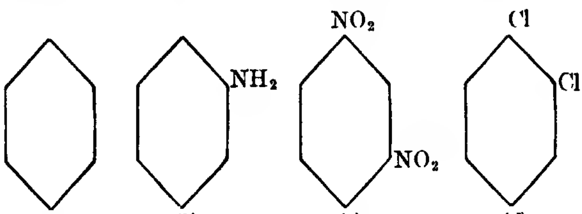

(a) (b) (c) (d)

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NOTE.—The paging begins with 1 in each section, and each Section has its number printed on the inside edge of the headline of each page. To find a reference, as "Acetaldehyde, §3, p139," glance through the volume until §3 is found, and then find page 139.

M

PREFACE

In numerous communities where night schools and extension classes have been started or planned, or where men wished to study privately, there has been difficulty in finding suitable textbooks. No books were available in English, which brought together the fundamental subjects of mathematics and elementary science and the principles and practice of pulp and paper manufacture. Books that treated of the processes employed in this industry were too technical, too general, out of date, or so descriptive of European machinery and practice as to be unsuitable for use on this Continent. Furthermore, a textbook was required that would supply the need of the man who must study at home because he could not or would not attend classes.

Successful men are constantly studying; and it is only by studying that they continue to be successful. There are many men, from acid maker and reel-boy to superintendent and manager, who want to learn more about the industry that gives them a livelihood and by study to fit themselves for promotion and increased earning power. Pulp and paper makers want to understand the work they are doing—the how and why of all the various processes. Most operations in this industry are, to some degree, technical, being essentially either mechanical or chemical. It is necessary, therefore, that the person who aspires to understand these processes should obtain a knowledge of the underlying laws of Nature through the study of the elementary sciences and mathematics, and be trained to reason clearly and logically.

After considerable study of the situation by the Committee on Education for the Technical Section of the Canadian Pulp and Paper Association and the Committee on Vocational Education for the Technical Association of the (U. S.) Pulp and Paper Industry, a joint meeting of these committees was held in Buffalo

in September, 1918, and a Joint Executive Committee was appointed to proceed with plans for the preparation of the text, its publication, and the distribution of the books. The scope of the work was defined at this meeting, when it was decided to provide for preliminary instruction in fundamental Mathematics and Elementary Science, as well as in the manufacturing operations involved in modern pulp and paper mill practice.

The Joint Educational Committee then chose an Editor, Associate Editor, and Editorial Advisor, and directed the Editor to organize a staff of authors consisting of the best available men in their special lines, each to contribute a section dealing with his specialty. A general outline, with an estimated budget, was presented at the annual meetings in January and February, 1919, of the Canadian Pulp and Paper Association, the Technical Association of the Pulp and Paper Industry and the American Paper and Pulp Association. It received the unanimous approval and hearty support of all, and the budget asked was raised by an appropriation of the Canadian Pulp and Paper Association and contributions from paper and pulp manufacturers and allied industries in the United States, through the efforts of the Technical Association of the Pulp and Paper Industry.

To prepare and publish such a work is a large undertaking; its successful accomplishment is unique, as evidenced by these volumes, in that it represents the cooperative effort of the Pulp and Paper Industry of a whole Continent.

The work is conveniently divided into sections and bound into volumes for reference purposes; it is also available in pamphlet form for the benefit of students who wish to master one part at a time, and for convenience in the class room. This latter arrangement makes it very easy to select special courses of study; for instance, the man who is specially interested, say, in the manufacture of pulp or in the coloring of paper or in any other special featuro of the industry, can select and study the special pamphlets bearing on those subjects and need not study others not relating particularly to the subject in which he is interested, unless he so desires. The scope of the work enables the man with but little education to study in the most efficient manner the preliminary subjects that are necessary to a thorough understanding of the principles involved in the manufacturing processes and operations; these subjects also afford an excellent review and reference textbook to others. The work

is thus especially adapted to the class room, to home study, and for use as a reference book.

It is expected that universities and other educational agencies will institute correspondence and class room instruction in Pulp and Paper Technology and Practice with the aid of these volumes. The aim of the Committee is to bring an adequate opportunity for education in his vocation within the reach of every one in the industry. To have a vocational education means to be familiar with the past accomplishments of one's trade and to be able to pass on present experience for the benefit of those who will follow.

To obtain the best results, the text must be diligently studied; a few hours of earnest application each week will be well repaid through increased earning power and added interest in the daily work of the mill. To understand a process fully, as in making acid or sizing paper, is like having a light turned on when one has been working in the dark. As a help to the student, many practical examples for practice and study and review questions have been incorporated in the text; these should be conscientiously answered.

The Editor extends his sincere thanks to the Committee and others, who have been a constant support and a source of inspiration and encouragement; he desires especially to mention Mr. George Carruthers, Chairman, and Mr. R. S. Kellogg, Secretary, of the Joint Executive Committee; Mr. J. J. Clark, Associate Editor, Mr. T. J. Foster, Editorial Advisor, and Mr. John Erhardt of the McGraw-Hill Book Company, Inc.

The Committee and the Editor have been generously assisted on every hand; busy men have written and reviewed manuscript, and equipment firms have contributed drawings of great value and have freely given helpful service and advice. Among these kind and generous friends of the enterprise are: Mr. O. Bache-Wiig, Mr. James Beveridge, Mr. J. Brooks Beveridge, Mr. H. P. Carruth, Mr. Martin L. Griffin, Mr. H. R. Harrigan, Mr. Arthur Burgess Larcher, Mr. J. O. Mason, Mr. Elis Olsson, Mr. George K. Spence, Mr. Edwin Sutermeister, Mr. F. G. Wheeler, and American Writing Paper Co., Dominion Engineering Works, E. I. Dupont de Nemours Co., F. C. Huyck & Sons, Hydraulic Machinery Co., Improved Paper Machinery Co., E. D. Jones & Sons Co., A. D. Little, Inc., National Aniline and Chemical Works, Process Engineers, Pusey & Jones Co., Rice, Barton &

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J. NEWELL STEPHENSON,
Editor

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VOLUME II

MECHANICS AND HYDRAULICS
ELEMENTS OF ELECTRICITY ELEMENTS OF CHEMISTRY

BY

J. J. CLARK, M.E., AND T. L. CROSSLEY, F.C.I.C.

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